UTJECAJNE LINIJE

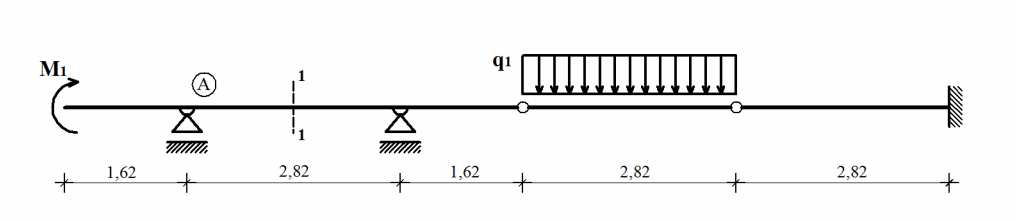
Zadatak C1

a) GERBEROV NOSAČ

Pomoću utjecajnih linija odredite unutarnje sile M i T u presjeku 1-1 te reakciju u ležaju A.

$M_1 = 92$ kNm

$q_1 = 30$ kN/m

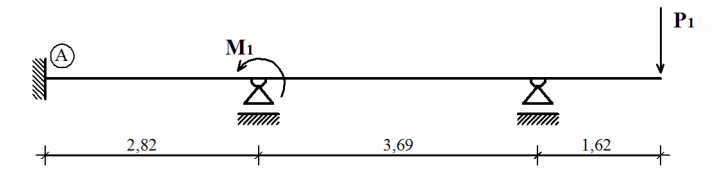


b) KONTINUIRANI NOSAČ

Pomoću utjecajnih linija odredite moment i vertikalnu reakciju u ležaju A.

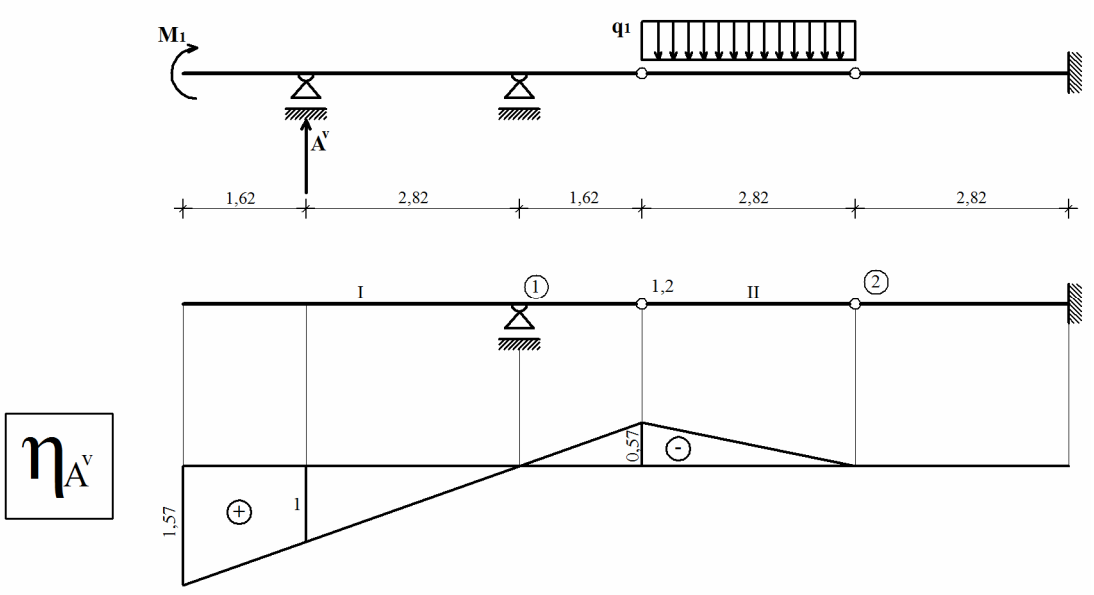
$M_1 = 92$ kNm

$P_1 = 80$ kN/m



a) **GERBEROV NOSAČ**

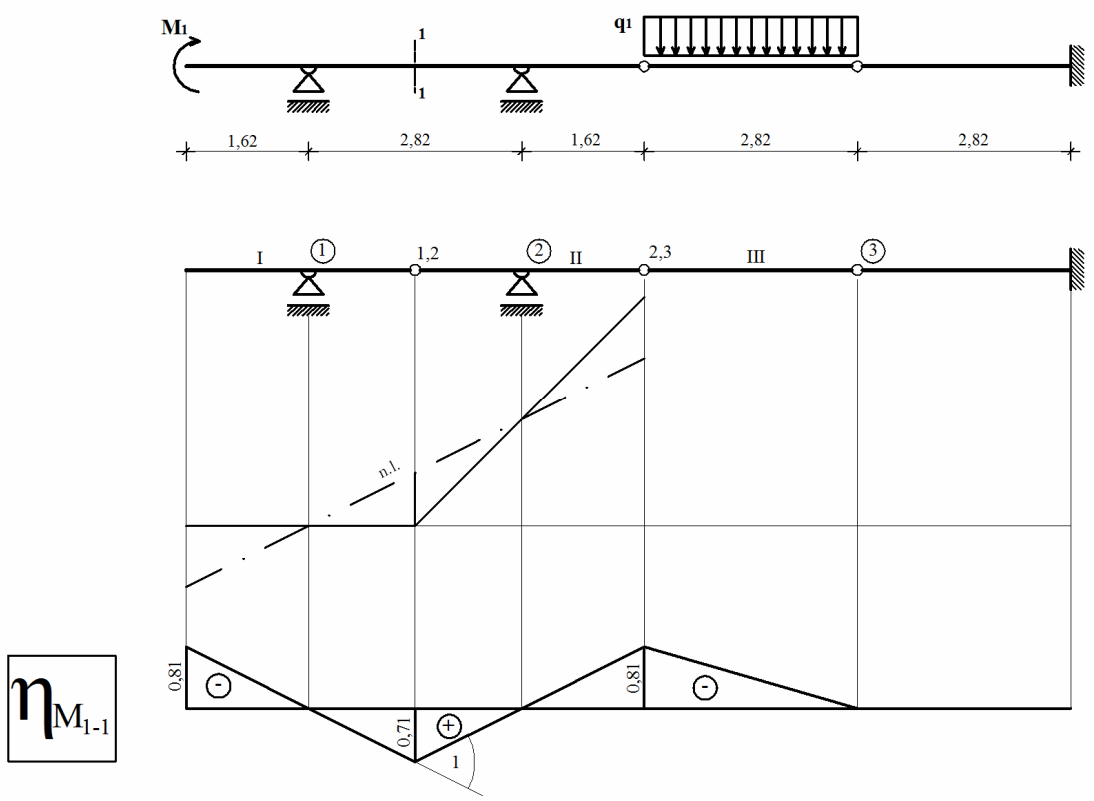
REAKCIJA A^v



$$A^v(M_1, q_1) = -M_1 \cdot tg\alpha + Q \cdot \eta_Q$$

$$A^v = -92 \cdot \tfrac{1}{2,82} + (30 \cdot 2,82) \cdot (-\tfrac{0,57}{2}) = -56,74\,\text{kN}$$

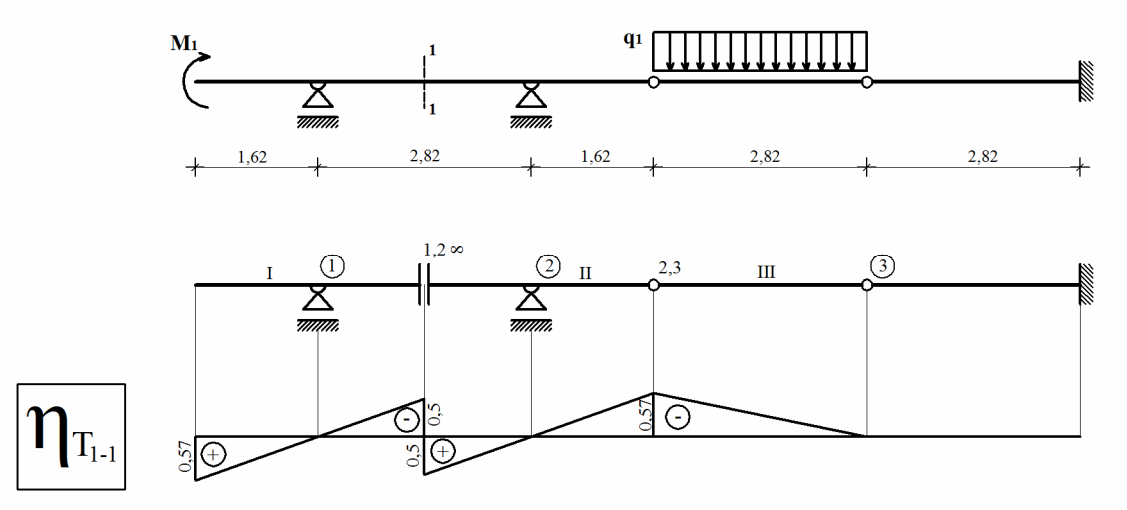
MOMENT U PRESJEKU 1-1 (M_{1-1})



$$M_{1-1}(M_1, q_1) = -M_1 \cdot tg\alpha + Q \cdot \eta_Q$$

$$M_{1-1} = 92 \cdot \tfrac{0,81}{1,62} + (30 \cdot 2,82) \cdot (-\tfrac{0,81}{2}) = 11,74\,\text{kNm}$$

POPREČNA SILA U PRESJEKU 1-1 (T_{1-1})

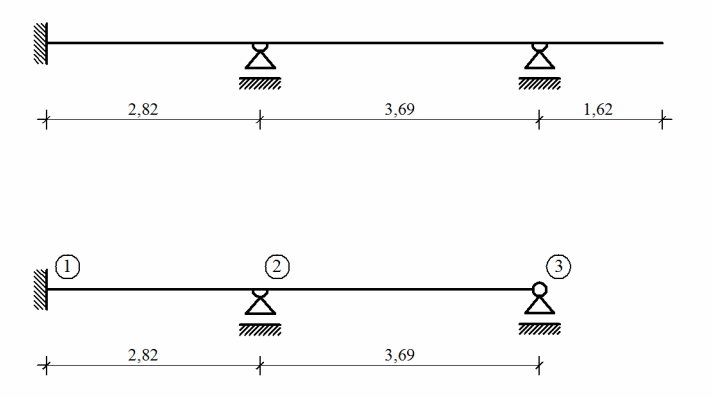


$$T_{1-1}(M_1, q_1) = -M_1 \cdot tg\alpha + Q \cdot \eta_Q$$

$$T_{1-1} = -92 \cdot \frac{0,57}{1,62} + (30 \cdot 2,82) \cdot (-\frac{0,57}{2}) = -56,48\,\text{kN}$$

b) KONTINUIRANI NOSAČ

CROSSOV POSTUPAK



- koeficijenti krutosti elemenata:

$$k_{12} = \frac{EI}{2,82}$$

$$k_{23} = \frac{EI}{3,69}$$

- koeficijent krutosti čvora 2:

$$k_2 = k_{12} + \frac{3}{4} \cdot k_{23} = EI \cdot \left(\frac{1}{2,82} + \frac{3}{4} \cdot \frac{1}{3,69}\right) = 0,56EI$$

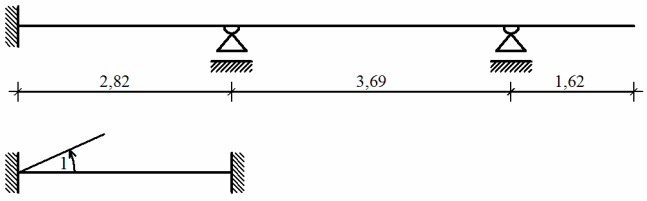
- razdjelni koeficijenti:

čvor 2 $\qquad \mu_{21} = \frac{k_{12}}{k_2} = \frac{\frac{EI}{2,82}}{0,56EI} = 0,63 \rightarrow 0,64$

$$\mu_{21} = \frac{\frac{3}{4}k_{12}}{k_2} = \frac{\frac{3}{4}\frac{EI}{3,69}}{0,56EI} = 0,36$$

$$\sum \mu = 1,0$$

MOMENT M_A



$\varphi_1 = 1$

- momenti upetosti:

$$\overline{M}_{12} = 4k_{12}\varphi_1 = 4 \cdot \frac{EI}{2,82} \cdot 1 = 1,42 \cdot EI$$

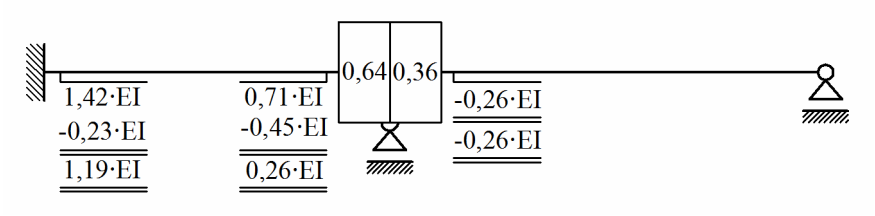
$$\overline{M}_{21} = 4k_{12}\varphi_1 = 2 \cdot \frac{EI}{2,82} \cdot 1 = 0,71 \cdot EI$$

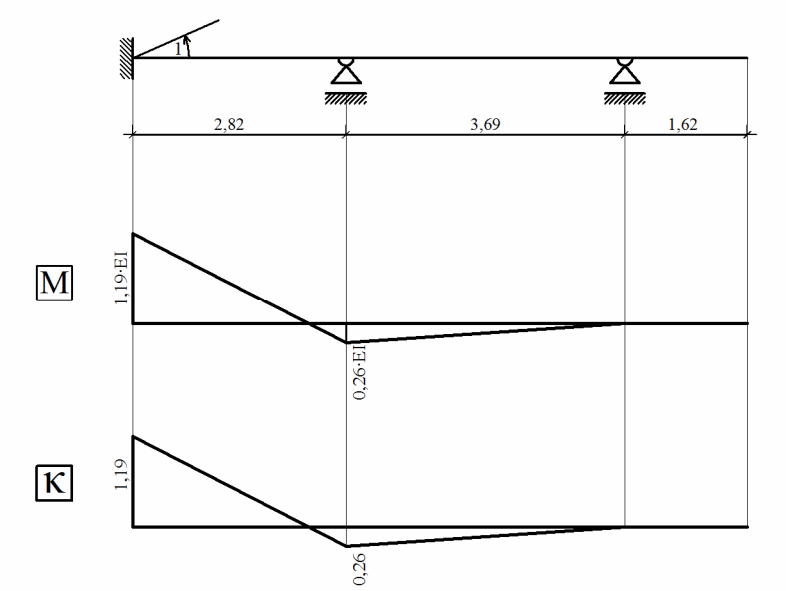
čvor **2** $\Delta M_{neurav.} = 0{,}71\cdot EI$

$-0{,}71\cdot EI \quad \cdot 0{,}64 = -0{,}45\cdot EI$

$-0{,}71\cdot EI \quad \cdot 0{,}36 = -0{,}26\cdot EI$

$-0{,}71\cdot EI$





$\kappa = \frac{M}{EI}$

- kutovi:

$$\varphi_1 = \tfrac{1}{2}\cdot 1{,}19\cdot 2{,}31 = 1{,}37$$
$$\varphi_2 = \tfrac{1}{2}\cdot 0{,}26\cdot 0{,}51 = 0{,}07$$
$$\varphi_3 = \tfrac{1}{2}\cdot 0{,}26\cdot 3{,}69 = 0{,}48$$

- mjerilo dužina: $1\,[cm] = \frac{L}{8{,}13}\,[m] \rightarrow \mathrm{m} = \frac{L}{8{,}13}$

- mjerilo kutova: $1\,[cm] = \frac{1}{6}\,[m^0] \rightarrow \mathrm{k} = \frac{1}{6}$

- kutovi:

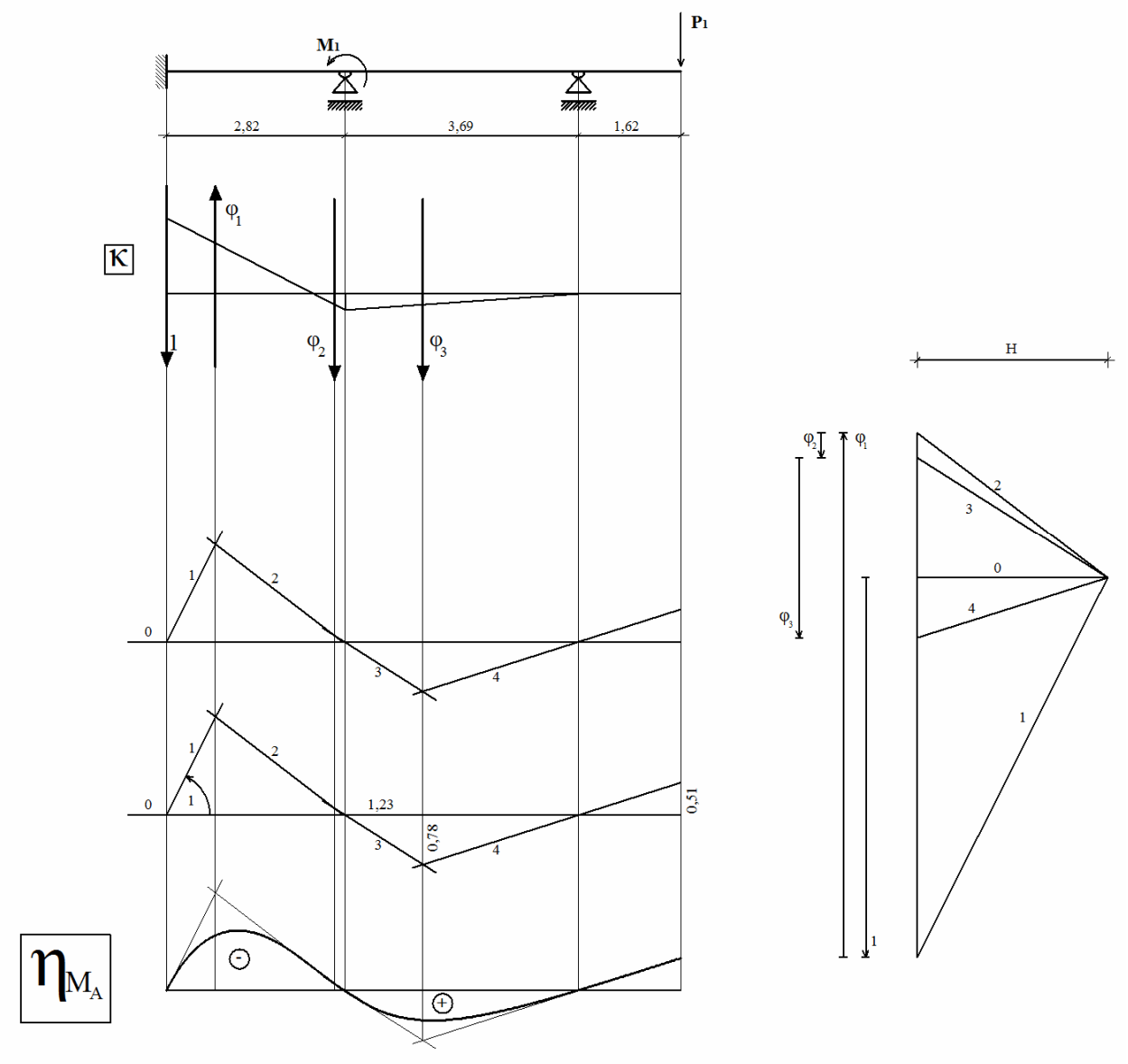
$$\varphi_1^* = \frac{\varphi_1}{k} = \frac{1{,}37}{\frac{1}{6}} = 8{,}22\ \mathrm{cm}$$
$$\varphi_2^* = \frac{\varphi_2}{k} = \frac{0{,}07}{\frac{1}{6}} = 0{,}42\ \mathrm{cm}$$
$$\varphi_3^* = \frac{\varphi_3}{k} = \frac{0{,}48}{\frac{1}{6}} = 2{,}88\ \mathrm{cm}$$
$$1^* = \frac{1}{k} = \frac{1}{\frac{1}{6}} = 6\ \mathrm{cm}$$

- polna udaljenost:

$$H^* = \frac{H}{k} \qquad H = H^* \cdot k \qquad n = \frac{1}{H}$$

$$H^* = 3 \text{ cm} \qquad H = 3 \cdot \frac{1}{6} \qquad n = \frac{1}{\frac{1}{2}}$$

$$H = \frac{1}{2} \qquad n = 2$$



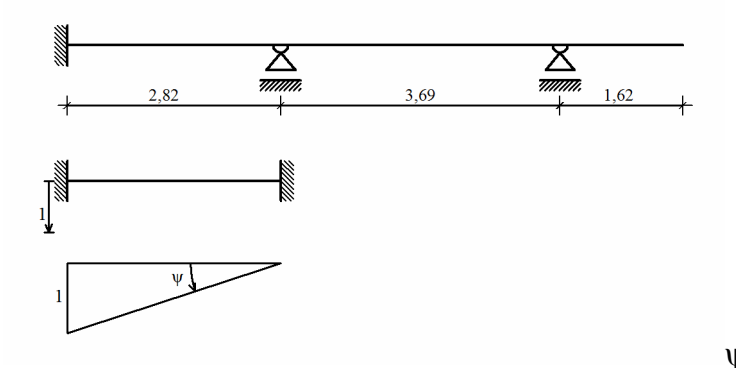
$$M_A(M_1, P_1) = -M_1 \cdot tg\alpha + P \cdot \eta_P$$

$$M_A = -M_1 \cdot tg\alpha^* \cdot \frac{\frac{m}{n}}{m} + P \cdot \eta_p^* \cdot \frac{m}{n}$$

$$M_A = -92 \cdot \frac{0,78}{1,23} \cdot \frac{1}{2} - 80 \cdot 0,51 \cdot \frac{1}{2}$$

$$M_A = -49,57 \text{ kNm}$$

REAKCIJA R_A



$$\psi_{12} = \frac{1}{2,82}$$

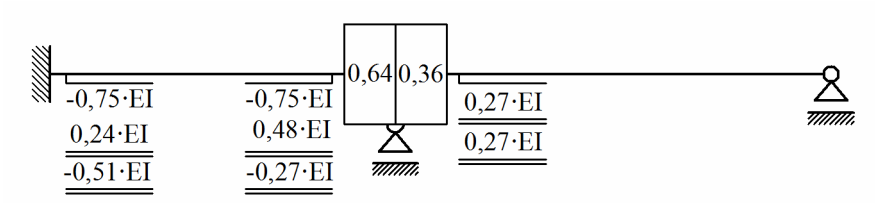
- momenti upetosti:

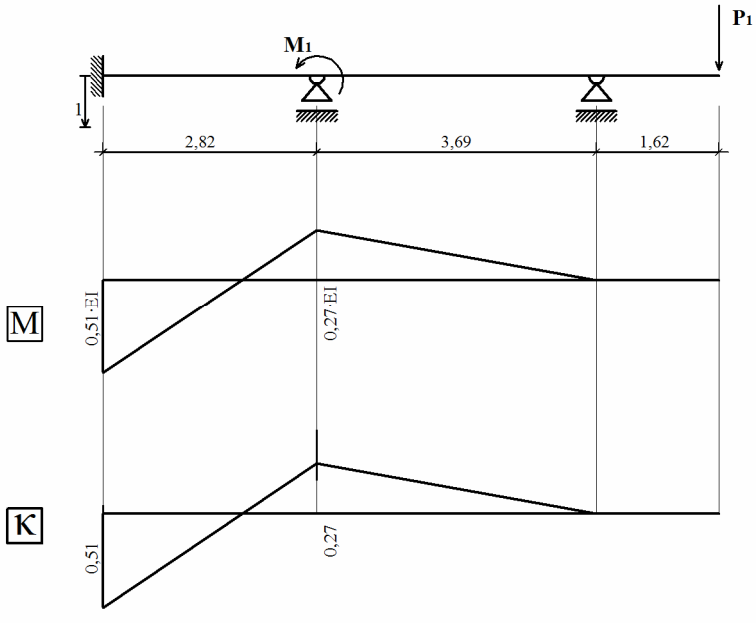
$$\overline{M}_{12} = -6k_{12}\psi_{12} = -6 \cdot \frac{EI}{2,82} \cdot \frac{1}{2,82} = -0,75 \cdot EI$$

$$\overline{M}_{21} = -6k_{12}\psi_{12} = -6 \cdot \frac{EI}{2,82} \cdot \frac{1}{2,82} = -0,75 \cdot EI$$

čvor **2** $\Delta M_{neurav.} = -0,75 \cdot EI$

$$0,75 \cdot EI \cdot 0,64 = 0,48 \cdot EI$$
$$0,75 \cdot EI \cdot 0,36 = 0,27 \cdot EI$$
$$0,75 \cdot EI$$





$$\kappa = \frac{M}{EI}$$

- kutovi:

$$\varphi_1 = \tfrac{1}{2} \cdot 0{,}51 \cdot 1{,}84 = 0{,}47$$
$$\varphi_2 = \tfrac{1}{2} \cdot 0{,}27 \cdot 0{,}98 = 0{,}13$$
$$\varphi_3 = \tfrac{1}{2} \cdot 0{,}27 \cdot 3{,}69 = 0{,}50$$

- mjerilo dužina: $1\,[cm] = \frac{L}{8{,}13}\ [m] \ \rightarrow \ \text{m} = \frac{L}{8{,}13}$

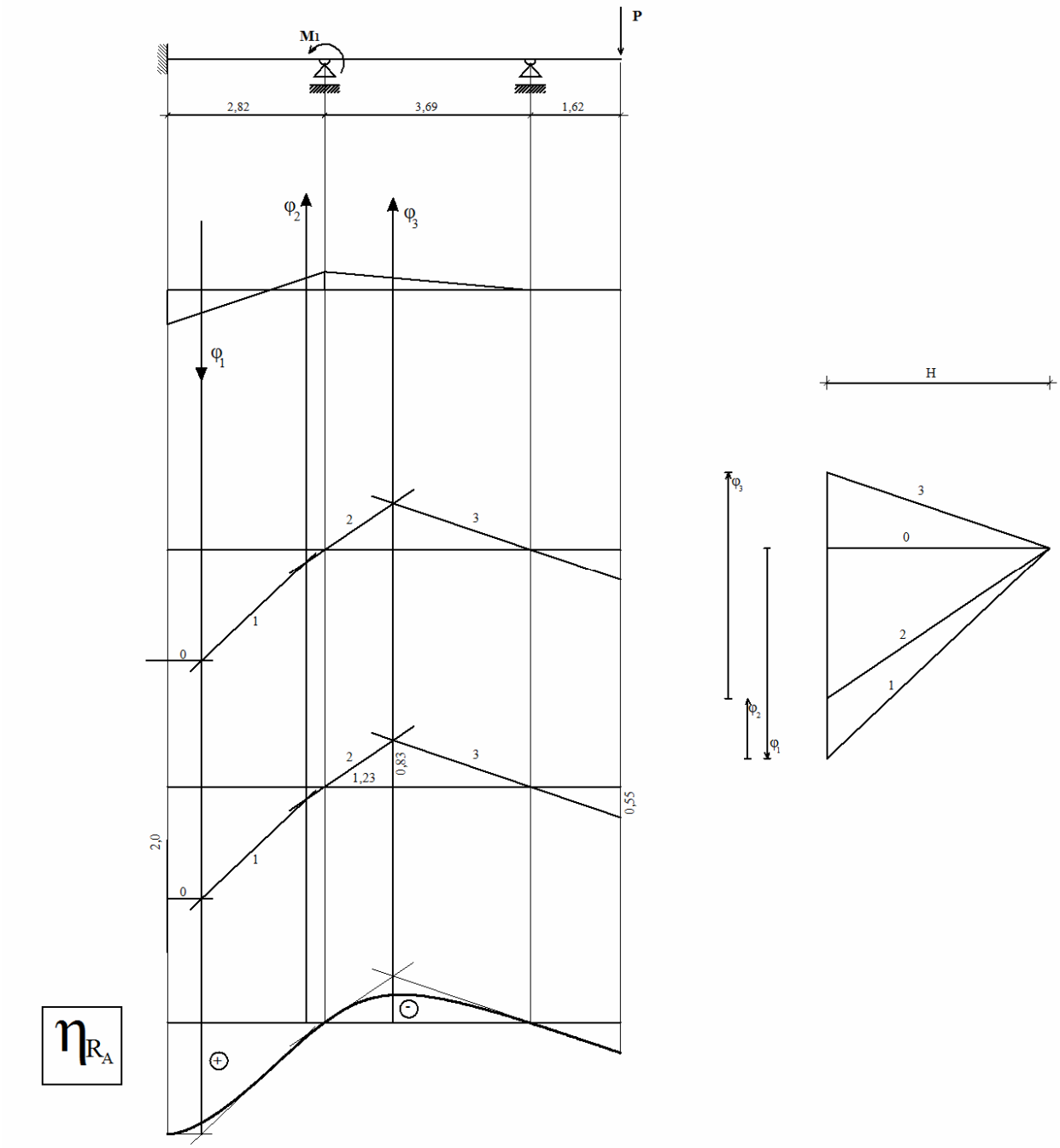
- mjerilo kutova: $1\,[cm] = \frac{1}{8}\ [m^0] \ \rightarrow \ \text{k} = \frac{1}{8}$

- kutovi:

$$\varphi_1^* = \frac{\varphi_1}{k} = \frac{0{,}47}{\frac{1}{8}} = 3{,}76 \text{ cm}$$
$$\varphi_2^* = \frac{\varphi_2}{k} = \frac{0{,}13}{\frac{1}{8}} = 1{,}04 \text{ cm}$$
$$\varphi_3^* = \frac{\varphi_3}{k} = \frac{0{,}50}{\frac{1}{8}} = 4{,}0 \text{ cm}$$

- polna udaljenost:

$$H^* = \tfrac{H}{k} \qquad H = H^* \cdot k \qquad n = \tfrac{1}{H}$$
$$H^* = 4 \text{ cm} \qquad H = 4 \cdot \tfrac{1}{8} \qquad n = \tfrac{1}{\frac{1}{2}}$$
$$H = \tfrac{1}{2} \qquad n = 2$$



η_{R_A}

$$M_A(M_1, P_1) = -M_1 \cdot tg\alpha + P \cdot \eta_P$$

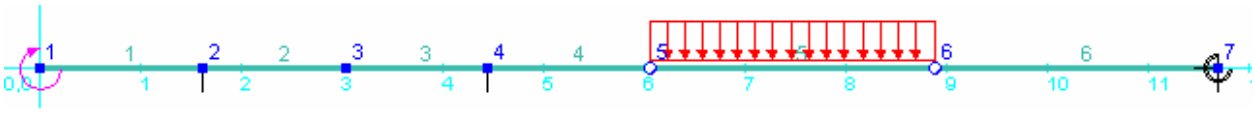
$$M_A = -M_1 \cdot tg\alpha^* \cdot \frac{\frac{m}{n}}{m} + P \cdot \eta_p^* \cdot \frac{m}{n}$$

$$M_A = 92 \cdot \frac{0,83}{1,23} \cdot \frac{1}{2} + 80 \cdot 0,55 \cdot \frac{1}{2}$$

$$M_A = 53,04\ \text{kNm}$$

REZULTATI – DIM:

a) GERBEROV NOSAČ



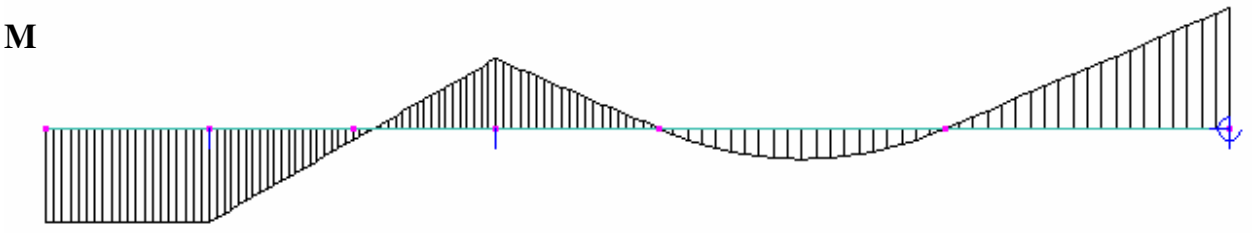
Nodes:

Label	x	y
1	0	0
2	1.62	0
3	3.03	0
4	4.44	0
5	6.06	0
6	8.88	0
7	11.7	0

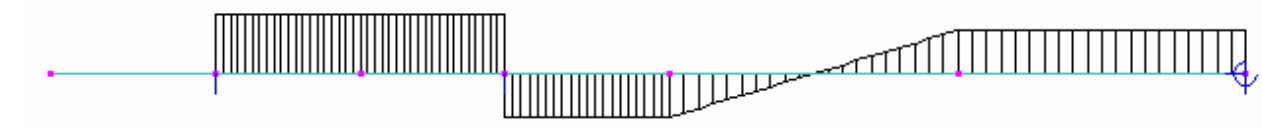
Elements:

Label	1st node	2nd node	Characteristics
1	1	2	G
2	2	3	G
3	3	4	G
4	4	5	G
5	5	6	G
6	6	7	G

M



T



Element end forces:

el	N_ij	T_ij	M_ij	N_ji	T_ji	M_ji
1:	0	0	-92	0	0	92
2:	0	-56.9241	-92	0	56.9241	11.737
3:	0	-56.9241	-11.737	0	56.9241	-68.526
4:	0	42.3	68.526	0	-42.3	0
5:	0	42.3	0	0	42.3	0
6:	0	-42.3	0	0	42.3	-119.286

Reactions:

nd	R_x	R_y	M
2:		-56.9241	
4:		99.2241	
7:	0	42.3	-119.286

b) KONTINUIRANI NOSAČ

Nodes:

Label	x	y
1	0	0
2	2.82	0
3	6.51	0
4	8.13	0

Elements:

Label	1st node	2nd node	Characteristics
1	1	2	K
2	2	3	K
3	3	4	K

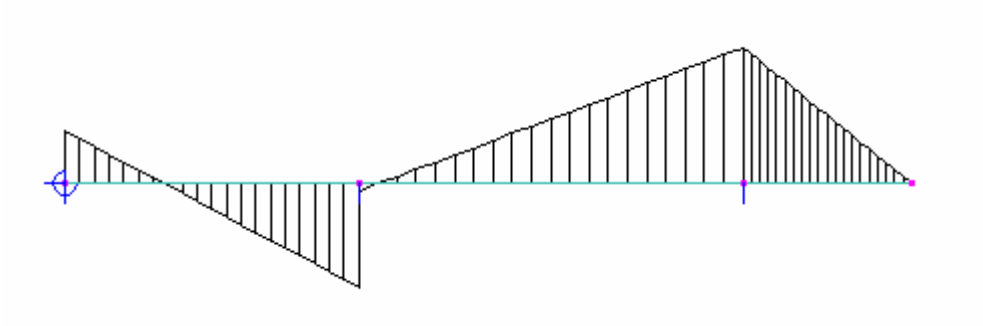
Element end forces:

el	N_ij	T_ij	M_ij	N_ji	T_ji	M_ji
1:	0	53.0167	49.8357	0	-53.0167	99.6713
2:	0	-37.2009	-7.67132	0	37.2009	-129.6
3:	0	80	129.6	0	0	0

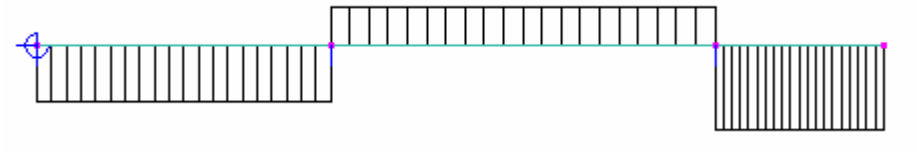
Reactions:

nd	R_x	R_y	M
1:	0	53.0167	49.8357
2:		-90.2176	
3:		117.201	

M



T



Usporedba rezultata:

	SILA	UL	DIM
GERBEROV NOSAČ	$\mathbf{A^v}$	- 56,74 kN	- 56,92 kN
	$\mathbf{M_{1-1}}$	11,74 kNm	11,74 kNm
	$\mathbf{T_{1-1}}$	- 56,48 kN	- 56,92 kN
KONTINUIRANI NOSAČ	$\mathbf{M_A}$	- 49,57 kNm	- 49,84 kNm
	$\mathbf{R_A}$	53,04 kN	53,02 kN