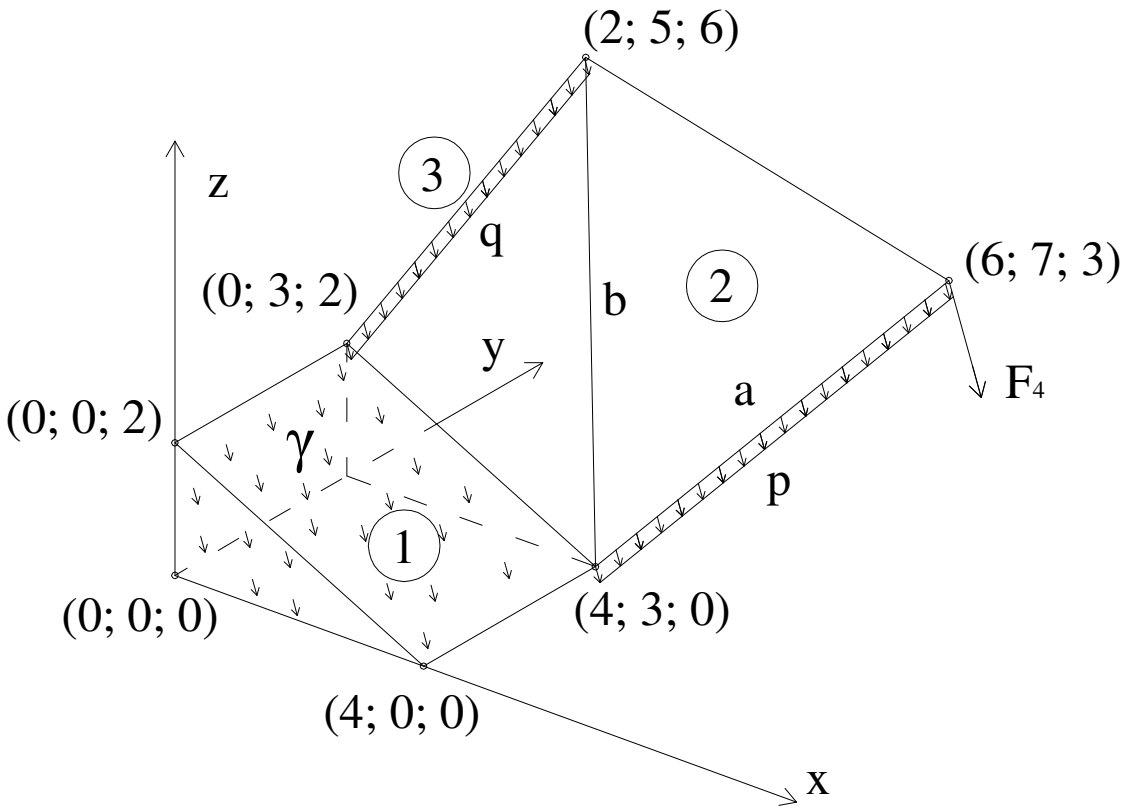
1. Zadan je sustav na koji djeluju različite vrste sila koje su jednako usmjerene. Na trostranu prizmu ¨1¨ djeluje volumenska sila $\gamma = 15$ kN/m^3, na trokut ¨2¨ površinska sila $p = 9{,}0$ kN/m^2, na dužinu ¨3¨ djeluje linijska sila $q = 23$ kN/m. U točki (6; 7; 3) djeluje koncentrirana sila $F_4 = 135$ kN. Treba odrediti težište.



DIO ¨1¨

Volumen: $V = \dfrac{4 \cdot 3 \cdot 2}{2} = 12 m^3$; **$F_1 = 12 \cdot 15 = 180$ kN**

Težište: $x_1 = 1{,}333$; $y_1 = 1{,}5$; $z_1 = 0{,}666$

DIO ¨2¨

Površina: $A_2 = \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$; $\left|\vec{a} \times \vec{b}\right| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 3 \\ -2 & 2 & 6 \end{vmatrix} = 18\vec{i} - 18\vec{j} + 12\vec{k}$

$A_2 = \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right| = \dfrac{1}{2}\sqrt{18^2 + 18^2 + 12^2} = 14{,}071$; **$F_2 = A_2 \cdot p = 126{,}64$ kN**

Težište: $x_2 = \dfrac{4+6+2}{3} = 4$; $y_2 = \dfrac{3+7+5}{3} = 5$; $z_2 = \dfrac{6+3+0}{3} = 3$

DIO ¨3¨

Dužina: $l_3 = \sqrt{2^2 + 2^2 + 4^2} = 4{,}899$; **$F_3 = l_3 \cdot q = 112{,}68$ kN**

Težište: $x_3 = \frac{0+2}{2} = 1$; $y_3 = \frac{3+5}{2} = 4$; $z_3 = \frac{2+6}{2} = 4$

DIO ¨4¨

Sila: $F_4 = 135$ kN;

Hvatište: $x_4 = 6$; $y_4 = 7$; $z_4 = 3$

i	F_i	x_i	y_i	z_i	$x_i \cdot F_i$	$y_i \cdot F_i$	$z_i \cdot F_i$
1	180,00	1,333	1,5	0,666	239,94	270,00	119,88
2	126,64	4,0	5,0	3,0	506,56	633,20	379,92
3	112,68	1,0	4,0	4,0	112,68	450,72	450,72
4	135,00	6,0	7,0	3,0	810,00	945,00	405,00
∑	554,32	/	/	/	1669,18	2298,92	1355,52

$$x_T = \frac{\sum x_i F_i}{\sum F_i} = 3{,}011; \qquad y_T = \frac{\sum y_i F_i}{\sum F_i} = 4{,}147; \qquad z_T = \frac{\sum z_i F_i}{\sum F_i} = 2{,}445$$

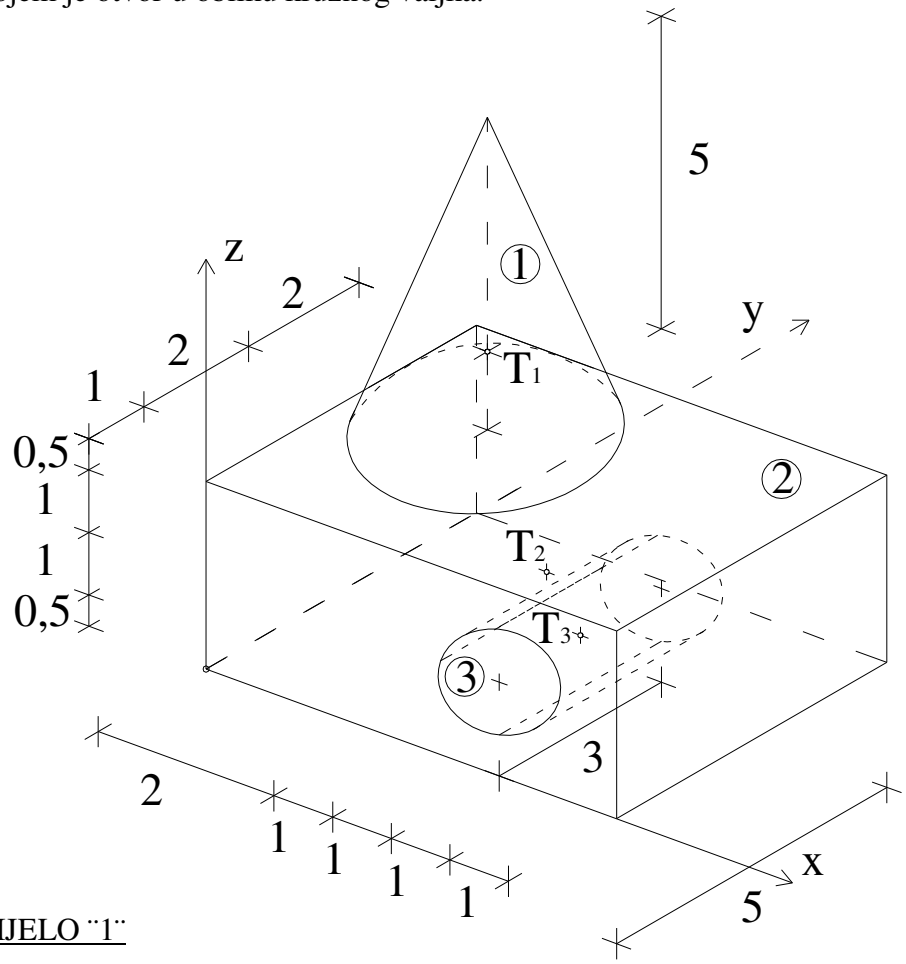
KONTROLA – Novo ishodište u dobiveno težište

i	F_i	x_i	y_i	z_i	$x_i \cdot F_i$	$y_i \cdot F_i$	$z_i \cdot F_i$
1	180,00	-1,678	-2,647	-1,779	-302,04	-476,46	-320,22
2	126,64	+0,989	+0,853	+0,555	+125,25	+108,02	+70,29
3	112,68	-2,011	-0,147	+1,555	-226,60	-16,56	+175,22
4	135,00	+2,989	+2,853	+0,555	+403,52	+385,16	+74,93
∑	554,32	/	/	/	+0,13	+0,16	+0,22

$$\Delta x_T = \frac{0{,}13}{554{,}32} = 0{,}0002; \qquad \Delta y_T = \frac{0{,}16}{554{,}32} = 0{,}0003; \qquad \Delta z_T = \frac{0{,}22}{554{,}32} = 0{,}0004$$

Najveća dužina u sustavu: 9,695 m → Δ_i su relativno vrlo mali.

2. Treba odrediti težište složenog tijela koje se sastoji od kružnog stošca i kvadra u kojem je otvor u obliku kružnog valjka.



TIJELO ¨1¨

Volumen: $V = \frac{2^2 \pi \cdot 5}{3} = 20{,}944$

Težište: $x_1 = 2{,}0$; $y_1 = 3{,}0$; $z_1 = 4{,}25$

TIJELO ¨2¨

Volumen: $V = 5 \cdot 6 \cdot 3 = 90{,}00$

Težište: $x_1 = 3{,}0$; $y_1 = 2{,}5$; $z_1 = 1{,}5$

TIJELO ¨3¨

Volumen: $V = \pi^2 \cdot 3 = 9{,}425$

Težište: $x_1 = 3{,}0$; $y_1 = 2{,}5$; $z_1 = 1{,}5$

Ukupni volumen = ∑volumena = 90 + 20,944 – 9,425 = 101,519

$$x_T = \frac{2 \cdot 20{,}944 + 3 \cdot 90 - 4 \cdot 9{,}425}{101{,}519} = 2{,}701m$$

$$y_T = \frac{3 \cdot 20{,}944 + 2{,}5 \cdot 90 - 1{,}5 \cdot 9{,}425}{101{,}519} = 2{,}696m$$

$$z_T = \frac{4{,}25 \cdot 20{,}944 + 1{,}5 \cdot 90 - 1{,}5 \cdot 9{,}425}{101{,}519} = 2{,}067m$$

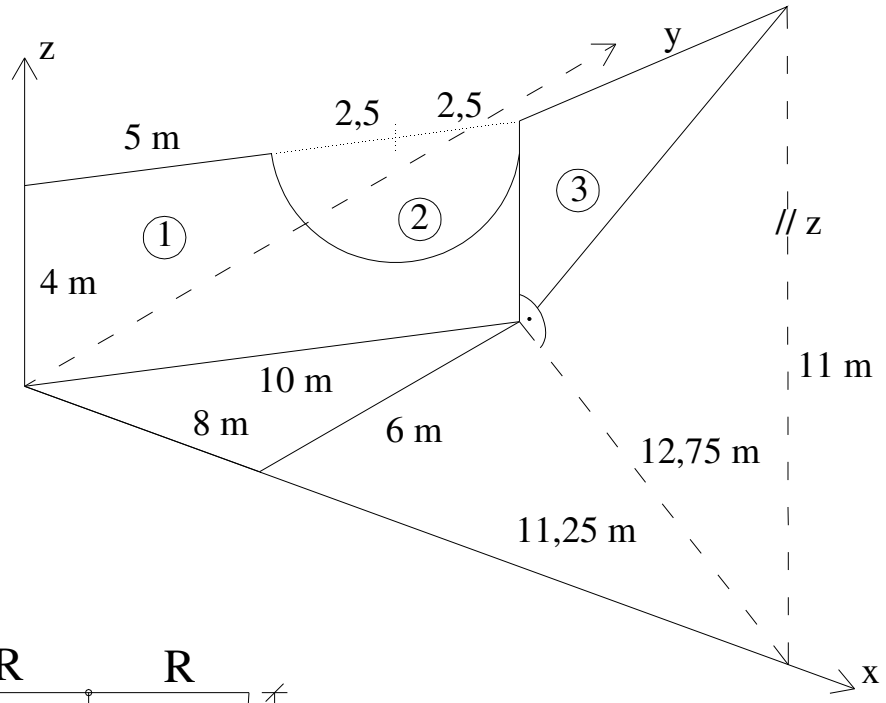
KONTROLA

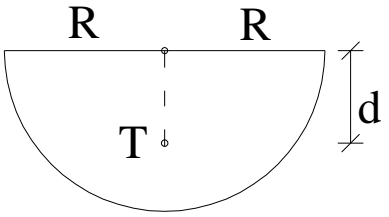
$$\Delta x_T = \frac{-0{,}701 \cdot 20{,}944 + 0{,}299 \cdot 90 - 1{,}299 \cdot 9{,}425}{101{,}519} = 0{,}0001$$

$$\Delta y_T = \frac{0{,}304 \cdot 20{,}944 - 0{,}196 \cdot 90 - (-1{,}196) \cdot 9{,}425}{101{,}519} = 0{,}0000$$

$$\Delta z_T = \frac{2{,}183 \cdot 20{,}944 - 0{,}567 \cdot 90 - (-0{,}567) \cdot 9{,}425}{101{,}519} = 0{,}0003$$

3. Treba odrediti težište složenog lika; iz pravokutnika je izrezan polukrug.





Udaljenost težišta

$$d = \frac{4 \cdot R}{3 \cdot \pi}$$

DIO ¨1¨ $A_1 = 10 \cdot 4 = 40$; $x_1 = 4$; $y_1 = 3$; $z_1 = 2$.

DIO ¨2¨ $A_2 = \frac{2{,}5^2 \pi}{2} = 9{,}81747$; $x_2 = 6$; $y_2 = 4{,}5$; $z_2 = 2{,}93897$.

DIO ¨3¨ $A_3 = \frac{4 \cdot 12{,}75}{2} = 25{,}5$;

$x_3 = \frac{8+8+19{,}25}{3} = 11{,}75$; $y_3 = \frac{6+6+0}{3} = 4{,}0$; $z_3 = \frac{0+4+11}{3} = 5{,}0$.

A = 40,0 – 9,81747 + 25,5 = 55,68253

$$x_T = \frac{4 \cdot 40 - 6 \cdot 9{,}81747 + 11{,}75 \cdot 25{,}5}{A} = 7{,}19652$$

$$y_T = \frac{3 \cdot 40 - 4{,}5 \cdot 9{,}81747 + 4{,}0 \cdot 25{,}5}{A} = 3{,}19349$$

$$z_T = \frac{2 \cdot 40 - 2{,}938975 \cdot 9{,}81747 + 5{,}0 \cdot 25{,}5}{A} = 3{,}20831$$

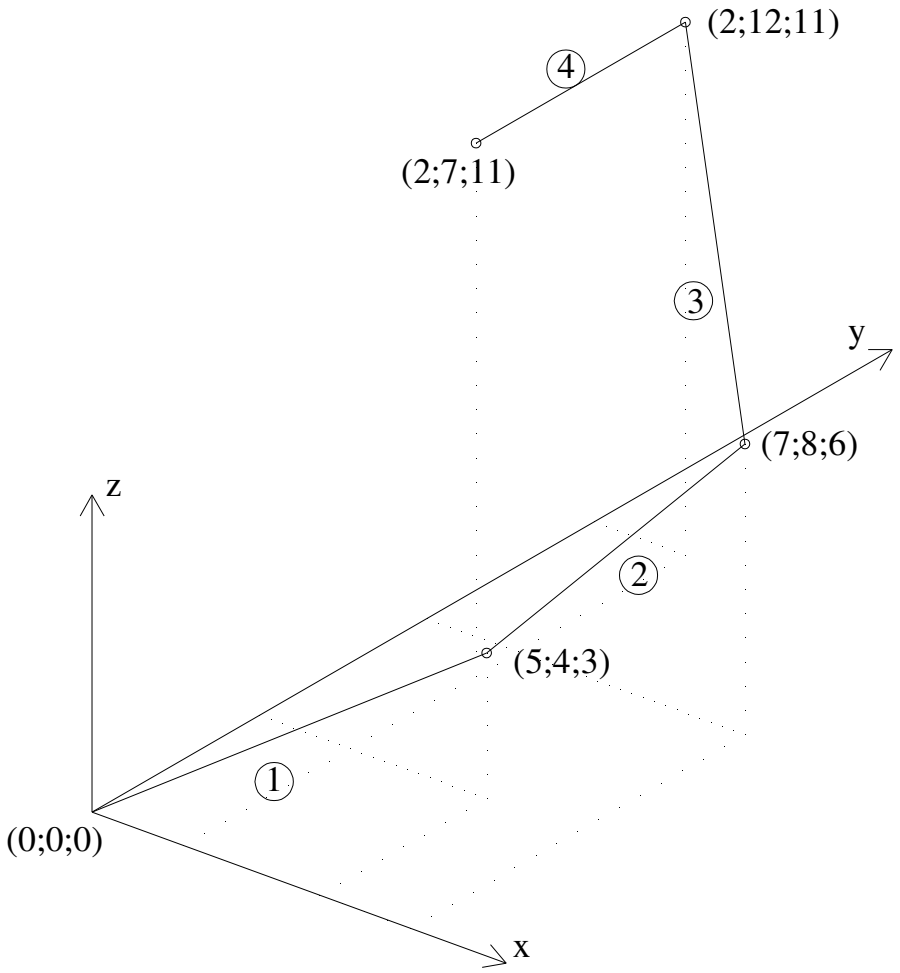
KONTROLA

$$\Delta x_T = \frac{-3{,}19652 \cdot 4{,}0 - (-1{,}19652) \cdot 9{,}81747 + 4{,}56348 \cdot 25{,}5}{A} = 0{,}00000$$

$$\Delta y_T = \frac{-0{,}19349 \cdot 40 - 1{,}30654 \cdot 9{,}81747 + 0{,}80651 \cdot 25{,}5}{A} = 0{,}00000$$

$$\Delta z_T = \frac{-1{,}20831 \cdot 40 - (-0{,}26934) \cdot 9{,}81747 + 1{,}79169 \cdot 25{,}5}{A} = 0{,}00000$$

4. Treba odrediti težište prostorne linije koja se sastoji od 4 dužine



i	l_i	x_i	y_i	z_i	$x_i \cdot l_i$	$y_i \cdot l_i$	$z_i \cdot l_i$
1	7,07106	2,5	2,0	1,5	17,67765	14,14212	10,60659
2	5,38516	6,0	6,0	4,5	32,31096	32,31096	24,23322
3	8,12404	4,5	10,0	8,5	36,55818	81,2404	69,05434
4	5,00000	2,0	9,5	11,0	10,0	47,5	55,0
$\sum$	25,58026	***3,77427***	***6,84878***	***6,21159***	96,54679	175,19348	158,89415
		x_T	y_T	z_T			

KONTROLA

i	l_i	$\bar{x}_i$	$\bar{y}_i$	$\bar{z}_i$	$\bar{x}_i \cdot l_i$	$\bar{y}_i \cdot l_i$	$\bar{z}_i \cdot l_i$
1	7,07106	-1,27427	-4,84878	-4,71159	-9,01044	-34,28601	-33,3159
2	5,38516	2,22573	-0,84878	-1,71159	11,98591	-4,57082	-9,21719
3	8,12404	0,72573	3,15122	2,28841	5,98586	25,60064	18,59113
4	5,00000	-1,77427	2,65122	4,78841	-8,87135	13,2561	23,94205
∑	25,58026	***0,00000***	***0,00000***	***0,00000***	-0,00002	-0,00009	0,00009
		Δx_T	Δy_T	Δz_T			

5.

$A_1 = \frac{4{,}5 \cdot 3}{2} = 6{,}75\,;$ $A_2 = 7 \cdot 4 = 28{,}0\,;$ $A_3 = 1{,}4^2 \cdot \pi = 6{,}16$