

$$E = 3 \cdot 10^7 \text{ kN/m}^2$$

$$b/h = 50/50 \text{ cm}$$

NEPOZNANICE: u_{23} , φ_2

$$l_{12} = 3,30 \text{ m}$$

$$l_{23} = 3,25 \text{ m}$$

$$l_{24} = 3,30 \text{ m}$$

$$l_{35} = \sqrt{4,2^2 + 1,65^2} = 4,512482687 \text{ m}$$

$$I = \frac{b \cdot h^3}{12} = \frac{0,5 \cdot 0,5^3}{12} = \frac{0,0625}{12} = 5,208333 \cdot 10^{-3} \text{ m}^4$$

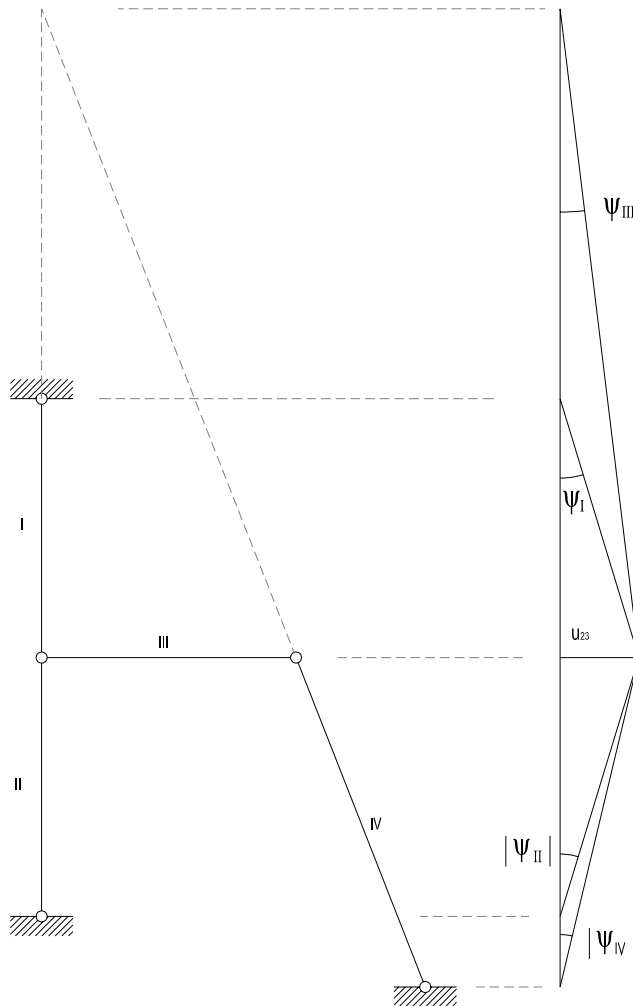
$$EI = EI = 3 \cdot 10^7 \cdot 5,208333 \cdot 10^{-3} = 156\,250 \text{ kNm}^2$$

$$k_{12} = k_{24} = EI / l_{12} = 47\,348,485 \text{ kNm}$$

$$k_{23} = EI / l_{23} = 48\,076,923 \text{ kNm}$$

$$k_{35} = EI / l_{35} = 34\,626,172 \text{ kNm},$$

PLAN POMAKA



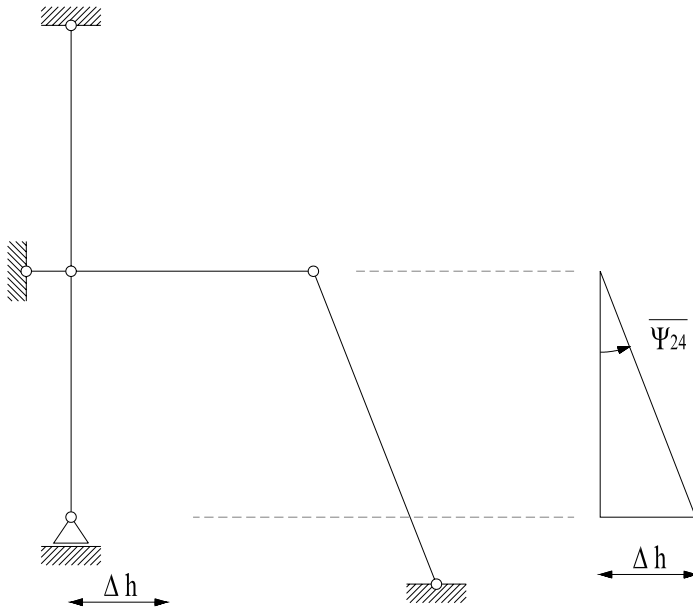
$$\Psi_{III} = \Psi_{23} = \frac{u_{23}}{\frac{4,2}{1,65} * 3,25} = \frac{u_{23}}{8,272727273} = 0,12087912 * u_{23}$$

$$\Psi_I = \Psi_{12} = \frac{u_{23}}{3,30} = 0,303030303 * u_{23}$$

$$\Psi_{II} = \Psi_{24} = - \frac{u_{23}}{3,30} = - 0,303030303 * u_{23}$$

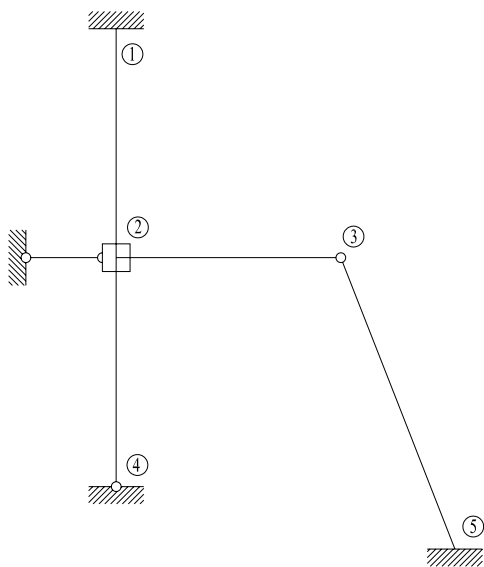
$$\Psi_{IV} = \Psi_{35} = - \frac{u_{23}}{4,2} = - 0,238095238 * u_{23}$$

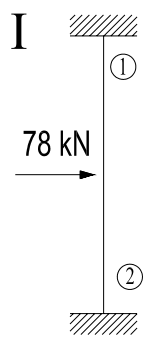
PRISILNI POMAK



$$\overline{\Psi}_{24} = \frac{\Delta h}{3,3} = \frac{0,005}{3,3} = 1,515151515 * 10^{-3}$$

OSNOVNI SISTEM



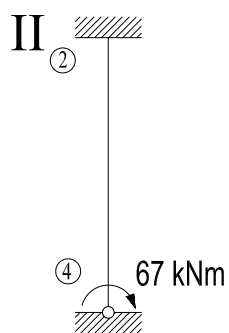


$$\overline{M}_{12} = -\frac{78 \cdot 3,3}{8} = -32,175 \text{ kNm}$$

$$\overline{M}_{21} = \frac{78 \cdot 3,3}{8} = 32,175 \text{ kNm}$$

$$M_{12} = 2 * k_{12} * \varphi_2 - 6 * k_{12} * \Psi_{12} + \overline{M}_{12}$$

$$M_{21} = 4 * k_{12} * \varphi_2 - 6 * k_{12} * \Psi_{12} + \overline{M}_{21}$$



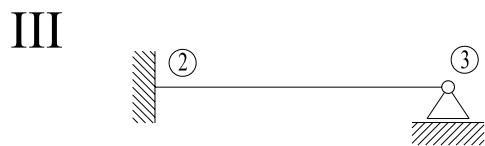
$$\overline{M}_{24}^c(M) = -\frac{67}{2} = -33,5 \text{ kNm}$$

$$\overline{M}_{24}^c(\Delta h) = -3 * k_{24} * \Psi_{24}$$

$$\overline{M}_{24}^c = \overline{M}_{24}^c(M) + \overline{M}_{24}^c(\Delta h)$$

$$M_{24} = 3 * k_{24} * \varphi_2 - 3 * k_{24} * \Psi_{24} + \overline{M}_{24}^c$$

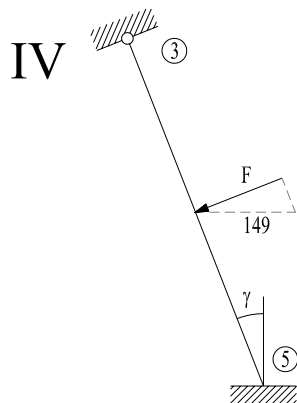
$$M_{42} = 0$$



$$\overline{M}_{23} = 0$$

$$M_{23} = 3 * k_{23} * \varphi_2 - 3 * k_{23} * \Psi_{23}$$

$$M_{32} = 0$$



$$\text{tg } \gamma = \frac{1,65}{4,2} \implies \gamma \approx 21,44773632^\circ$$

$$F = 149 * \cos \gamma = 138,6819725 \text{ kN}$$

$$\overline{M}_{53}^c = -3 \frac{F * l_{35}}{16} = -117,3375 \text{ kNm}$$

$$M_{35} = 0$$

$$M_{53} = -3 * k_{35} * \Psi_{35} + \overline{M}_{53}^c$$

JEDNADŽBA RAVNOTEŽE ČVORA 2

$$\sum M_2 = 0$$

$$- M_{21} - M_{24} - M_{23} = 0$$

$$M_{21} + M_{24} + M_{23} = 0$$

$$4 k_{12} \varphi_2 - 6 k_{12} \psi_{12} + 32,175 + 3 k_{24} \varphi_2 - 3 k_{24} \psi_{24} + (-33,5 - 3 k_{24} \overline{\psi_{24}} + 3 k_{23} \varphi_2 - 3 k_{23} \psi_{23}) = 0$$

$$4 * 47\,348,485 * \varphi_2 - 6 * 47\,348,485 * 0,303030303 * u_{23} + 32,175 + 3 * 47\,348,485 * \varphi_2 + 3 * 47\,348,485 * 0,303030303 * u_{23} - 33,5 - 3 * 47\,348,485 * 1,515151515 * 10^{-3} + 3 * 48\,076,923 * \varphi_2 - 3 * 48\,076,923 * 0,12087912 * u_{23} = 0$$

$$189\,393,94 * \varphi_2 - 86\,088,15454 * u_{23} + 32,175 + 142\,045,455 * \varphi_2 + 43\,044,07727 * u_{23} + -33,5 - 215,2203863 + 144\,230,769 * \varphi_2 - 17\,434,48843 * u_{23} = 0$$

$$475\,670,164 * \varphi_2 - 60\,478,56614 * u_{23} = 216,5453863$$

VIRTUALNI RAD

$$\delta W = 0$$

$$P_1 * \delta \psi_{12} * 1,65 + P_2 * \delta \psi_{35} * 2,1 + M_{21} * \delta \psi_{12} + M_{23} * \delta \psi_{23} + M_{24} * \delta \psi_{24} +$$

$$M_{53} * \delta \psi_{35} + M_{12} * \delta \psi_{12} - M * \delta \psi_{24} = 0$$

$$\frac{78 * 1,65}{3,3} * u_{23} - \frac{149 * 2,1}{4,2} * u_{23} + \frac{M_{21}}{3,3} * u_{23} + \frac{M_{23}}{8,272727273} * u_{23} - \frac{M_{24}}{3,3} * u_{23} - \frac{M_{53}}{4,2} * u_{23} +$$

$$+ \frac{M_{12}}{3,3} * u_{23} + \frac{67}{3,3} * u_{23} = 0 \quad / : u_{23}$$

$$\frac{M_{21}}{3,3} + \frac{M_{23}}{8,272727273} - \frac{M_{24}}{3,3} - \frac{M_{53}}{4,2} + \frac{M_{12}}{3,3} = 15,1969697 \quad / * 3,3$$

$$M_{21} + 0,39890198 * M_{23} - M_{24} - 0,785714285 * M_{53} + M_{12} = 50,15$$

$$4 k_{12} \varphi_2 - 6 k_{12} \psi_{12} + 32,175 + 0,39890198 (3 k_{23} \varphi_2 - 3 k_{23} \psi_{23}) - 3 k_{24} \varphi_2 + 3 k_{24} \psi_{24} - \overline{M}_{24}^c +$$

$$- 0,785714285 (-3 k_{35} \psi_{35} + \overline{M}_{53}^c) + 2 k_{12} \varphi_2 - 6 k_{12} \psi_{12} - 32,175 = 50,15$$

$$4 * 47\,348,485 * \varphi_2 - 6 * 47\,348,485 * \frac{u_{23}}{3,3} + 32,175 + 57\,533,93933 * \varphi_2 +$$

$$- 57\,533,93933 * 0,12087912 * u_{23} - 3 * 47\,348,485 * \varphi_2 - 3 * 47\,348,485 * \frac{u_{23}}{3,3} +$$

$$- \left(-33,5 - 3 * 47\,348,485 * \frac{0,005}{3,3} \right) - 2,357142855 * 34\,626,172 * 0,238095238 * u_{23} +$$

$$+ 92,19374992 + 94\,696,97 * \varphi_2 - 284\,090,91 * \frac{u_{23}}{3,3} = 82,325$$

$$\begin{aligned}
&189\,393,94 * \varphi_2 - 86\,088,15455 * u_{23} + 32,175 + 57\,533,93933 * \varphi_2 - 6954,651956 * u_{23} + \\
&-142\,045,455 * \varphi_2 - 43\,044,07727 * u_{23} + 33,5 + 215,2203864 - 19\,433,05569 * u_{23} + \\
&+92,19374992 + 94\,696,97 * \varphi_2 - 86\,088,15455 * u_{23} = 82,325
\end{aligned}$$

$$199\,579,3943 * \varphi_2 - 241\,608,094 * u_{23} = -290,7641363$$

SUSTAV JEDNADŽBI :

$$475\,670,164 * \varphi_2 - 60\,478,56614 * u_{23} = 216,5453863 \quad (1)$$

$$199\,579,3943 * \varphi_2 - 241\,608,094 * u_{23} = -290,7641363 \quad (2)$$

$$\text{iz (1) : } \varphi_2 = \frac{216,5453863 + 60\,478,56614 * u_{23}}{475\,670,164}$$

φ_2 uvrstimo u (2) :

$$199\,579,3943 * \left(\frac{216,5453863 + 60\,478,56614 * u_{23}}{475\,670,164} \right) - 241\,608,094 * u_{23} = -290,7641363$$

$$u_{23} = \frac{-290,7641363 - \frac{199\,579,3943 * 216,5453863}{475\,670,164}}{\frac{199\,579,3943 * 60\,478,56614}{475\,670,164} - 241\,608,094}$$

$$u_{23} = \frac{-381,6212056}{-216\,232,7939}$$

KONAČNA RJEŠENJA NEPOZNANICA

$$u_{23} = 0,001764862761 \text{ m}$$

$$\varphi_2 = 0,0006796342928 \text{ rad}$$

ODSTUPANJE OD RJEŠENJA PO DiM-u :

DiM	$u_{23} = 0,00176626 \text{ m}$	relativna pogreška = 0,079 %
	$\varphi_2 = 0,000681179 \text{ rad}$	relativna pogreška = 0,227 %

MOMENTI U ČVOROVIMA :

$$\begin{aligned}
 M_{12} &= 2 * k_{12} * \varphi_2 - 6 * k_{12} * \Psi_{12} + \overline{M}_{12} = \\
 &= 2 * 47\,348,485 * 0,0006796342928 - 6 * 47\,348,485 * \frac{0,001764862761}{3,3} - 32,175 = \\
 &= \underline{-119,75 \text{ kNm}}
 \end{aligned}$$

$$\begin{aligned}
 M_{21} &= 4 * k_{12} * \varphi_2 - 6 * k_{12} * \Psi_{12} + \overline{M}_{21} = \\
 &= 4 * 47\,348,485 * 0,0006796342928 - 6 * 47\,348,485 * \frac{0,001764862761}{3,3} + 32,175 = \\
 &= \underline{8,96 \text{ kNm}}
 \end{aligned}$$

$$\begin{aligned}
 M_{24} &= 3 * k_{24} * \varphi_2 - 3 * k_{24} * \Psi_{24} + \overline{M}_{24}^c = \\
 &= 3 * 47\,348,485 * 0,0006796342928 + 3 * 47\,348,485 * \frac{0,001764862761}{3,3} \\
 &\quad - 33,5 - 3 * 47\,348,485 * 1,515151515 * 10^{-3} = \\
 &= \underline{-76,21 \text{ kNm}}
 \end{aligned}$$

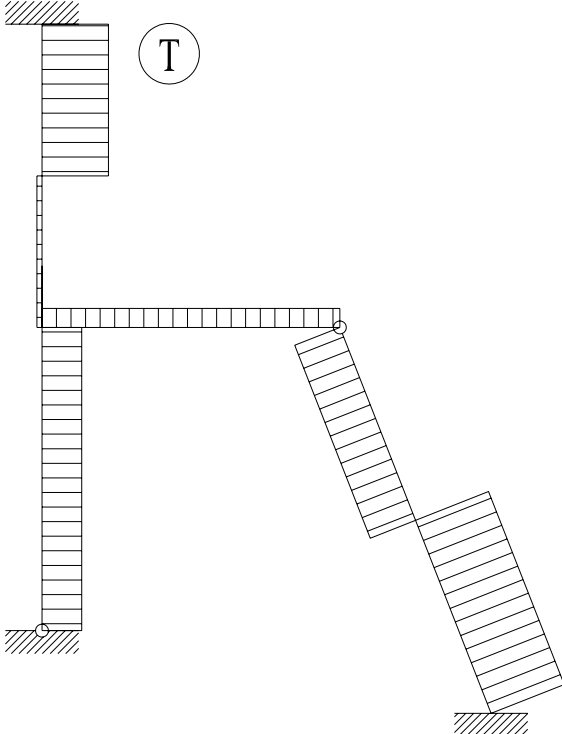
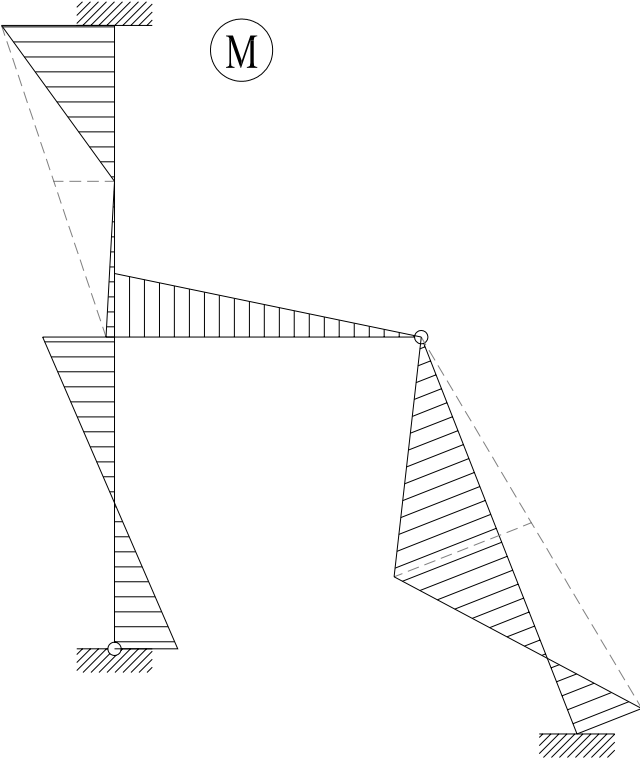
$$M_{42} = 0$$

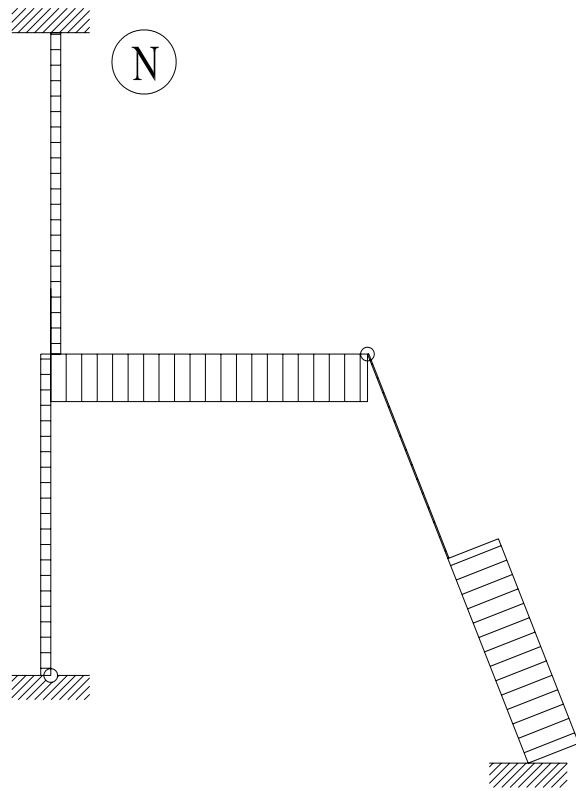
$$\begin{aligned}
 M_{23} &= 3 * k_{23} * \varphi_2 - 3 * k_{23} * \Psi_{23} = \\
 &= 3 * 48\,076,923 * 0,0006796342928 - 3 * 48\,076,923 * \frac{0,001764862761}{8,272727273} = \\
 &= \underline{67,26 \text{ kNm}}
 \end{aligned}$$

$$M_{32} = 0$$

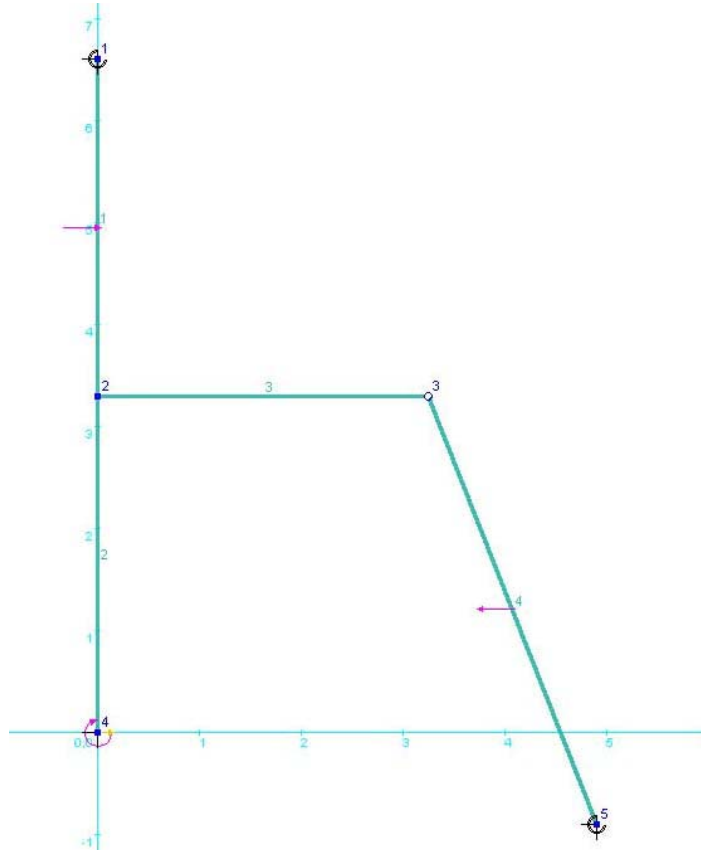
$$M_{35} = 0$$

$$\begin{aligned}
 M_{53} &= -3 * k_{35} * \Psi_{35} + \overline{M}_{53}^c = \\
 &= +3 * 34\,626,172 * \frac{0,001764862761}{4,2} - 117,3375 \\
 &= \underline{-73,69 \text{ kNm}}
 \end{aligned}$$





PRILOG: PRORAČUN U DiM-u



Degrees of Freedom: 6

Nodal displacements:

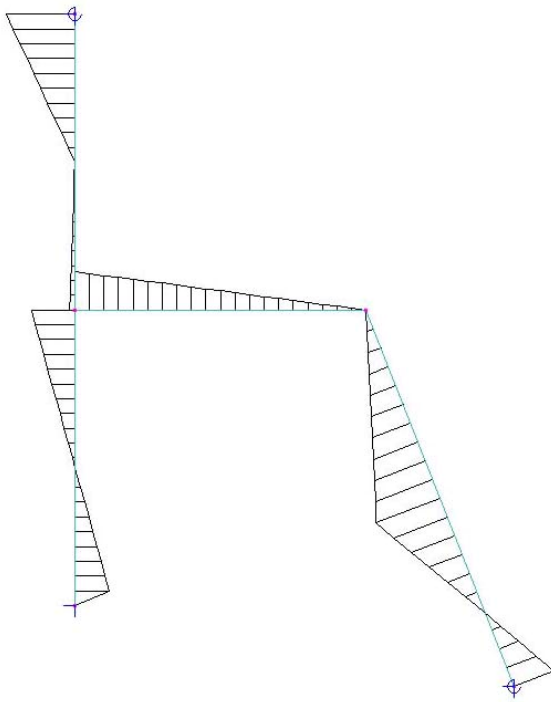
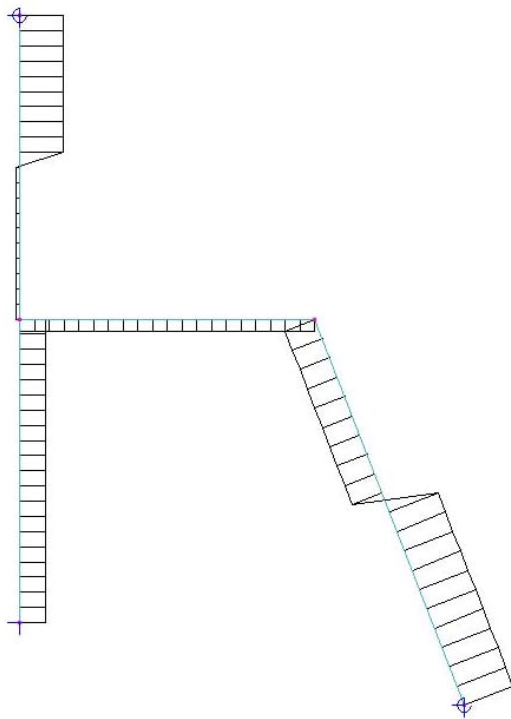
nd	u _i	v _i	phi _i
1:	0	0	0
2:	0.00176626	-4.52203e-06	0.000681179
3:	0.00174511	0.000704022	0
4:	0.005	0	0.000775531
5:	0	0	0

Element end forces:

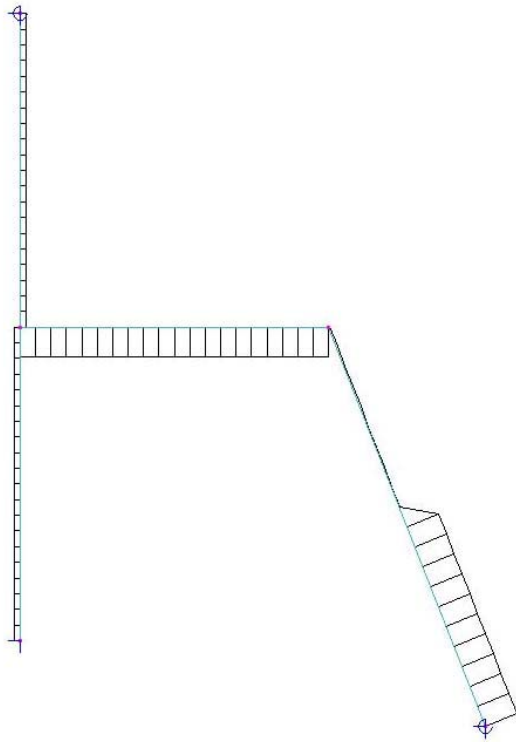
el	N _{ij}	T _{ij}	M _{ij}	N _{ji}	T _{ji}	M _{ji}
1:	-10.2773	-72.5125	-119.723	10.2773	-5.48745	9.13209
2:	10.2773	-43.3136	-75.9348	-10.2773	43.3136	0
3:	48.801	20.5547	66.8027	-48.801	-20.5547	0
4:	-1.28709	52.9375	0	55.7693	85.7445	-74.0204

Reactions:

nd	R _x	R _y	M
1:	-72.5125	10.2773	-119.723
4:	43.3136	10.2773	
5:	100.199	-20.5547	-74.0204

M - dijagramT - dijagram

N - dijagram



Pomaci

