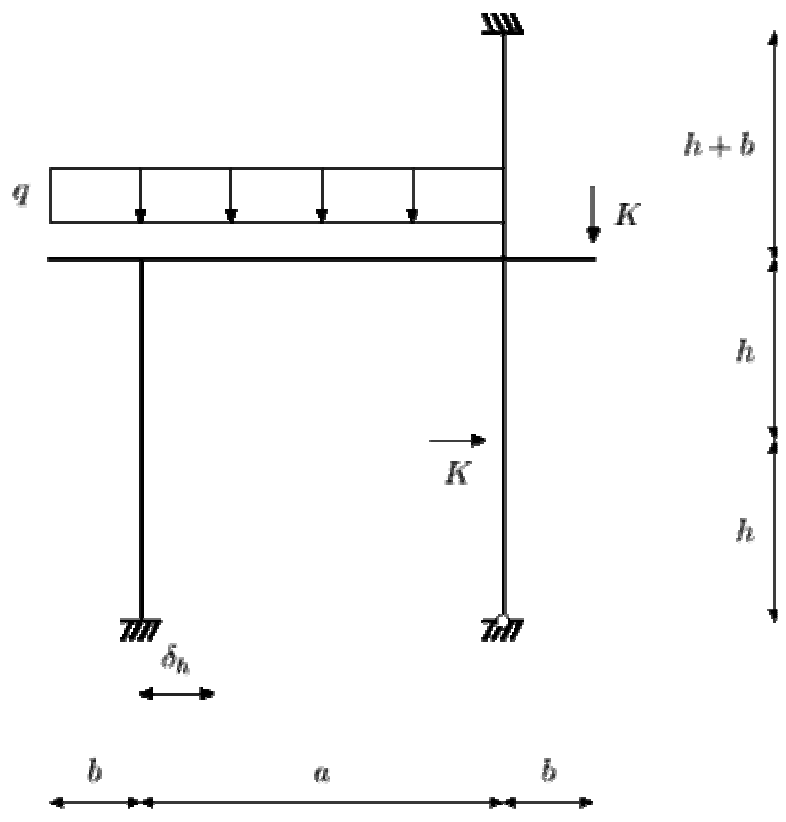


Zadatak

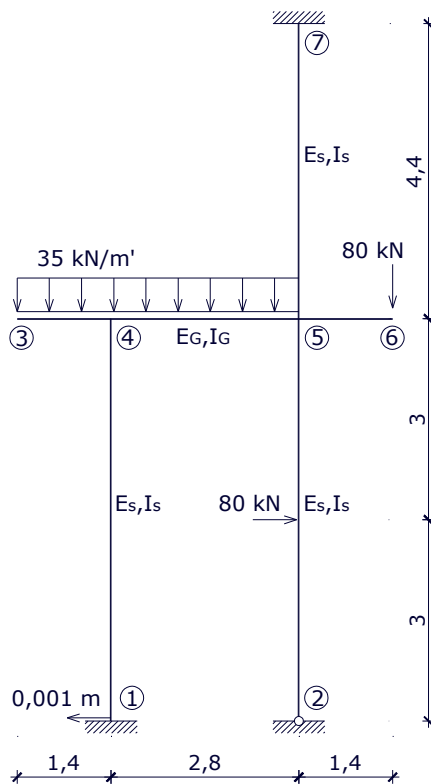
Za zadani sistem metodom pomaka odrediti MTN dijagrame.

Greda: $b/h = 36/72$ [cm], $E = 3 \cdot 10^7$ kN/m²;

Stupovi: $b/h = 36/36$ [cm], $E = 3 \cdot 10^7$ kN/m².



	Student	Zadatak	a	b	h	q	K	$\delta_{n,h}/\varphi_t/t$
32	Vedran Slunjski	2	2.8	1.4	3	35	80	-1



Geometrijske i materijalne karakteristike:

Greda:

$$J_G = \frac{b \cdot h^3}{12} = \frac{0,36 \cdot 0,72^3}{12} = 1119,744 \cdot 10^{-5} m^4$$

$$E_G I_G = 335923,2 kNm^2$$

Stupovi:

$$J_S = \frac{b \cdot h^3}{12} = \frac{0,36 \cdot 0,36^3}{12} = 139,968 \cdot 10^{-5} m^4$$

$$E_S I_S = 41990,4 kNm^2$$

Koeficijenti krutosti:

$$k_{14} = \frac{E_S \cdot I_S}{l_{14}} = \frac{41990,4}{6} = 6998,4 kNm$$

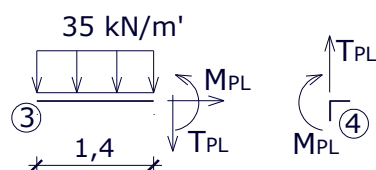
$$k_{25} = \frac{E_S \cdot I_S}{l_{25}} = \frac{41990,4}{6} = 6998,4 kNm$$

$$k_{45} = \frac{E_G \cdot I_G}{l_{45}} = \frac{335923,2}{2,8} = 119972,57 kNm$$

$$k_{57} = \frac{E_S \cdot I_S}{l_{57}} = \frac{41990,4}{4,4} = 9543,27 kNm$$

POJEDNOSTAVLJENJE SISTEMA

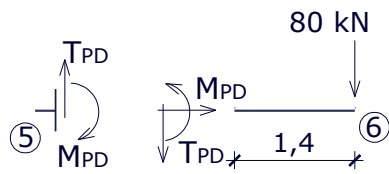
KONZOLA 3-4, djelovanje stavimo u čvor 4



$$M_{PL} = \frac{-35 \cdot 1,4^2}{2} = -34,3 kNm$$

$$T_{PL} = -35 \cdot 1,4 = -49,0 kN$$

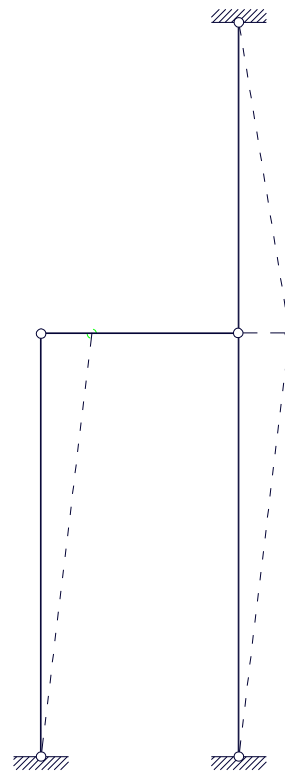
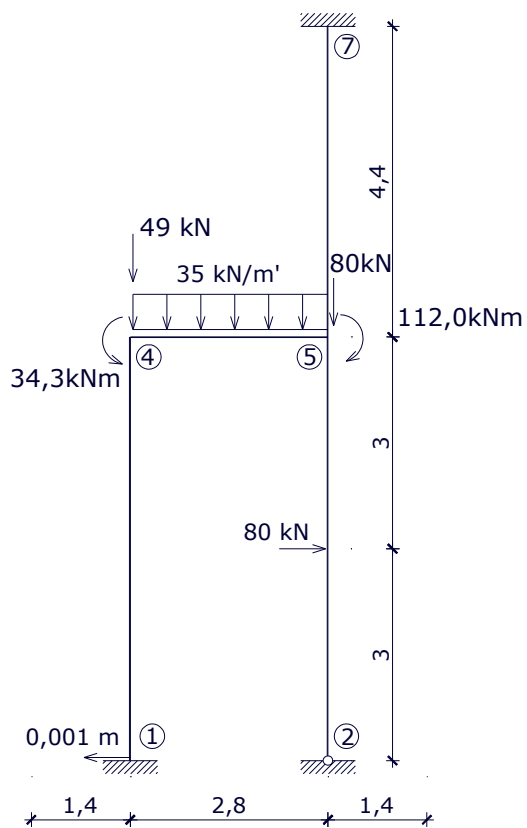
KONZOLA 5-6, djelovanje stavimo u čvor 5



$$M_{PD} = 80 \cdot 1,4 = 112 \text{ kNm}$$

$$T_{Pd} = -80,0 \text{ kN}$$

POJEDNOSTAVLJENI SISTEM – ZGLOBNA SCHEMA



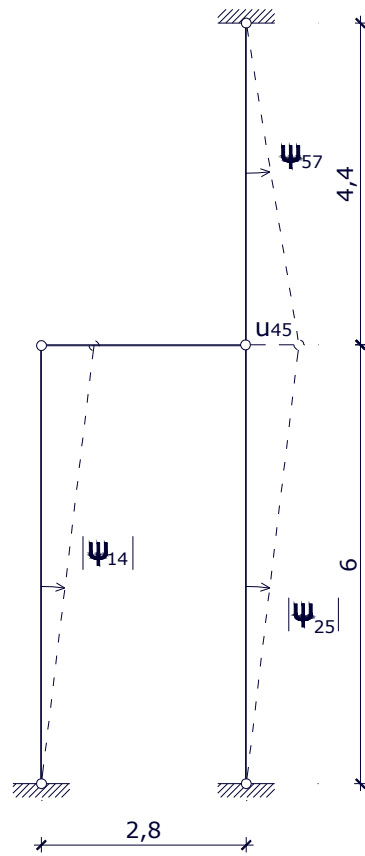
$$s = 2n_C - n_S$$

$$s = 4 - 4 = 0$$

zglobna shema ima 1 stupanj slobode

NEPOZNANICE: φ_4 ; φ_5 ; u_{45}

PLAN POMAKA

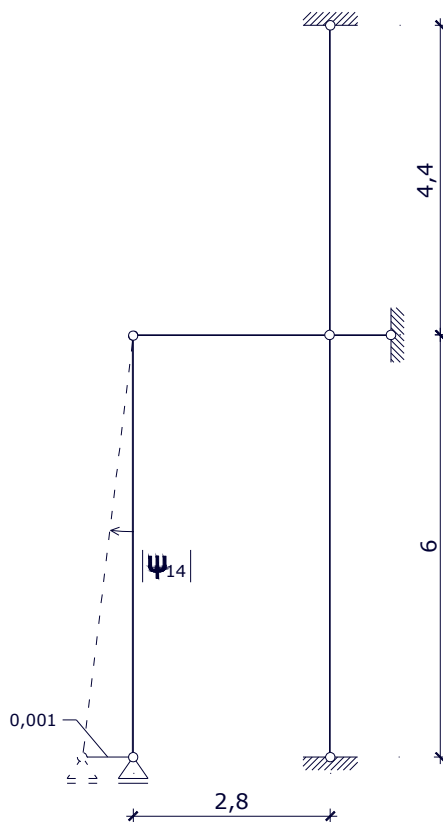


$$\psi_{14} = \frac{-u_{45}}{l_{14}} = \frac{-u_{45}}{6}$$

$$\psi_{25} = \frac{-u_{45}}{l_{25}} = \frac{-u_{45}}{6}$$

$$\psi_{57} = \frac{u_{45}}{l_{57}} = \frac{u_{45}}{4,4}$$

PLAN POMAKA OD PRISILNOG POMAKA



$$\psi_{14} = \frac{-0,001}{6}$$

MOMENTI NA KRAJEVIMA ELEMENATA

$$M_{14} = \overline{M}_{14} + m_{14} = -6k_{14}\overline{\psi}_{14} + 2k_{14}\varphi_4 - 6k_{14}\psi_{14} = \frac{-6k_{14}(-0,001)}{6} + 2k_{14}\varphi_4 - 6k_{14}\left(\frac{-u_{45}}{6}\right)$$

$$M_{14} = 13996,8\varphi_4 + 6998,4u_{45} + 6,9984$$

$$M_{41} = \overline{M}_{41} + m_{41} = -6k_{14}\overline{\psi}_{14} + 4k_{14}\varphi_4 - 6k_{14}\psi_{14} = \frac{-6k_{14}(-0,001)}{6} + 4k_{14}\varphi_4 - 6k_{14}\left(\frac{-u_{45}}{6}\right)$$

$$M_{41} = 27993,6\varphi_4 + 6998,4u_{45} + 6,9984$$

$$M_{45} = \overline{M}_{45} + m_{45} = \frac{g \cdot l^2}{12} + 4k_{45}\varphi_4 + 2k_{45}\varphi_5 = \frac{35 \cdot 2,8^2}{12} + 4k_{45}\varphi_4 + 2k_{45}\varphi_5$$

$$M_{45} = 479890,28\varphi_4 + 239945,15\varphi_5 + 22,867$$

$$M_{54} = \overline{M}_{54} + m_{54} = -\frac{g \cdot l^2}{12} + 2k_{45}\varphi_4 + 4k_{45}\varphi_5 = -\frac{35 \cdot 2,8^2}{12} + 2k_{45}\varphi_4 + 4k_{45}\varphi_5$$

$$M_{54} = 239945,14\varphi_4 + 479890,2856\varphi_5 - 22,867$$

$$M_{52}^c = \overline{M}_{52}^c + m_{52}^c = -\frac{P \cdot l}{8} - \frac{1}{2} \cdot \frac{P \cdot l}{8} + 3k_{25}\varphi_5 - 3k_{25}\psi_{25} = -90 + 3k_{25}\varphi_5 - 3k_{25}\left(\frac{-u_{45}}{6}\right)$$

$$M_{52}^c = 20995,2\varphi_5 + 3499,2u_{45} - 90,0$$

$$M_{57} = \overline{M}_{57} + m_{57} = 4k_{57}\varphi_5 - 6k_{57}\psi_{57} = 4k_{57}\varphi_5 - 6k_{57}\left(\frac{u_{45}}{4,4}\right)$$

$$M_{57} = 38173,09\varphi_5 - 13013,55u_{45}$$

$$M_{75} = \overline{M}_{75} + m_{75} = 2k_{57}\varphi_5 - 6k_{57}\psi_{57} = 4k_{57}\varphi_5 - 6k_{57}\left(\frac{u_{45}}{4,4}\right)$$

$$M_{75} = 19086,55\varphi_5 - 13013,55u_{45}$$

JEDNADŽBE RAVNOTEŽE

ČVORA 4

$$\sum M = 0 \Rightarrow -M_{41} - M_{45} + M_{PL} = 0$$

$$27993,6\varphi_4 + 6998,4u_{45} + 6,9984 + 479890,28\varphi_4 + 239945,15\varphi_5 - 34,3 = 0$$

$$507883,89\varphi_4 + 239945,15\varphi_5 + 6998,4u_{45} = 4,435 \dots \dots \dots (1)$$

ČVORA 5

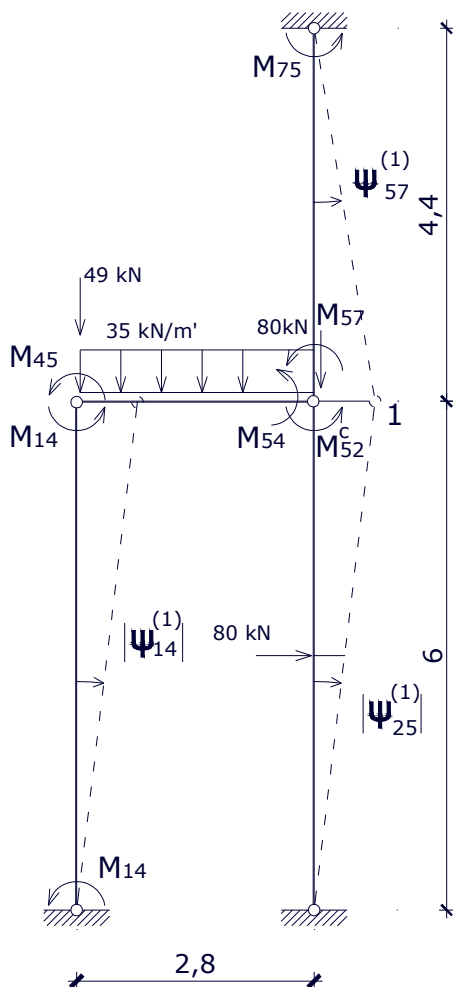
$$\sum M = 0 \Rightarrow -M_{52}^c - M_{54} - M_{57} - M_{Pd} = 0$$

$$20995,2\varphi_5 + 3499,2u_{45} - 90,0 + 239945,14\varphi_4 + 479890,2856\varphi_5 - 22,867 +$$

$$38173,09\varphi_5 - 13013,55u_{45} + 112 = 0$$

$$239945,14\varphi_4 + 539058,58\varphi_5 - 9514,35u_{45} = 0,86 \dots \dots \dots (2)$$

VIRTUALNOG RADA



$$\psi_{14}^1 = \frac{-1}{l_{14}} = \frac{-1}{6}$$

$$\psi_{25}^1 = \frac{-1}{l_{25}} = \frac{-1}{6}$$

$$\psi_{57}^1 = \frac{1}{l_{57}} = \frac{1}{4,4}$$

$$\sum W = 0 \Rightarrow M_{14}\psi_{14}^1 + M_{41}\psi_{14}^1 + M_{57}\psi_{57}^1 + M_{75}\psi_{57}^1 + M_{52}^c\psi_{25}^1 + P \cdot 0,5 = 0$$

$$(13996,8\varphi_4 + 6998,4u_{45} + 6,9984 + 27993,6\varphi_4 + 6998,4u_{45} + 6,9984)\left(\frac{-1}{6}\right) +$$

$$(38173,09\varphi_5 - 13013,55u_{45} + 19086,55\varphi_5 - 13013,55u_{45})\left(\frac{1}{4,4}\right) +$$

$$(20995,2\varphi_5 + 3499,2u_{45} - 90,0)\left(\frac{-1}{6}\right) + 40 = 0$$

$$-6998,4\varphi_4 + 9514,35\varphi_5 - 8831,25u_{45} = -52,67 \quad / \quad (-1)$$

$$6998,4\varphi_4 - 9514,35\varphi_5 + 8831,25u_{45} = 52,67 \dots\dots\dots(3)$$

KONAČNI SUSTAV JEDNADŽBI

$$\begin{bmatrix} 507883,88 & 239945,14 & 6998,4 \\ 239945,14 & 539058,58 & -9514,35 \\ 6998,4 & -9514,35 & 9122,85 \end{bmatrix} \begin{bmatrix} \varphi_4 \\ \varphi_5 \\ u_{45} \end{bmatrix} = \begin{bmatrix} 4,439 \\ 0,867 \\ 52,667 \end{bmatrix}$$

RJEŠENJE SUSTAVA JEDNADŽBI

$$\{\{\varphi_4 \rightarrow -0.000160298, \quad \varphi_5 \rightarrow 0.000180345, \quad u_{45} \rightarrow 0.00608416\}\}$$

KONAČNI MOMENTI NA KRAJEVIMA ELEMENATA

$$M_{14} = 13996,8(-0.000160298) + 6998,4(0.00608416) + 6,9984 = 47,33kNm$$

$$M_{41} = 27993,6(-0.000160298) + 6998,4(0.00608416) + 6,9984 = 45,09kNm$$

$$M_{45} = 479890,28(-0.000160298) + 239945,15(0.000180345) + 22,867 = -10,79kNm$$

$$M_{54} = 239945,14(-0.000160298) + 479890,2856(0.000180345) - 22,867 = 25,22kNm$$

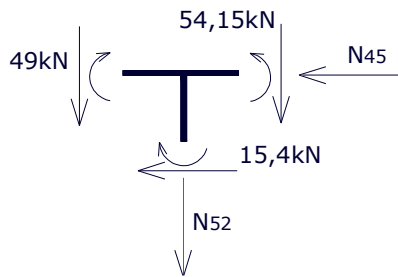
$$M_{52}^c = 20995,2(0.000180345) + 3499,2(0.00608416) - 90,0 = -64,92kNm$$

$$M_{57} = 38173,09(0.000180345) - 13013,55(0.00608416) = -72,29kNm$$

$$M_{75} = 19086,55(0.000180345) - 13013,55(0.00608416) = -75,73kNm$$

ODREĐIVANJE UZDUŽNIH SILA

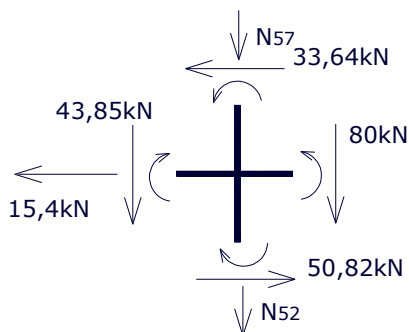
RAVNOTEŽA ČVORA 4



$$N_{45} = -15,4 \text{ kN}$$

$$N_{41} = -49 - 54,15 = -103,15 \text{ kN}$$

RAVNOTEŽA ČVORA 5



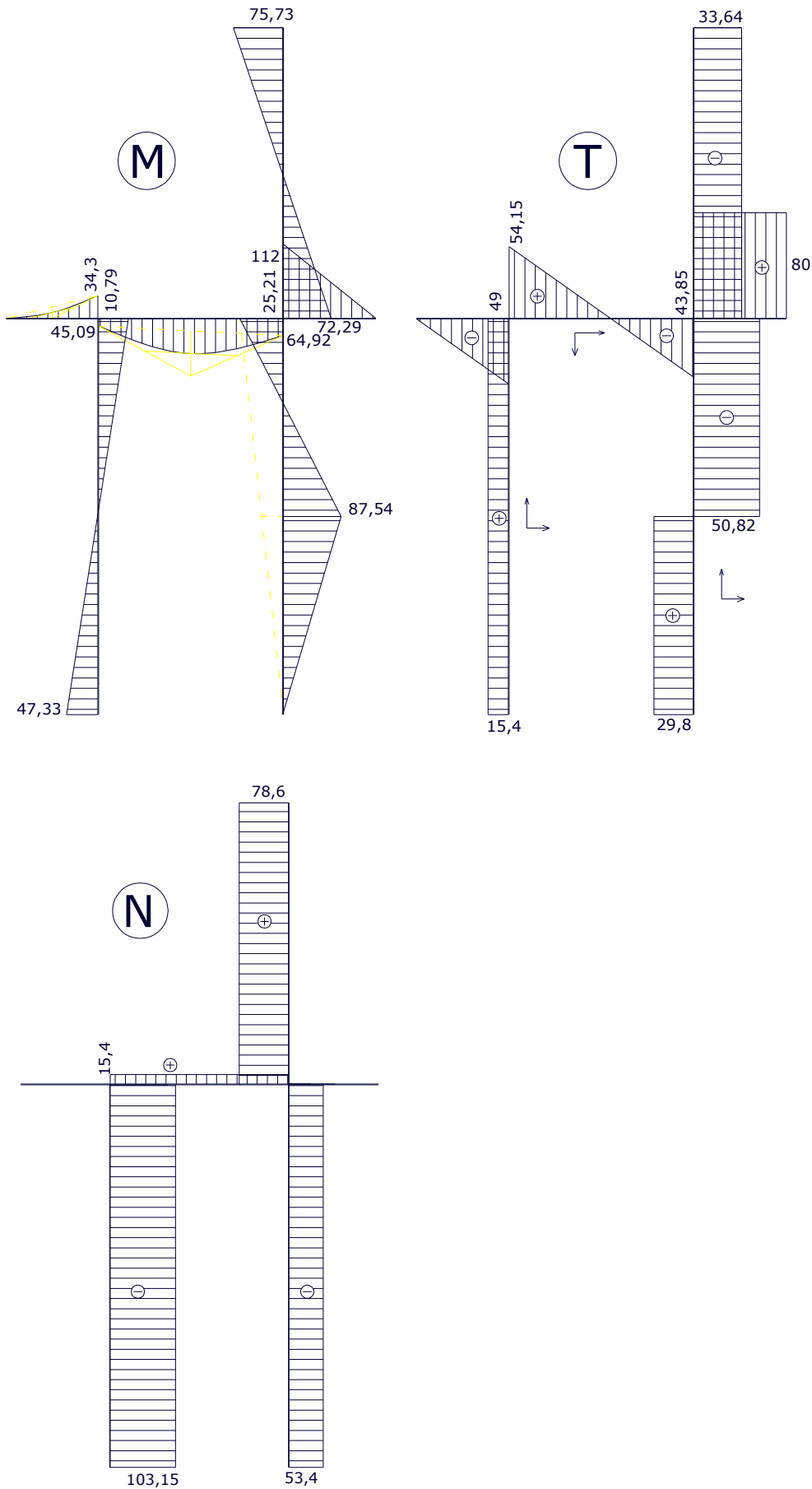
$$\frac{N_{57}}{N_{52}} = \frac{EA}{l_{57}} \cdot \frac{l_{25}}{EA} = \frac{6}{4,4}$$

$$N_{52} + \frac{6}{4,4} N_{52} = -80 - 43,85$$

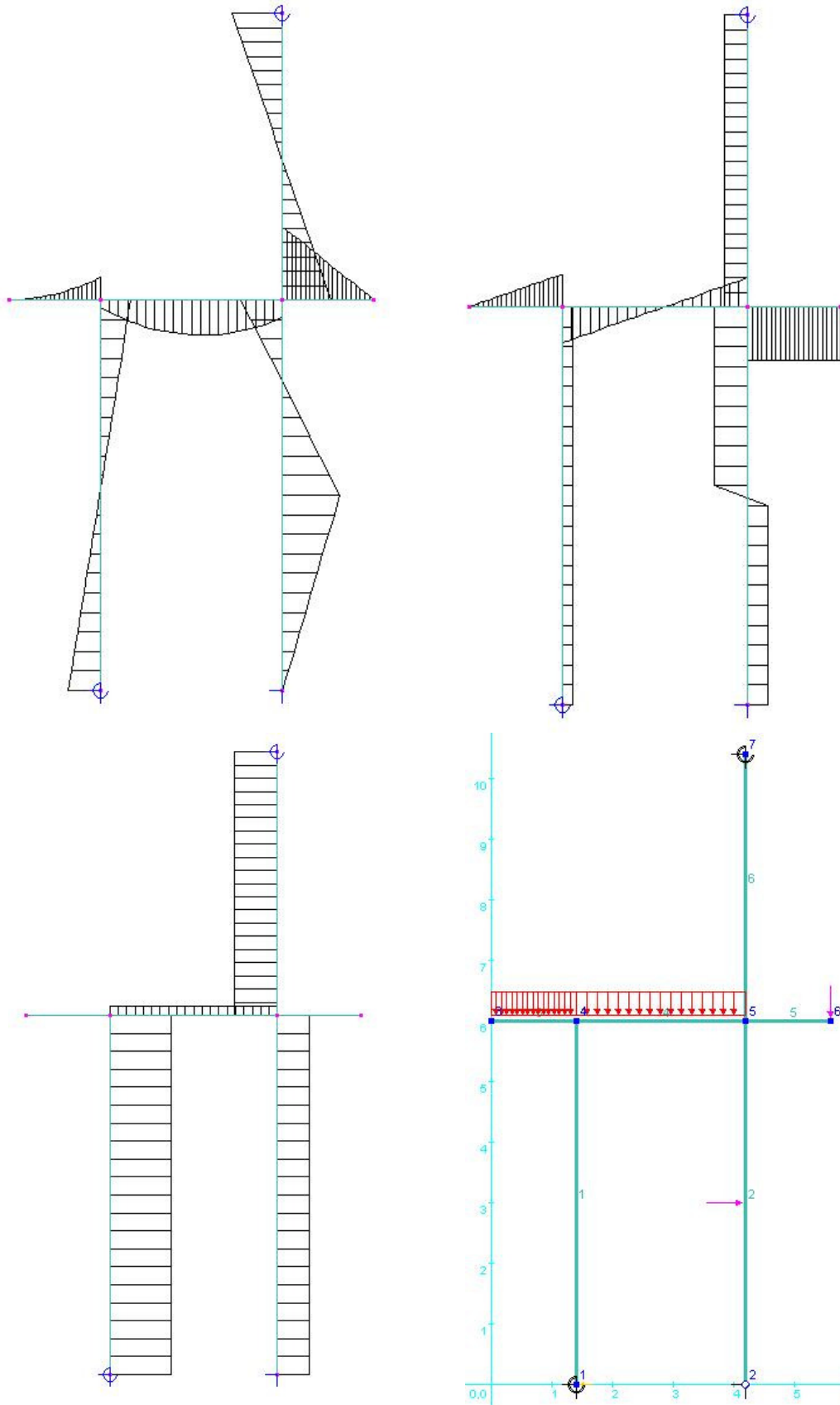
$$N_{52} = -53,40 \text{ kN}$$

$$N_{57} = \frac{6}{4,4} (-53,4) = -78,6 \text{ kN}$$

DIJAGRAMI M,T,N



RJEŠENJE U DIM-u



Degrees of Freedom: 12

Nodal displacements:

nd	u _i	v _i	phi _i
1:	-0.001	0	0
2:	0	0	0
3:	0.00629668	0.000182733	-0.000118612
4:	0.00629668	-1.03267e-09	-0.000166262
5:	0.00629668	-5.23485e-10	0.000186751
6:	0.00629668	4.3622e-05	-4.66369e-05
7:	0	0	0

Element end forces:

el	N _{ij}	T _{ij}	M _{ij}	N _{ji}	T _{ji}	M _{ji}
1:	103.267	15.8581	48.738	-103.267	-15.8581	46.4108
2:	52.3485	29.3257	0	-52.3485	50.6743	-64.0458
3:	0	0	0	0	49	-34.3
4:	-15.8581	54.2673	-12.1108	15.8581	43.7327	26.8591
5:	0	80	112	0	-80	0
6:	-71.3843	-34.8162	-74.8134	71.3843	34.8162	-78.3778

Reactions:

nd	R _x	R _y	M
1:	-15.8581	103.267	48.738
2:	-29.3257	52.3485	
7:	-34.8162	71.3843	-78.3778