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ABSTRACTS

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Plenary lectures

On students' geometrical knowledge and its measurement

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By a famous book of Eric Temple Bell (a mathematician), Mathematics is the Queen and Servant of Science. Similarly, Geometry can be the Princess and Maid of Engineering Sciences. Indeed, no one can think that you may be a good engineer without geometrical knowledge of different kind.

In the last decade the European tertiary education system changed radically. The main reason was the Bologna process, that needn't to be introduced. As to Hungary, parallelly with the changes in higher education, a new, two-level secondary school final examination (Matura) was introduced, that also reformed the upper secondary education. Beside some advantages of the new system (the exam is recognized by tertiary level institutions as entrance examination, etc) it involves a radical alteration of perspective towards competence-based education. This approach arose as the opposite of encyclopedic knowledge that characterized the old education system. In the mathematic education, it caused the abandonment of more complex (mainly geometrical) exercises that require knowledge of theorems and their proofs. Instead, e.g. the role of statistics and probability theory increased, whilst geometry became one of the biggest losers of the change. It suffered great reduction in its content and processing time.

In this talk we give a survey on geometrical knowledge of today first year civil engineering and architecture students, measured by different standard and non-standard tests. First we refer to the conclusions of the survey carried out by colleagues at the Budapest University of Technology and Economics, when the problem solving skill of first year architects was measured. In the second part of the presentation we summarize some results from the Ybl Miklós Faculty of Architecture and Civil Engineering. In the last years my team conducted surveys on the improvement of spatial abilities, where we used the world-wide Mental Cutting Test and Mental Rotation Test. In the analysis statistical methods were used; the conclusions extracted from data evaluation were submitted to hypothesis testing and the results were interpreted.

Key words: teaching geometry, Mental Cutting Test, Mental Rotation Test, descriptive geometry

MSC 2010: 51N05

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Porisms

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This talk shall give an overview on porisms, *i.e.*, geometric figures and configurations of geometric objects that close in some sense. The most famous example of a porism is that of Poncelet: Given two conic sections (in general position). If an n-sided polygon with vertices on one conic section and edges tangent to the other conic section closes for one certain starting point, then it closes for any choice of the starting point.

A well-known example is given by a triangle with its incircle and circumcircle. (The triangle can be rotated freely such that its vertices trace the circum circle and its edges are tangent to the incircle.) Note that this is not a rigid body motion.

There are many such closing theorems in geometry. We shall give some examples of Poncelet like porisms and others. Further we want to collect some more or less known results from elementary and algebraic geometry dealing with the case of two circles. Finally we want to gain insight into the mathematics behind the proof of Poncelet's theorem.

The role of projective geometry in computer vision

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We give a review of various topics in computer vision. It starts from one hundred years old areal photogrammetry, used to obtain topographic maps. Contemporary applications include robotics, 3D photography and television, panoramic vision, medical image processing, 3D gesture control and much more. In the lecture we emphasize the geometrical nature of the underlaying algorithms.

Key words: computer vision, projective geometry, photogrammetry, multiple view geometry, panoramic vision

MSC 2010: 68T45

Contributed talks

Application of elevational projection in defining scope of construction pit excavation

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One of the frequent problems that the civil engineering experts have to deal with during planning the process of building is to define the scope of construction pit excavation. The most common method to solve this problem is to use the so-called elevational projection. In this article we show the phases of defining the scope of construction pit excavation together with one example from the civil engineering practice and how to use some tasks from basic theory of elevational projection to solve such problems.

Key words: elevational projection, scope of construction pit excavation

MSC 2010: 51N05

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Problem solving and e-learning: challenge and pitfalls

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A problem is only a problem (as mathematicians use the word) if you don't know how to go about solving it. A problem that has no "surprises" in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) it is an exercise [3].

Problem solving has been studied very thoroughly from Polya's time till today. It plays an important role in mathematics and should have a prominent role in the mathematics education. On the other hand, there is little or no research on implementing problem solving into e-learning.

We are going to give a brief overview of problem solving from Polya's time till today in two directions. In the first direction are characteristics of good problem solving process - what is good problem and teaching process? In the second direction are skills of a good problem solver - what are the adequate patience level, confidence, willingness and attitude towards problem solving?

Then we will present our first steps in dealing with problem solving through e-learning. Our idea was to expose students to greater number of problem tasks (that they didn't expect in the beginning of the semester) with an opportunity to get the feedback and discussion with teacher through e-learning. We wanted to research three things: how many students would use this way of problem solving, whether this extra engagement would yield better results for similar tasks in mid-term tests, and students' attitude towards problem solving.

We came to conclusion that students cannot become successful problem solvers overnight. Helping students to become successful problem solvers should be a long-term goal, so effort should be made to reach this goal in every mathematical topic and in every lesson.

Key words: mathematics, problem solving, e-learning, attitude

MSC 2010: 97D70

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Isoptic curves of the conic sections in the hyperbolic and elliptic plane

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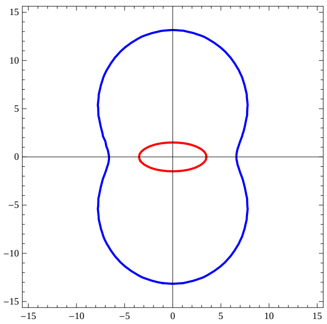
In this talk, we consider the isoptic curves on the hyperbolic and elliptic plane. This topic is widely investigated in the Euclidean geometry (see e.g. [1]), but in the non-Euclidean geometries there are no similar results.

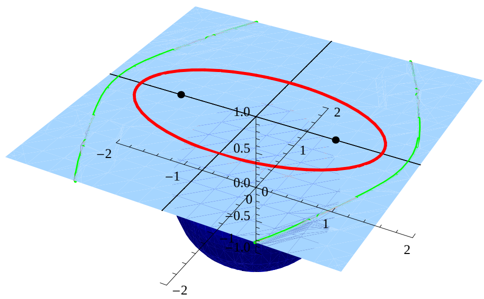
Our goal is to give a procedure to determine the equations of the isoptic curves of the conic sections in the hyperbolic and elliptic planes and visualize them on the Euclidean screen of computer (see [2], [3]).

We will use for computation the classical Cayley-Klein model of the hyperbolic geometry and the projective model of the elliptic geometry.

Key words: isoptic curve, hyperbolic geometry, elliptic geometry, projective model

MSC 2010: 51M10, 14H50, 51M09, 53A35.





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The Golden Section's heptagonal conections

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It is well known that the Golden Section plays an important role in the geometry of several poygons and polyhedra; the best known example is the length of a diagonal in the regular pentagon with unit side. In this talk we show how the Golden Section appears as the solution of an enumerative problem connected with heptagons, more precisely, with heptagonal tilings. The results are then generalized to other types of tilings.

Paradoxical buildings

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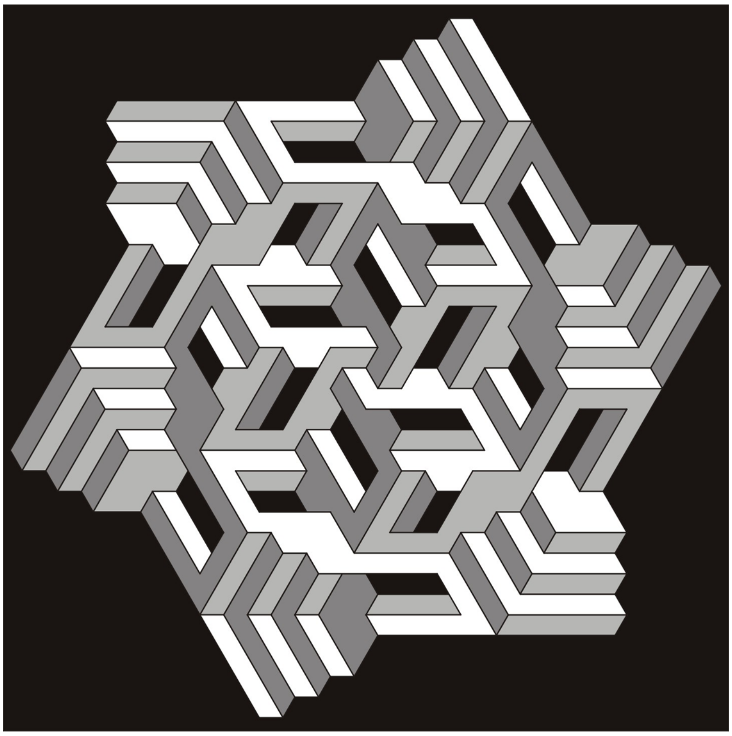
The presentation gives a survey on the last year investigations from the Geometry of Paradoxical Buildings. Complex, matching spatial constructions of six side views will be shown. The presentation fits to the new researches on systematical paradoxical researches. The examination of the pictures develops the spatial imagination and is helpful in students' visual training.

Key words: geometry in fine art

MSC 2010: 00A66, 97M80



Motion of a line segment whose endpoint paths have equal arc length

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We study motions in the Euclidean plane and in Euclidean 3-space where two points A and B have paths of equal arc length. Trivial examples are curved translations, where all point paths are congruent by translation or screw motions, where all points on a right cylinder coaxial with the screw motion have congruent point paths. In the planar case there is only one non-trivial type: If A and B have paths of equal arc length the motion is generated by the rolling of a straight line, namely the bisector of AB on an arbitrary curve. This can be easily shown by means of the projection theorem of kinematics. In 3-space there is a nice relation to the ruled surface ϕ generated by the line AB: The path of the midpoint M of AB is the striction curve on ϕ.

This is also the key to the solution to the following interpolation problem: Given a set of discrete positions A_iB_i of the segment AB find a smooth motion that moves AB through the given positions and additionally guarantees that the paths of A and B have equal arc length.

Beside trivial cases (rotations, curved translations) there are no planar motions where more than two points have paths of equal arc length. In 3-space there do exist non-trivial motions where three or four given points have paths of equal arc length. The problem to determine such motions is closely related to the question of finding the set $\mathcal{L}$ of all lines whose distances to the three or four given points are equal. In case of three points $\mathcal{L}$ is a line congruence (2 parametric set of lines) whereas in case of four points $\mathcal{L}$ is a ruled surface. The moving axode of the motion that moves the three or four given points on paths of equal arc length is a ruled surface contained by $\mathcal{L}$.

Key words: paths of equal arc length, motion of a line, ruled surface, striction curve, projection theorem

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About midsurfaces of space curves

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We consider two space curves c_1 and c_2 and define a surface of translation as the manifold of the midpoints of all possible chords in between the curves. A simple example for such an approach is the generation of a hyperbolic paraboloid. There is another – less known – example with a helix $c_1 = c_2$ as generating space curve, where the result is in a non-trivial way the helicoid. Thus, the helicoid can also be interpreted as a surface of translation (Figure 1). If c_1 and c_2 are coaxial and reversely congruent helices, the midsurface is a remarkable surface that is at the same time a surface of translation and a surface of revolution (Figure 2) that carries two pencils of helices.

Key words: chord midpoint surface, helicoid, surface of translation

MSC 2010: 51-Nxx, 53-Axx

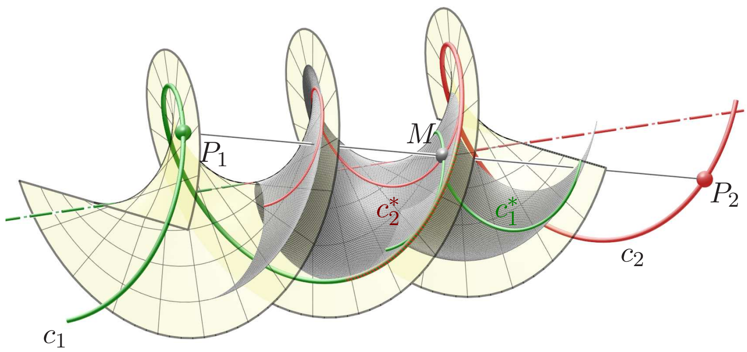

Figure 1: A helicoid is not only a helical surface but can also be generated by translating a helix along another congruent helix.

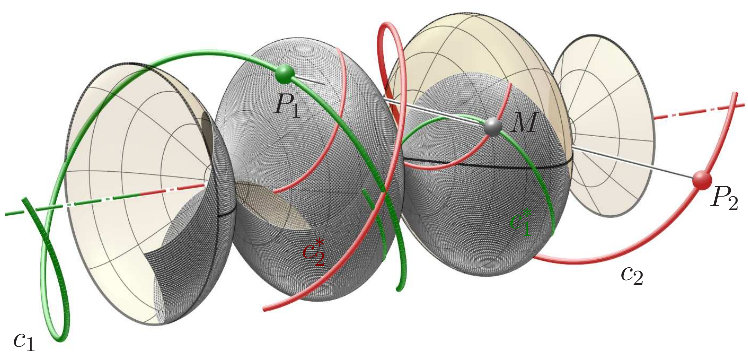

Figure 2: This is an example of a translation surface that is at the same time a surface of rotation. It is the midsurface of two reversely congruent coaxial helices.

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Introducing 3D modeling in teaching of geometry at technical faculties at University of Zagreb

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Present day teaching of constructive geometry at technical faculties in Croatia is to a large degree based on the principles which professor Vilko Niče established at the former Technical Faculty in Zagreb, as far back as the mid-20th century. Due to the high quality of his method of teaching, geometric contents have remained (after undergoing all types of reforms, including the Bologna Process) an important part of fundamental education at Zagreb technical faculties. Niče's successors have modernized educational materials (exercises and textbooks) maintaining relatively high level of our students' geometric knowledge, intuitive understanding of space and strictly logical geometric reasoning. Unfortunately, we are lagging behind the EU when it comes to using 3D CAD (except for the Faculty of Mining, Geology and Petroleum Engineering at no other technical faculty in Zagreb students use CAD programme in geometric courses) and implementing educational materials in the system of e-learning.

In this sense, with the purpose to enhance collaboration among teachers and improve teaching of geometrical courses at the technical faculties in Zagreb, this year we have started the project *Introducing 3D modeling in the teaching of geometry at technical faculties* (3D GEOMTEH) supported by the Fund for the Development of the University of Zagreb.

The Project will cover, among other activities, creating of the basic repository of educational materials related to shared teaching topics and those customized to profiles of individual faculties. Such a repository does not exist within university and polytechnic centers in Croatia. We hope that its formation will result in improving teaching, harmonization of approaches and methods of using e-learning on colleges within Universities, as well as ensure adoption of common standards and recommendations for the creation and use of educational materials for e -learning.

Here we will present some parts of the working material which is made as the main repository of the educational material connected to the geometrical subject. We used the programs *Rhinoceros* 4.0, *GeoGebra*^{4} and *Jing*.

Key words: geometrical education, 3D modeling, GeoGebra, Jing, Rhinoceros

MSC 2010: 97G80, 97U50

Blending of two spheres by a surface with minimal distortion

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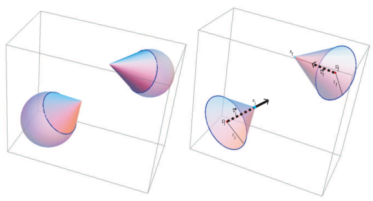
Starting from a problem of an earlier work of skinning of spheres [1] here we consider two spheres with two given circles on them, which have to be blended by a surface. This surface touches the spheres along the given circles. Finally the input data can be considered as two circular cones, the touching cones of the spheres. This problem has also been solved by Dupin cyclides and their generalizations (see [2], [3]), but with restrictions to the input data, or, in terms of its generalizations, sometimes with less satisfactory results. Here we apply the rotational minimizing frame [4], an alternative of the well-known Frenet-frame, with the help of which a blending surface with minimized rotational distortion can be defined.

Key words: blending of spheres, rotational minimizing frame

MSC 2010: 65U17



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Focal curve of pencil of 2^{nd} class curves in pseudo-Euclidean plane

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A focus of a plane curve in the Euclidean plane is defined as the intersection of tangent lines to that curve drawn from the absolute points of the plane (so-called isotropic tangent lines). A curve of class m has m^2 foci. A focus of a plane curve in the pseudo-Euclidean plane is defined analogously. In the projective model of the pseudo-Euclidean plane we will consider pencils of 2^{nd} class curves and construct their focal curves. It will be shown that the focal curve of a pencil of 2^{nd} class curves is generally a circular cubic. We will show the analogy with the Euclidean plane, but also some types of focal curves that do not exist in the Euclidean plane.

Key words: pseudo-Euclidean plane, foci, pencil of 2^{nd} class curves, entirely circular cubic

MSC 2010: 51A05, 51M15

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Geometry of ARO–quasigroups

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In this presentation a new class of idempotent medial quasigroups will be introduced, the so–called ARO–quasigroups. A quasigroup will be called ARO–quasigroup if it satisfies the identities of idempotency and mediality, i.e. we have the identities $aa = a$, $ab \cdot cd = ac \cdot bd$, and besides that if the identity $ab \cdot b = ba \cdot a$ is also valid. Some examples of ARO–quasigroups will be given as well. These quasigroups are interesting because of the possibility of defining affine–regular octagons and to study them by means of formal calculations in a quasigroup. The "geometrical" concepts of a parallelogram and midpoint will be introduced in a general ARO–quasigroup. Some results about the introduced geometric concepts will be proved and a number of statements about new points obtained from the vertices of an affine–regular octagon will also be studied.

Key words: ARO–quasigroup, affine–regular octagon

MSC 2010: 20N05

On two families of curves

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As a starting point of our study, we introduce a fixed circle Φ with radius r and one of its tangents t. Then, by fixing $d \in \mathbb{R}$, we define a one-parametric family $\mathcal{T}_{(r,d)}$ of triangles whose sides are tangential to Φ and it holds,

$$\Delta ABC \in \mathcal{T}_{(r,d)} \Longleftrightarrow A, B \in t, \quad d = \pm|\overrightarrow{AB}|.$$

Since the introduced motion is not rigid for the other two side lengths of the triangles, the properties of the triangles of the family are being continuously changed.

We show that, if the side $\overline{AB}$ runs on a given tangent t, the path of the third triangle vertex, not lying on t, lies on a conic Γ. The obtained conic Γ hyperosculates the given circle Φ and is symmetrical with respect to the axis o_C, the circle diameter perpendicular to t. In the same way we analyze the path of the triangle midpoints M_{AC} and M_{BC} and prove that their paths lie on two symmetrical cubics sharing one of the asymptotes.

Furthermore, by varying d, we obtain families of conics and cubics whose properties are being analyzed.

Key words: family of triangles, pencil of conics, family of cubics

MSC 2010: 51M04, 51M15

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Forming of velaroidal surfaces on ring plan with two families of sinusoids

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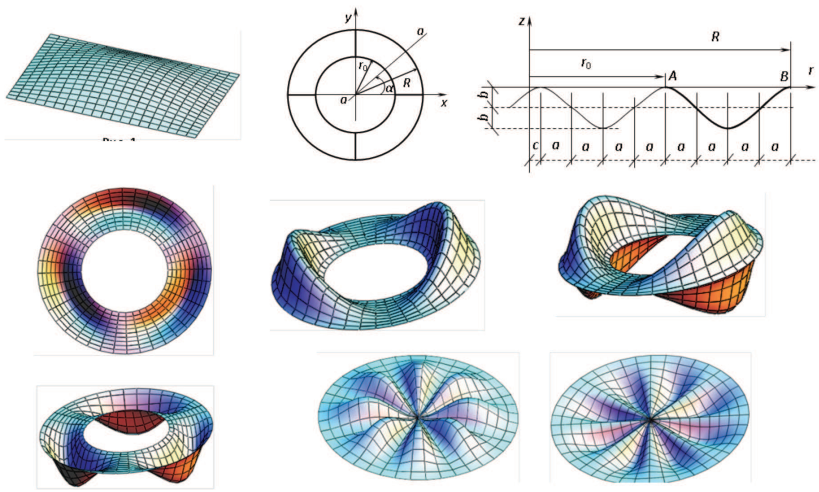
Surfaces, limited by two flat concentric circumferences with two sinusoids as two families of co-ordinate lines, are examined. These surfaces belong to the group of velaroidal surfaces [1], [2], [3]. The considering surfaces can find their application in landscape architecture, and also when designing buildings and some details or objects of interior, as these surfaces consist of cyclic repetitive identical elements.

Key words: velaroidal surfaces, sinusoids, architecture



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Isoptics of free-form curves

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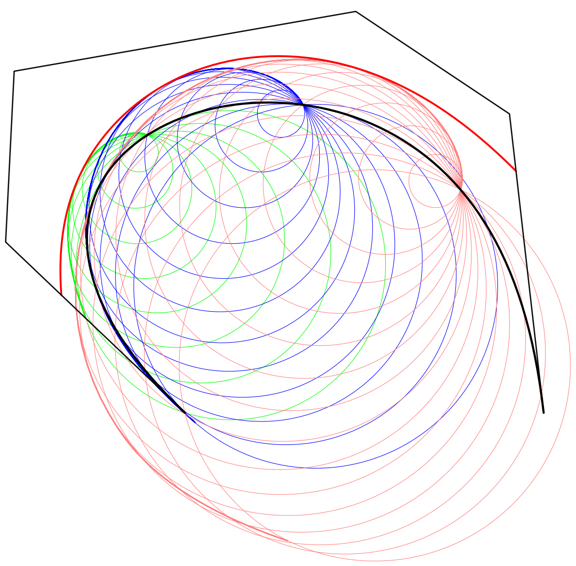
If we have a planar curve, the locus of the points from which the mentioned curve can be seen under a given angle is called isoptic curve. These curves are well-studied and very important in geometry, but they are generally very hard to compute even for computer algebra systems.

We present a method to determine isoptics of a free-form curve. The computation is based on the envelopes of special families of α-isoptic circular arcs. There is a possibility to extend the problem to higher dimensions.

Key words: isoptic curve, free-form curve, envelope

MSC 2010: 65D17



Geometry in the architecture of the acclaimed architects

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Pritzker architectural prize is known as the world's highest recognition for architects. It is awarded each year to a living architect for significant achievement. The awards have been presented since 1979. Among the winners were Frank O'Ghery, Oscar Niemeyer, Kenzo Tange, Hans Hollein, Zaha Hadid, Norman Forstner, Renzo Piano and others. In recent years, the prize winners were:

- Jean Nouvel (2008)
- Peter Zumtor (2009)
- Kazuyo Sejima and Ryue Nishizawa (2010)
- Eduardo Souto de Moura (2011)
- Wang Shu (this year).

Pritzker prize is a prestigious award and the winners normally guide the development of architecture. It is reasonable to ask, what is the role of the geometry and spatial perception in the eyes of the winners. We analyzed the last 5 winners' work and found out that simple geometric shapes are given priority, and that is a block element in achieving the overarching objectives of the architecture. Reasons for this are several. One reason is the rational use of resources from idea to execution, which should take into account of sthe ustainable quality architecture which respect three Vitruvious' postulates (strength, beauty and utility).

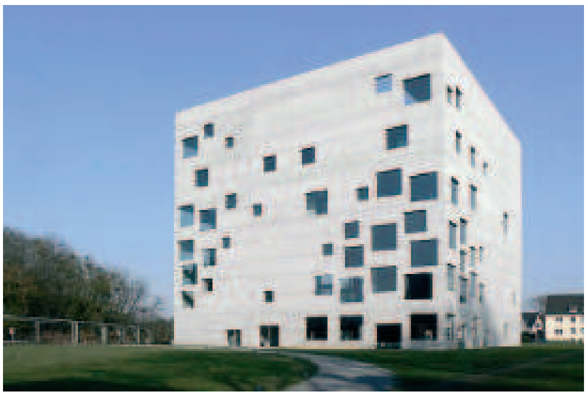

Figure 1: Sejima, Nishizava: Zollverein School of Management and Design, Essen, Germany, 2006
`http://archidialog.com/2011/02/11/le-corbusier-sanaa-walls-with-holes\%E2\%80\%9D/`

Mathematical concepts in graphics of M. C. Escher

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This article is inspired by Dutch graphic artist M. C. Escher and his works. Although he had no formal mathematical education, this famous and established artist used in his work some mathematical ideas as well as some mathematical concepts and properties, such as infinity and the problem of tessellations. In his woodcuts, which are also very popular, he displays elaborate concern about pattern and symmetry. The regular solids also held a spetial interest for him.

He discovered and made illustrations for tilings for 17 symmetry groups in plane.While studying symmetry, he conntacted the great Hungarian mathematican Polya, who influenced him imenesely. Penrose and Coxeter, two great mathematicians he cooperated with, also made a great impact on his work. The relationship between Escher and the mathematicians were reciprocal since his works served as an inspiration for some mathematical research.

Key words: tessalation, symmetry groups, polyhedra, impossible figueras, non-Euclidian geometry, projective geometry, eternity, duality

An analytical approach to Kiepert Conics in regular *Cayley-Klein* geometries

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Recently we reported on Kiepert conics in hyperbolic CK-geometry. The results were obtained by synthetic considerations. In this presentation we employ an analytical approach and consider both regular CK-geometries – hyperbolic as well as elliptic – again focussing on Kiepert conics and related objects.

Key words: Cayley-Klein geometries, triangle, isogonal transformation, Kiepert conics

MSC 2010: 51M09, 51N15

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On projection of Euclidean regular 4-polytopes onto the computer 2-screen with visibility and shading

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joint work with István Prok

In previous works (see [1], [2], [3]) the authors extended the method of central projection to higher dimensions, namely, for $\mathbf{E}_4 \to \mathbf{E}_2$ projection from a one dimensional centre figure, together with a natural visibility algorithm. All these are presented in the linear algebraic machinery of real projective sphere PS_4 or space $P_4(\mathbf{V}_5, \boldsymbol{V}_5, \sim)$ over a real vector space $\mathbf{V}_5$ for points, and its dual $\boldsymbol{V}_5$ for hyperplanes up to the usual equivalence $\sim$ (expressed by multiplication by positive real numbers or non-zeros, respectively).

In this presentation we further develop the exterior (Grassmann) algebra method (with scalar product) by computer to illumination effects, e.g., visibility and shading for the Euclidean regular 4-polytopes by the homepage of István Prok:
`http://www.math.bme.hu/~prok`

I hope you will enjoy an esthetical attraction (nearly the first time in the world!?).

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Leonardo's polyhedrons

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The (star) rhombicuboctahedron: it was first illustrated by Leonardo in Pacioli's book. The artist probably had real models. Maybe he made simpler examples by using reeds, rattans or straw. If he used these instruments he had to make the polyhedron rigid.

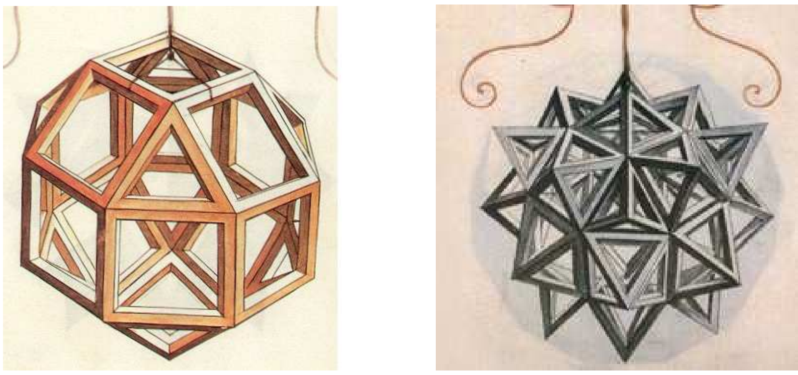

Figure 1: The right picture shows that da Vinci's drawing might be wrong: The pyramid at the bottom of his picture has square base instead of triangle base.

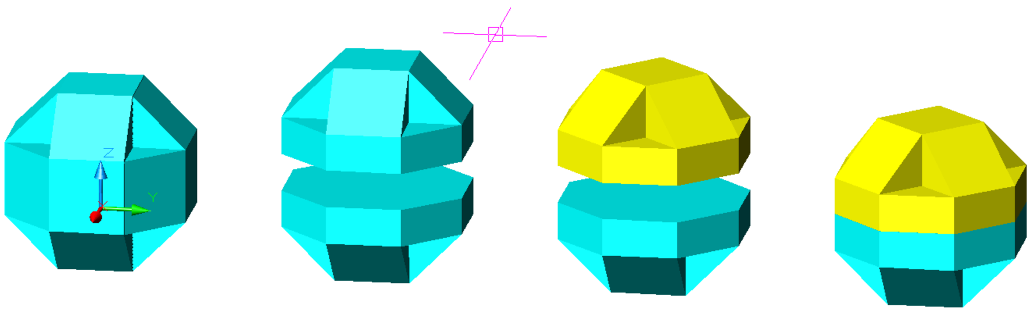

Figure 2: The above pictures solved the problems. We consider the star polyhedron's rigidity, and try to solve the secret of the star polyhedron.

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Equidistant surfaces in $\mathbf{H}^2\times\mathbf{R}$ space

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The $\mathbf{H}^2\times\mathbf{R}$ geometry is derived by direct product of the spherical plane $\mathbf{H}^2$ and the real line $\mathbf{R}$. E. Molnár has showed in [1], that the homogeneous 3-spaces have a unified interpretation in the projective 3-sphere $\mathcal{PS}^3(\mathbf{V}^4, \boldsymbol{V}_4, \mathbf{R})$. In our work we shall use this projective model of $\mathbf{H}^2\times\mathbf{R}$ geometry and in this manner the geodesic lines, geodesic spheres can be visualized on the Euclidean screen of computer ([3]).

Moreover, we shall define the notion of the equidistant surface of two points, determine its equation and we shall visualize it in some cases ([2]). We shall also show a possible way of making the computation simpler and obtain the equation of an equidistant surface with more possible geometric meaning. The pictures were made using the Wolfram *Mathematica* software.

Key words: non-Euclidean geometry, geodesic curve, equidistant surface, Beltrami-Cayley-Klein model

MSC 2010: 53A35, 51M10, 52C17, 52C22

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Optimization of skinning of circles by energy minimization

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Interpolation of geometric data sets has special importance in Computer Aided Geometric Design. If the data set consists of circles then a so called skinning curve can be determined which touches each of the objects. G. Slabaugh has an iterative way to construct these skinning curves using an energy formulation to minimize the convex combination of arc length and curvature [1]. A new technique has been worked out to compute skins by R. Kunkli and M. Hoffmann [2] applying classical geometric methods and a G1 continuous skin constructed by Hermite interpolation curves.

We optimize the result of [2] for the given touching points and tangent directions using Slabaugh's energy minimization technique [1].

Key words: skinning, optimization, circle, minimization, energy function

MSC 2010: 65D17

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Constructive elaboration of translating surfaces in the virtual 3D space

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Translation surfaces are type of surfaces that are deduced from two curves, sliding one curve along the other in such a way that a point on the first curve traces the second curve. Bohemian dome is a translation algebraic closed quartic surface. Surface modeling and constructive treatment in 3D space will be elaborated for some translation surfaces in virtual environment, using computer system. Virtual 3D space is used as a Euclidean model of the projective space, where we can apply projective transformations.

Surface properties are very important in architectural and civil engineering structure shaping. Translating surfaces are used for shells in engineering. Shells are curved thin structures, which can take load as the membrane. In terms of the curvature, shells are classified into three groups, according on Gaussian curvature K. Shell have compression stresses following the convex curvature. The tension stresses follow the concave curvature. Geometry of surface is of fundamental importance for behavior of the structure under load.

In the design of structures such as membranes (hyperbolic paraboloid), domes, cable grids, barrel vaults, *tensegrity* structures ("tensional integrity"), foldable structures and so on, important areas are modeling, form-finding and displacement analysis.

Key words: translating surfaces, modelling, CAD, shells

MSC 2010: 00A67

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Results overview of GEFFA Summer School

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The GEFFA Summer School contribution to modernization and development of curricular elements will be presented. The aim is elaboration of geometric courses for students at civil engineering and architectural faculties. Here an overview of the results and outcomes of workshops will be given.

Key words: engineering education, CAD

MSC 2010: 97Uxx

Collision detection with toleranced motions

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In [1], the authors presented a tolerancing concept for affine and Euclidean displacements. One of the main results is that the orbit of a point under a ball of affine displacements (with respect to a suitable metric) is a Euclidean ball. Moreover, the orbit of a line is a one-sheeted hyperboloid of revolution and the orbit of a plane is a two-sheeted hyperboloid.

The simple shape of these orbits suggests applications in collision detection with polyhedral objects. In our contribution, we present some preliminary results in this direction. The main idea consists of subdividing a given (affine or Euclidean) motion and compute a minimal bounding ball for each segment. For each bounding ball, we compute the toleranced image of a polyhedral object and test for collision with fixed objects. This collision test can be performed efficiently by means of bounding sphere sets. We demonstrate how to compute them and adapt them to changing radius of the displacement ball in case of a refined subdivision.

Key words: collision detection, bounding spheres, motion subdivision

MSC 2010: 70F35, 70B10, 68U05

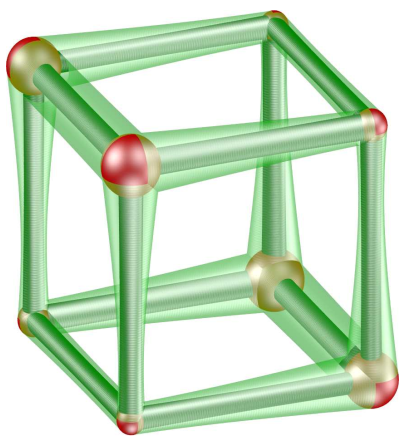

Figure 1: Toleranced position of a cube

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Equidistant surfaces in Nil-space

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W. Heisenberg's real matrix group provides a non-commutative translation group of an affine 3-space. The **Nil**-geometry, which is one of the eight Thurston 3-geometries, can be derived from this group. It was proved by E. Molnár in [1] that the maximal simply connected homogeneous Riemannian 3-geometries have a unified interpretation in the 3-dimensional projective spherical space that can be embedded into the Euclidean 4-space.

The equidistant surfaces play a major role in a lot of the discret geometrical problems, for example in the examination of the Dirichlet-Voronoi cells. In this lecture we introduce the notion of the equdistant surfaces in **Nil**-space, analogously to the Euclidean case. Moreover, we develope a procedure to visualize and examine these surfaces.

Key words: Thurston geometries, **Nil** geometry, equidistant surface, Dirichlet-Voronoi cell.

MSC 2010: 53A35, 51M10, 52C17, 52C22.

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Some old and some new geometry stories

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Some beautiful, old ideas from the field of synthetic geometry will be presented by using modern geometry tools. We will present a few ideas, based on the duality principle, for the further study in the quasi-hyperbolic plane which will hopefully result in some interesting geometry papers.

Short preview of an educational experiment's plan

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As a PhD student of Mathematics and Didactics I am required to develop and carry out an experiment during the time of one term. The topic of my experiment is the development and developability of the spatial ability of our architecture and civil engineering students. As the success of this experiment greatly depends on it being thoroughly prepared, I would like to talk in my presentation about the details of planning, related literature and the results of previous studies carried out by others. My intention is to gain more knowledge from my colleagues' helpful advice.

Key words: educational experiment, spatial ability

MSC 2010: 51N05, 97D40

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Stitching B-spline curves

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We present an algorithm for stitching B-spline curves which is different from the generally used least square method. Our aim is to find a symbolic solution for unifying the control polygons of arcs separately described as 4th degree B-spline curves.

Key words: B-spline curves

MSC 2010: 65D17, 65D05, 65D07, 68U05, 68U07

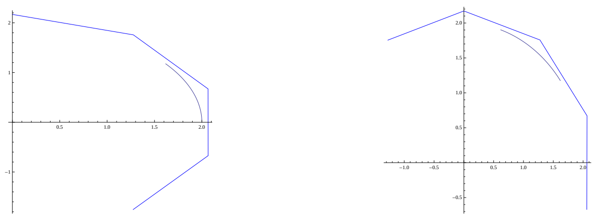

Figure 1: Two B-spline arcs with control polygons

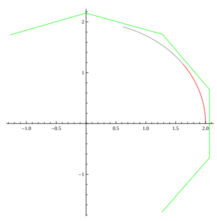

Figure 2: Merged arcs and the unified control polygon

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New upper bounds on a problem of covering a d-cube by rectangular boxes of smaller diameter

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Joint work with Á. Jenei, College of Dunaújváros, Hungary.

Let $f(d,n)$ be the smallest positive real number for which the d-dimensional unit cube can be covered by n rectangular boxes of diameter at most $f(d,n)$.

In [1], the author presented an upper bound on $f(d,n)$ for every $d \geq 2$, $n \geq 2$. In a subsequent paper [2], Joós improved on the upper bound of $f(d,n)$ when $n = 3 \cdot 2^{d-2}$ for $d \geq 3$, and $n = 3 \cdot 2^{d-3}$ for $d \geq 4$.

Now we present new upper bounds on $f(d,n)$ when $n = 2^d - 1$, $n = 2^d - 2$, $n = 2^{d-1} + 1$, $n = 2^{d-1} + 2$ for various values of d, and in some other cases.

Key words: covering, extremum problem, diameter

MSC 2010: 52C17, 52A40

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How to teach the contemporary architect engineer - Catalan surfaces

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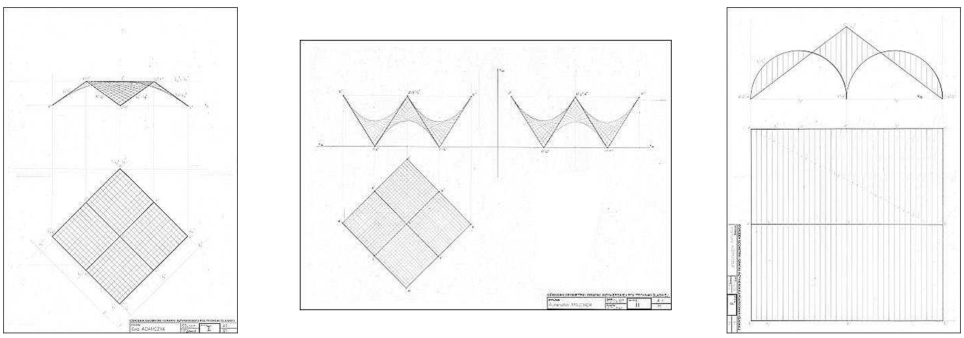
Within the subject of Descriptive Geometry at the Faculty of Architecture the teaching staff quite often face a problem of a new approach to space - its shaping and transformation. However, there is one thing they are sure of: sticking with traditional methods of object creation, based on projections it is necessary to focus stronger on modeling. There is an issue of subject responsibility for development of skills in using traditional tools for drawing in 2D - projections and 3D - axonometry, perspective as well as preparing models. At the conceptual level of designing, an architect engineer will not give up a piece of paper or a pencil, and abilities to make a model of the object is especially valuable in designing offices and is a favorable way of presenting designs to investors.

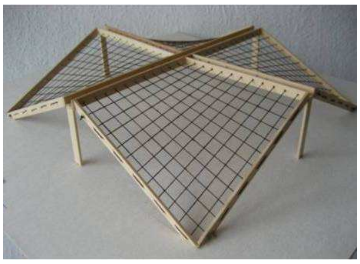
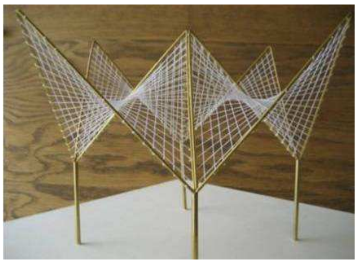
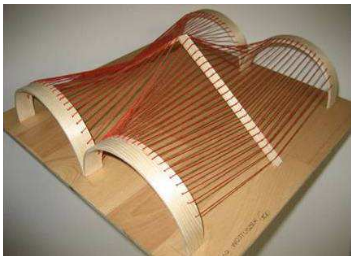
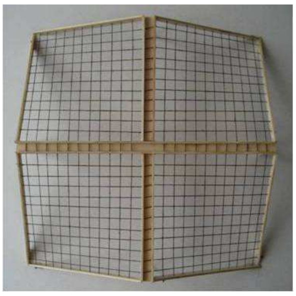
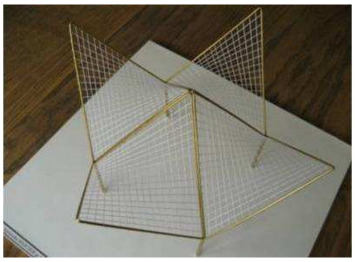
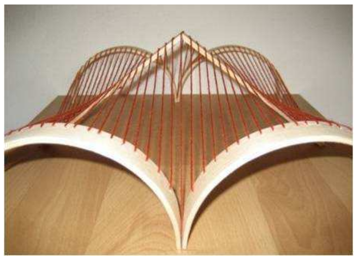
Carrying classes gradually - from idea to making a 2D drawing to presenting a design in a 3D form leads to better understanding of the relations of mutual location of object elements. It makes one aware of advantages and disadvantages of descriptive and space method of mapping an object.

Education in any domain always aims at developing abilities of rational problem solving, using knowledge on laws and rules governing a given discipline. While teaching descriptive geometry, emphasis should be put on knowledge and skills necessary in the future engineering activity, as well as on development of creative inventiveness of a future graduate. The presentation will discuss issues connected with a designing task of construction of roofs designed on the basis of Catalan structures.

Key words: descriptive geometry, models, Catalan surfaces, didactic solutions



Figures 1, 2, 3: Three or two projections of designed roof surfaces

Figures 4, 5, 6, 7, 8, 9: The models of Catalan surface made by first year students of Faculty of Architecture.

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Minkowski set operators

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Concept of Minkowski set combination is presented with possible modifications as generalizations of well defined set operations of Minkowski sum and Minkowski product of point sets in the n-dimensional Euclidean space $\mathbb{E}^n$. Minkowski linear set operator, Minkowski matrix set operator and Minkowski arithmetic set operator are introduced and their particular properties are discussed. Examples of smooth manifolds in higher dimensional Euclidean spaces created by means of Minkowski operators applied to point sets, discrete sets of points determined by coordinates or infinite sets determined as smooth manifolds by vector representations, are visualized by means of their orthographic projections to several three dimensional subspaces of the respective space. Presented geometric forms created by means of Minkowski combinations of point sets in $\mathbb{E}^n$ are regarded as certain artistic visualizations of abstract algebraic relations, concepts and patterns.

Key words: Minkowski set combination, Minkowski linear set operator, Minkowski matrix set operator, Minkowski arithmetic set operator

MSC 2010: 51N20, 65D17

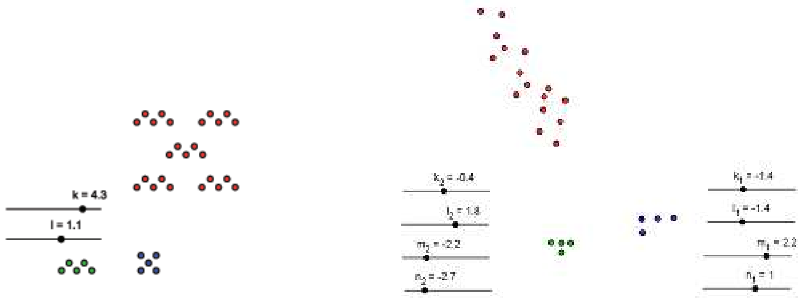

Figure 1: Minkowski linear and matrix combinations of two point sets in $\mathbb{E}^2$

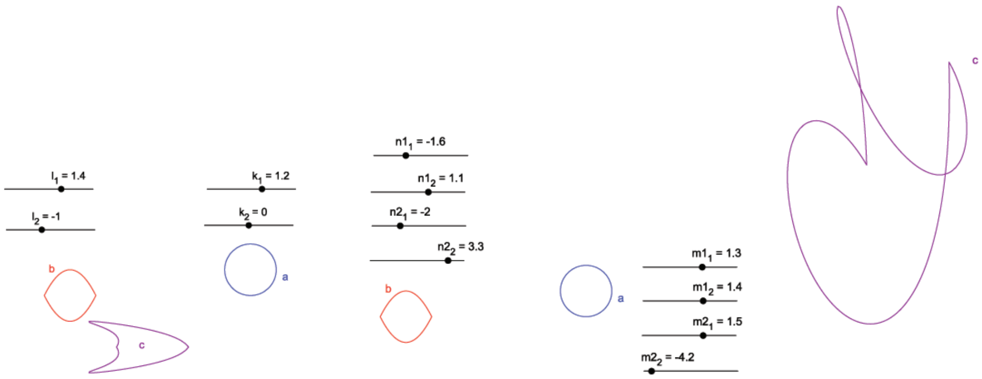

Figure 2: Minkowski linear and matrix combinations of two equally parameterized smooth curves in $\mathbb{E}^2$

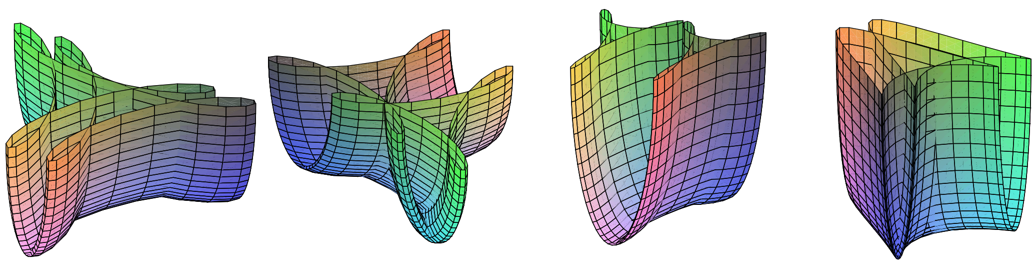

Figure 3: Minkowski linear combinations of two smooth curves in $\mathbb{E}^3$

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Engineering problem. Various methods of its decision

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Descriptive geometry is a branch of mathematics that studies various methods of representing spatial objects in the plane and their transformations. It is one of fundamental disciplines in education of future engineers. Its importance follows from the fact that it enables one to express his or her thoughts in an orderly manner, by using the language of graphic, to prepare drawings and to interpret them. The knowledge of descriptive geometry allows us to solve various engineering problems. Besides, the wealth of methods of descriptive geometry allows for different ways of solving the problem and for the choice of optimal approach.

An example: Given are three mine galleries, a, b, and c. The task is to project the fourth gallery d, intersecting galleries a and b, and running parallel to c (see Fig. 1).

We present four methods of solving this problem based on different geometric models.

Solving problems using different methods facilitates the development of spatial perception and logical thinking. By comparing different methods of descriptive geometry, future engineers develop professional competences such as the ability to analyze the proposed solutions and to choose the most suitable ones.

Key words: descriptive geometry, engineering problems, geometric models

MSC 2010: 51N05, 97G80

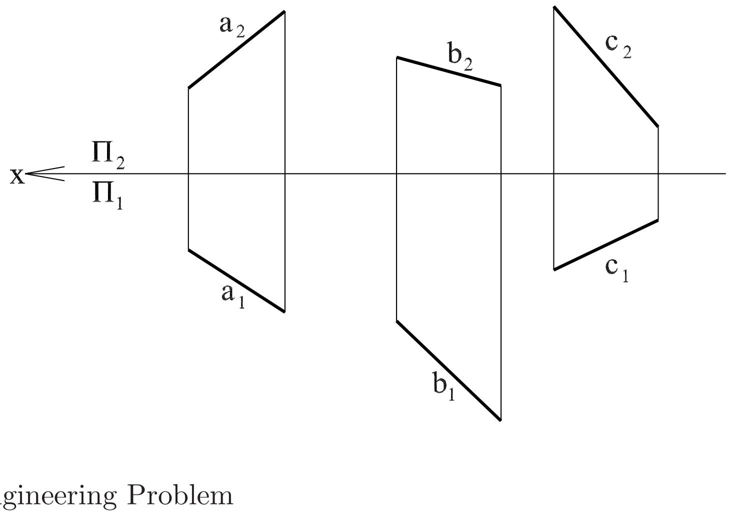

Figure 1: Engineering Problem

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Projections of projections onto a common plane

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We discuss linear projections of objects onto a common or parallel planes, as well as projections of these projections onto a common image plane in the classical projective 3-dimensional space. The unit object of our projections is the cube. Based on all conditions, we can formulate the following statement: Assume that we have two differently projected images of an object in a common plane, and we know - or we can reconstruct - the orientation of given images related to each other, as well as the trace point of the connecting straight line of the two proper or ideal center points of projection - line l. In this case, new images of the given object can be projected from two different proper or ideal points of any straight lines of the image plane if this line contains the trace point of l, i.e. the coinciding images of the initial two center points of projection. Initial and gained images can be paired in interest of further constructions of newer images.

In several cases, it can be proved that the above method is also applicable if the two given images were projected onto parallel planes. We can use the properly oriented but arbitrarily placed images in any magnification on a common plane. The two center points of the planar construction of the new images join the connecting straight line of the images of the trace points of the former described line l on the parallel image planes.

Our way of construction may be regarded as a generalization of a special case of Eckhart's method, due to the common or parallel image planes. In a further special case of our construction, the first two center points of projection are the opposite vertices of the cube, joining a diagonal of the solid [2]. (This method may be generalized for projections of 3-dimensional models of more-dimensional cubes.) It can be deduced from our way of construction that Ceva's theorem is valid for images of the cube's vertices and relevant trace triangles as well.

Key words: classical projective space, linear projections, Eckhart's method, Ceva's theorem

MSC 2010: 51N05, 51N15

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The rational trigonometry of a tetrahedron

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This talk will outline a new approach to the trigonometry of a general tetrahedron in Euclidean space, although in fact the formulas work for any affine space with a metrical structure given by a quadratic form, so in particular the formulas hold also for relativistic geometry. While this is a long standing subject, the classical approach using distances and angles encounters very serious obstacles; our approach is to use a combination of planar affine rational trigonometry ([5]) and planar projective trigonometry ([4]) (which is the basis for Universal Hyperbolic Geometry ([1],[2],[3])).

In particular the key metrical notions are quadrance between two points, spread between two lines or two planes, quadreas of triangular faces, the solid spread made by a tripod of three concurrent lines in three dimensional space, and the quadrume of the tetrahedron itself. These notions will be reviewed, but it is perhaps useful to summarize them here.

Suppose we work in a three dimensional space over a field (the rational numbers are always the best and most natural example) with a symmetric bilinear form $v \cdot w \equiv vMw^T$ between (row) vectors v and w, for a symmetric non-degenerate matrix M.

The **quadrance** of the vector v is the number

$$Q(v) \equiv v \cdot v.$$

The **spread** between the vectors v and w is the number

$$S(v, w) \equiv 1 - \frac{(v \cdot w)^2}{Q(v) Q(w)}.$$

If P and R are two planes with normal vectors p and r respectively, then the **spread** $s(P, R)$ between the planes is

$$s(P, R) \equiv S(p, r).$$

If a triangle has side vectors v, w and u (so that say $v+w+u = 0$) with respective quadrances Q_v, Q_u and Q_w, then the **quadrea** of the triangle is

$$\mathcal{A} \equiv (Q_v + Q_u + Q_w)^2 - 2\left(Q_v^2 + Q_u^2 + Q_w^2\right).$$

In Euclidean geometry, by a rational version of Heron's formula which we prefer to call Archimedes formula, $\mathcal{A}$ is 16 times the square of the area of the triangle.

If v, w and u are three row vectors which are the directions of three collinear lines forming a tripod, then the **solid spread** $\mathcal{S}$ of the tripod is

$$\mathcal{S} \equiv \frac{\det^2 \begin{pmatrix} v \\ u \\ w \end{pmatrix}}{Q(v)\, Q(u)\, Q(w)}.$$

So altogether a general tetrahedron has 6 edge quadrances (one for each edge), 12 face spreads (three for each face), 6 edge spreads (between faces which meet at an edge), 4 solid spreads (one at each vertex) and one quadrume $\mathcal{V}$, a multiple of the square of the volume. These are the main quantities, but there are others, and we try to outline some of the fascinating relationships between these numbers. Some of the formulas are highly non-obvious!

Key words: Tetrahedron, rational trigonometry, projective geometry, quadrance, spread, volume, bilinear form.

MSC 2010: 51M10, 14N99, 51E99.

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Effective use of e-learning in teaching mathematics in higher education environment (2000–2010)

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Information technologies have deeply integrated themselves in contemporary society, especially in the last decade. Their rapid development has caused a significant change in educational system at all levels. Here we try to give a brief insight into technology use in teaching mathematics in higher education environment. Because of the field specificity, chalk and board have been the only instrument for teaching mathematics at faculties for ages, so the question is how we can keep that chalk and board tradition. Another question that arises here is how e-learning and mathematics supplement each other. This research provides a review of literature concerning that subject and tries to answer the aforementioned questions. It includes 24 papers, published in the period of 2000 – 2010. The results of the analysis identify many positive effects of e-learning in higher education mathematics, as well as student reaction on implemented technology and their mastering the same.

Key words: e-learning, mathematics, higher education

MSC 2010: 97U50

Posters

Visualization of special circular surfaces

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Let $\mathcal{C}(p)$ be a congruence of circles that consists of circles in Euclidean space E^3 passing through two given points P_1, P_2. The points P_1, P_2 lie on the axis z and are given by the coordinates $(0, 0, \pm p)$, where $p = \sqrt{q}$, $q \in \mathrm{R}$. If q is greater, equal or less than zero, $\mathcal{C}(p)$ is an elliptic, parabolic or hyperbolic congruence, respectively. For a given congruence $\mathcal{C}(p)$ and a given curve α a circular surface $\mathcal{CS}(\alpha, p)$ is defined as the system of circles from $\mathcal{C}(p)$ that intersect α.

In this poster we construct a special class of the circular surfaces, called *generalized rose surfaces*, where α is a cyclic-harmonic curve $CH(n, d, a)$ given by the polar equation $r(\varphi) = \cos \dfrac{n}{d}\varphi + a$, $\varphi \in [0, 2d\pi]$, where $\frac{n}{d}$ is a positive rational number in lowest terms and $a \in \mathbb{R}^+$. We study their algebraic properties and visualize their numerous forms with the program *Mathematica*.

Key words: circular surface, cyclic-harmonic curve, singular point, congruence of circles

MSC 2010: 51N20, 51M15

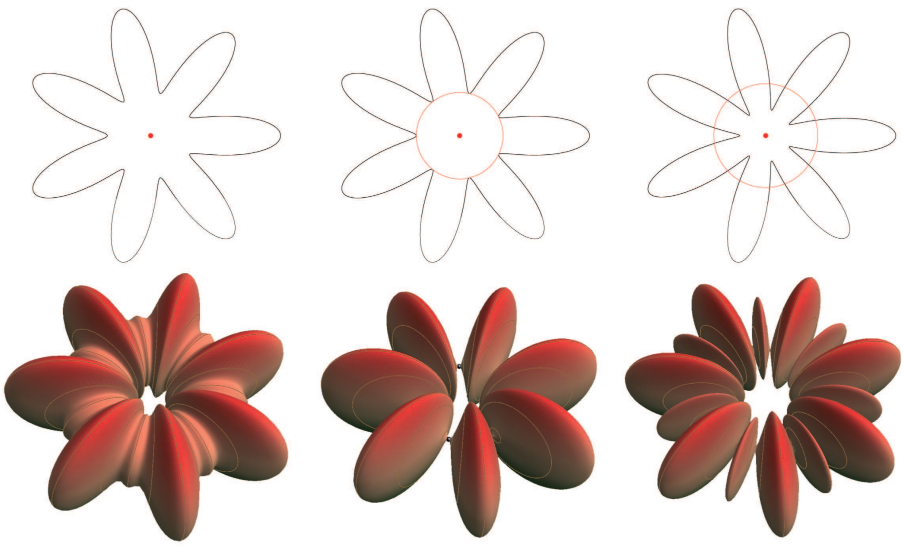

Figure 1: Three examples of generalized rose surfaces defined by corresponding cyclic-harmonic curves and $p = i$.

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Looking at the past and preparing for the future

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This poster gives an overview of teaching the course of Descriptive geometry in the last 40 years at the Faculty of Architecture, University of Zagreb.

As educators it is our responsibility to continually reflect on what we teach and how we present new material to our students.

Due to the decreasing share of Descriptive geometry in the curriculum of architectural and design studies, it is no longer possible to teach Descriptive geometry in its full extent.

In order to determine the topics and methods of Descriptive geometry which are relevant for professional architects and designers, we clarify the value of architectural drawings, so the students can apply them to various situations at the end of the course.

The results can be used for developing a curriculum for teaching Descriptive geometry in architecture and design in the future.

Key words: descriptive geometry, graphics education, architecture, design

Defining metric for visual smoothness of skinning algorithms

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This poster considers the visual smoothness of interpolating curves. It will examine skinning algorithms in detail. Especially the 2D ball skinning algorithms will be covered. Slabaugh introduced an energy function [1] and Kunkli defined a process to find the touching points [2] and made an elegant skinning method based on classical geometry [3]. I will try to give a simple metric for visual smoothness based on the number of direction changes of the obtained interpolation curve. Minimizing this metric will give the best visual result.

Key words: metric, visual smoothness, interpolation, skinning, circles

MSC 2010: 68U05

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