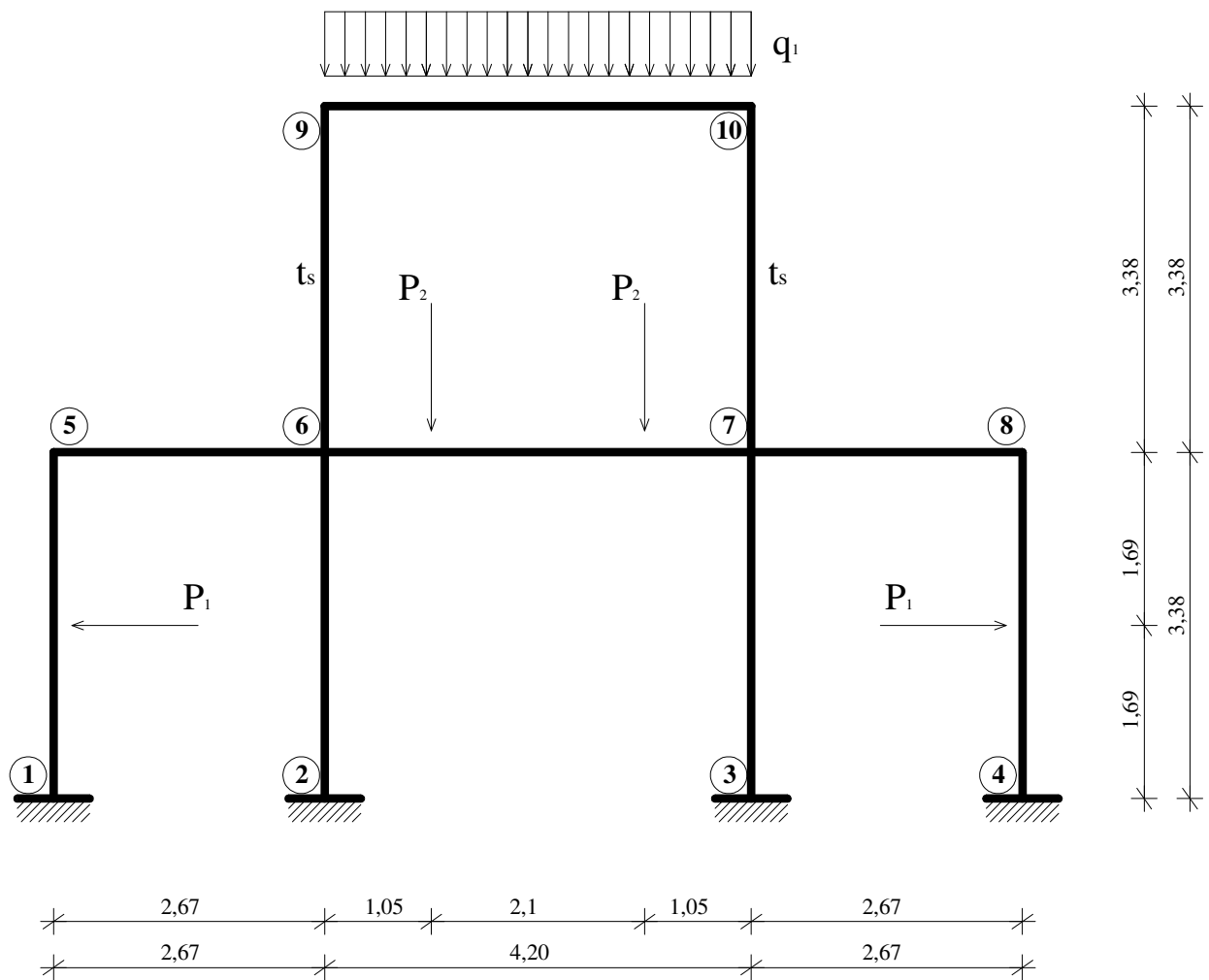


# RELAKSACIJSKI POSTUPCI

Zadatak C2



$$E = 3 \cdot 10^7 \text{ kN/m}^2$$

$$\alpha_t = 10^{-5} \text{ K}^{-1}$$

grede:  $b/h = 30/60$  [cm]

stupovi:  $b/h = 30/30$  [cm]

$$P_1 = 60 \text{ kN}$$

$$P_2 = 91 \text{ kN}$$

$$q_1 = 11,6 \text{ kN}$$

$$t_s = 11 \text{ }^\circ\text{C}$$

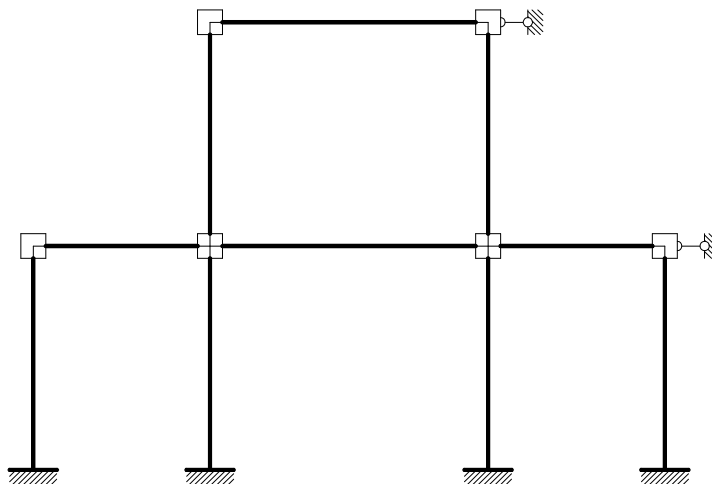
## GEOMETRIJSKE I MATERIJALNE KARAKTERISTIKE:

- grede:  $I_G = \frac{b \cdot h^3}{12} = \frac{0,3 \cdot 0,6^3}{12} = 0,0054 m^4$   
 $EI_G = 3 \cdot 10^7 \cdot 0,0054 = 162000 kNm^2$
- stupovi:  $I_S = \frac{b \cdot h^3}{12} = \frac{0,3 \cdot 0,3^3}{12} = 0,000675 m^4$   
 $EI_S = 3 \cdot 10^7 \cdot 0,000675 = 20250 kNm^2$

$$\frac{EI_G}{EI_S} = \frac{162000}{20250} = 8 \quad \rightarrow \quad EI_G = 8EI_S \quad \rightarrow \quad EI_S = EI$$
$$EI_G = 8EI$$

## CROSSOV POSTUPAK:

OSNOVNI  
SISTEM:



- koeficijenti krutosti elemenata:

$$k_{15} = k_{26} = k_{37} = k_{48} = k_{69} = k_{7,10} = \frac{EI_S}{3,38} = \frac{EI}{3,38}$$

$$k_{56} = k_{78} = \frac{EI_G}{2,67} = \frac{8EI}{2,67}$$

$$k_{67} = k_{9,10} = \frac{EI_G}{4,20} = \frac{8EI}{4,20}$$

- **koeficijenti krutosti čvorova:**

$$k_5 = k_{15} + k_{56} = EI \cdot \left( \frac{1}{3,38} + \frac{8}{2,67} \right) = 3,29EI$$

$$k_6 = k_{56} + k_{67} + k_{26} + k_{69} = EI \cdot \left( \frac{8}{2,67} + \frac{8}{4,20} + \frac{1}{3,38} + \frac{1}{3,38} \right) = 5,49EI$$

$$k_7 = k_{67} + k_{78} + k_{37} + k_{7,10} = EI \cdot \left( \frac{8}{4,20} + \frac{8}{2,67} + \frac{1}{3,38} + \frac{1}{3,38} \right) = 5,49EI$$

$$k_8 = k_{78} + k_{48} = EI \cdot \left( \frac{8}{2,67} + \frac{1}{3,38} \right) = 3,29EI$$

$$k_9 = k_{69} + k_{9,10} = EI \cdot \left( \frac{1}{3,38} + \frac{8}{4,20} \right) = 2,20EI$$

$$k_{10} = k_{9,10} + k_{7,10} = EI \cdot \left( \frac{8}{4,20} + \frac{1}{3,38} \right) = 2,20EI$$

- **razdjelni koeficijenti:**

čvor 5  $\mu_{51} = \frac{k_{15}}{k_5} = \frac{\frac{EI}{3,38}}{3,29EI} = 0,09$

$$\mu_{56} = \frac{k_{56}}{k_5} = \frac{\frac{8EI}{2,67}}{3,29EI} = 0,91$$

$$\sum \mu = 1,0$$

čvor 6  $\mu_{65} = \frac{k_{56}}{k_6} = \frac{\frac{8EI}{2,67}}{5,49EI} = 0,55$

$$\mu_{67} = \frac{k_{67}}{k_6} = \frac{\frac{8EI}{4,20}}{5,49EI} = 0,35$$

$$\mu_{62} = \frac{k_{26}}{k_6} = \frac{\frac{EI}{3,38}}{5,49EI} = 0,05$$

$$\mu_{69} = \frac{k_{69}}{k_6} = \frac{\frac{EI}{3,38}}{5,49EI} = 0,05$$

$$\sum \mu = 1,0$$

čvor 7  $\mu_{76} = \frac{k_{67}}{k_7} = \frac{\frac{8EI}{4,20}}{5,49EI} = 0,35$

$$\mu_{78} = \frac{k_{78}}{k_7} = \frac{\frac{8EI}{2,67}}{5,49EI} = 0,55$$

$$\mu_{73} = \frac{k_{37}}{k_7} = \frac{\frac{EI}{3,38}}{5,49EI} = 0,05$$

$$\mu_{7,10} = \frac{k_{7,10}}{k_7} = \frac{\frac{EI}{3,38}}{5,49EI} = 0,05$$

$$\sum \mu = 1,0$$

$$\text{čvor 8} \quad \mu_{87} = \frac{k_{78}}{k_8} = \frac{\frac{8EI}{2,67}}{3,29EI} = 0,91$$

$$\mu_{84} = \frac{k_{48}}{k_8} = \frac{\frac{EI}{3,38}}{3,29EI} = 0,09$$

$$\overline{\sum \mu} = 1,0$$

$$\text{čvor 9} \quad \mu_{96} = \frac{k_{69}}{k_9} = \frac{\frac{EI}{3,38}}{2,20EI} = 0,13$$

$$\mu_{9,10} = \frac{k_{9,10}}{k_9} = \frac{\frac{8EI}{4,20}}{2,20EI} = 0,87$$

$$\overline{\sum \mu} = 1,0$$

$$\text{čvor 10} \quad \mu_{10,9} = \frac{k_{9,10}}{k_{10}} = \frac{\frac{8EI}{4,20}}{2,20EI} = 0,87$$

$$\mu_{10,7} = \frac{k_{7,10}}{k_{10}} = \frac{\frac{EI}{3,38}}{2,20EI} = 0,13$$

$$\overline{\sum \mu} = 1,0$$

## MOMENTI UPETOSTI:

$$\overline{M}_{15} = -\frac{P_1 \cdot l_{15}}{8} = -\frac{60 \cdot 3,38}{8} = -25,35 \text{ kNm} \approx -25,4 \text{ kNm}$$

$$\overline{M}_{51} = \frac{P_1 \cdot l_{15}}{8} = \frac{60 \cdot 3,38}{8} = 25,35 \text{ kNm} \approx 25,4 \text{ kNm}$$

$$\overline{M}_{48} = \frac{P_1 \cdot l_{48}}{8} = \frac{60 \cdot 3,38}{8} = 25,35 \text{ kNm} \approx 25,4 \text{ kNm}$$

$$\overline{M}_{84} = -\frac{P_1 \cdot l_{48}}{8} = -\frac{60 \cdot 3,38}{8} = -25,35 \text{ kNm} \approx -25,4 \text{ kNm}$$

$$\overline{M}_{67} = \frac{P_2 \cdot 1,05 \cdot 3,15^2}{l_{67}^2} + \frac{P_2 \cdot 1,05^2 \cdot 3,15}{l_{67}^2} = \frac{91 \cdot 1,05 \cdot 3,15^2}{4,2^2} + \frac{91 \cdot 1,05^2 \cdot 3,15}{4,2^2} = 71,66 \text{ kNm} \approx 71,7 \text{ kNm}$$

$$\overline{M}_{67} = -\frac{P_2 \cdot 1,05 \cdot 3,15^2}{l_{67}^2} + \left( -\frac{P_2 \cdot 1,05^2 \cdot 3,15}{l_{67}^2} \right) = -\frac{91 \cdot 1,05 \cdot 3,15^2}{4,2^2} + \left( -\frac{91 \cdot 1,05^2 \cdot 3,15}{4,2^2} \right) = -71,66 \text{ kNm} \approx -71,7 \text{ kNm}$$

$$\overline{M}_{9,10} = \frac{q_1 \cdot l_{9,10}^2}{12} = \frac{11,6 \cdot 4,20^2}{12} = 17,05 \text{ kNm} \approx 17,1 \text{ kNm}$$

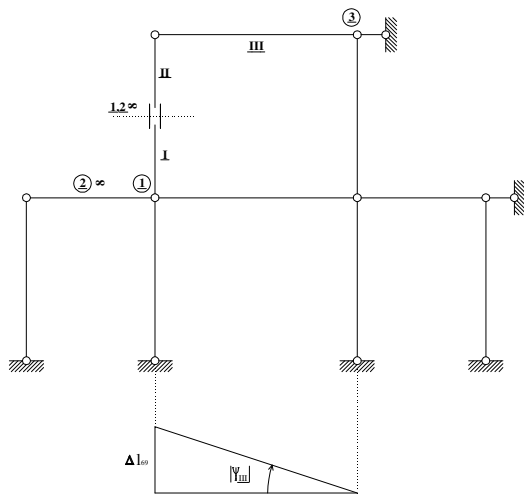
$$\overline{M}_{9,10} = -\frac{q_1 \cdot l_{9,10}^2}{12} = -\frac{11,6 \cdot 4,20^2}{12} = -17,05 \text{ kNm} \approx -17,1 \text{ kNm}$$

$$\overline{M}_{9,10}(t_s) = 0 \text{ kNm}$$

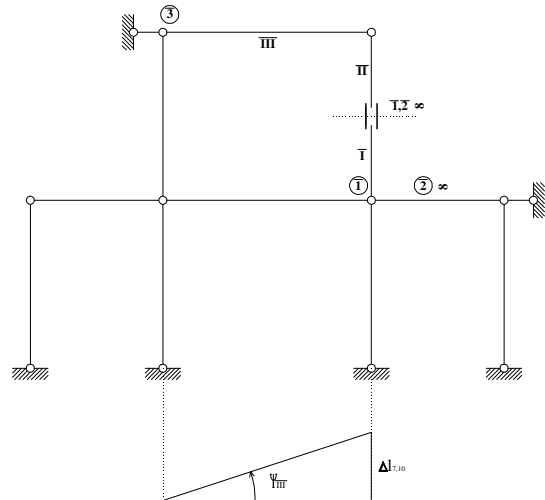
$$\overline{M}_{10,9}(t_s) = 0 \text{ kNm}$$

Iako je očito da su momenti upetosti na krajevima elementa 9-10 zbog zadane srednje temperature jednaki nuli jer je kut zaokreta elementa kao krutog tijela ( $\Psi_{9,10}$ ) jednak nuli (elementi 6-9 i 7-10 se jednako produljuju);

## PLANOVI POMAKA od produljenja zbog temperature:



$$\underline{\Psi}_{III} = \underline{\Psi}_{9,10} = -\frac{\Delta l_{69}}{4,20}$$



$$\overline{\Psi}_{III} = \overline{\Psi}_{9,10} = \frac{\Delta l_{7,10}}{4,20}$$

$$\Delta l_{69} = \alpha_t \cdot t_s \cdot l_{69}$$

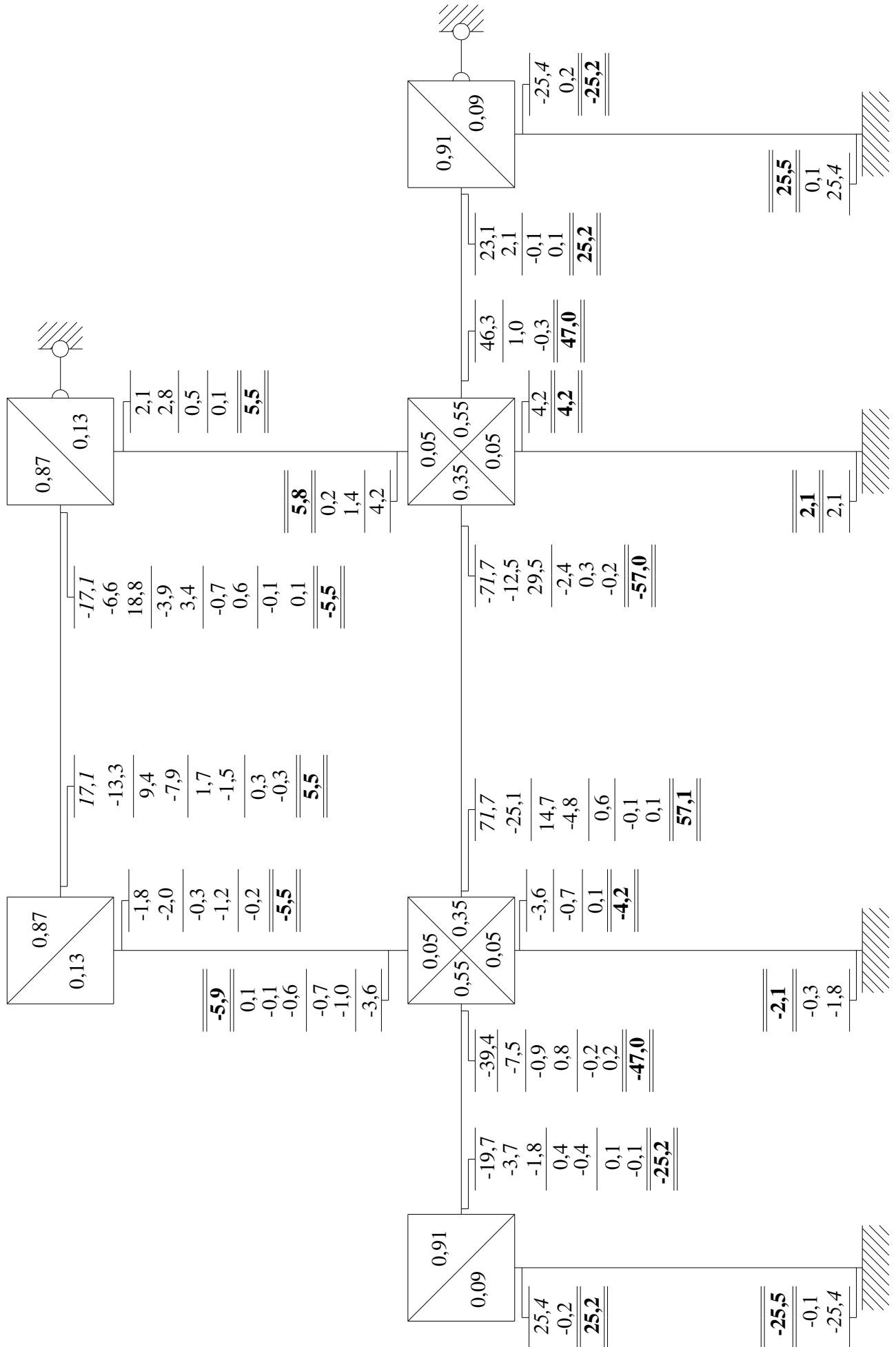
$$\Delta l_{7,10} = \alpha_t \cdot t_s \cdot l_{7,10}$$

$$l_{69} = l_{7,10} \rightarrow \Delta l_{69} = \Delta l_{7,10}$$

$$\Psi_{9,10} = \underline{\Psi}_{9,10} + \overline{\Psi}_{9,10} = -\frac{\Delta l_{69}}{4,20} + \frac{\Delta l_{7,10}}{4,20} = 0$$

$$\overline{M}_{9,10}(t_s) = -6k_{9,10} \Psi_{9,10} = 0$$

$$\overline{M}_{10,9}(t_s) = -6k_{9,10} \Psi_{9,10} = 0$$



## ITERACIJE:

čvor 6  $\Delta M_{\text{neurav.}} = 71,7 \text{ kNm}$

$$\begin{array}{r} -71,7 \cdot 0,55 = -39,4 \\ -71,7 \cdot 0,35 = -25,1 \\ -71,7 \cdot 0,05 = -3,6 \\ -71,7 \cdot 0,05 = -3,6 \\ \hline -71,7 \end{array}$$

čvor 7  $\Delta M_{\text{neurav.}} = -71,7 - 12,5 = -84,2 \text{ kNm}$

$$\begin{array}{r} 84,2 \cdot 0,55 = 46,3 \\ 84,2 \cdot 0,35 = 29,5 \\ 84,2 \cdot 0,05 = 4,2 \\ 84,2 \cdot 0,05 = 4,2 \\ \hline 84,2 \end{array}$$

čvor 9  $\Delta M_{\text{neurav.}} = 17,1 - 1,8 = 15,3 \text{ kNm}$

$$\begin{array}{r} -15,3 \cdot 0,87 = -13,3 \\ -15,3 \cdot 0,13 = -2,0 \\ \hline -15,3 \end{array}$$

čvor 10  $\Delta M_{\text{neurav.}} = -17,1 - 6,6 + 2,1 = -21,6 \text{ kNm}$

$$\begin{array}{r} 21,6 \cdot 0,87 = 18,8 \\ 21,6 \cdot 0,13 = 2,8 \\ \hline 21,6 \end{array}$$

čvor 6  $\Delta M_{\text{neurav.}} = 14,7 - 1,0 = 13,7 \text{ kNm}$

$$\begin{array}{r} -13,7 \cdot 0,55 = -7,5 \\ -13,7 \cdot 0,35 = -4,8 \\ -13,7 \cdot 0,05 = -0,7 \\ -13,7 \cdot 0,05 = -0,7 \\ \hline -13,7 \end{array}$$

čvor 9  $\Delta M_{\text{neurav.}} = 9,4 - 0,3 = 9,1 \text{ kNm}$

$$\begin{array}{r} -9,1 \cdot 0,87 = -7,9 \\ -9,1 \cdot 0,13 = -1,2 \\ \hline -9,1 \end{array}$$

čvor 10  $\Delta M_{\text{neurav.}} = -3,9 \text{ kNm}$

$$\begin{array}{r} 3,9 \cdot 0,87 = 3,4 \\ 3,9 \cdot 0,13 = 0,5 \\ \hline 3,9 \end{array}$$

čvor 8  $\Delta M_{\text{neurav.}} = 23,1 - 25,4 = -2,3 \text{ kNm}$

$$\begin{array}{r} 2,3 \cdot 0,91 = 2,1 \\ 2,3 \cdot 0,09 = 0,2 \\ \hline 2,3 \end{array}$$

čvor 5  $\Delta M_{\text{neurav.}} = 25,4 - 19,7 - 3,7 = 2,0 \text{ kNm}$

$$\begin{array}{r} -2,0 \cdot 0,91 = -1,8 \\ -2,0 \cdot 0,09 = -0,2 \\ \hline -2,0 \end{array}$$

čvor 9  $\Delta M_{\text{neurav.}} = 1,7 \text{ kNm}$

$$\begin{array}{r} -1,7 \cdot 0,87 = -1,5 \\ -1,7 \cdot 0,13 = -0,2 \\ \hline -1,7 \end{array}$$

$$\begin{aligned} \text{čvor } \underline{6} \quad \Delta M_{\text{neurav.}} &= -0,9 - 0,1 - 0,6 = -1,6 \text{ kNm} \\ & \quad 1,6 \cdot 0,55 = 0,9 \rightarrow 0,8 \\ & \quad 1,6 \cdot 0,35 = 0,6 \\ & \quad 1,6 \cdot 0,05 = 0,1 \\ & \quad 1,6 \cdot 0,05 = 0,1 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad 1,6 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{10} \quad \Delta M_{\text{neurav.}} &= -0,7 \text{ kNm} \\ & \quad 0,7 \cdot 0,87 = 0,6 \\ & \quad 0,7 \cdot 0,13 = 0,1 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad 0,7 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{7} \quad \Delta M_{\text{neurav.}} &= -2,4 + 0,3 + 0,2 + 1,4 + 1,0 = 0,5 \text{ kNm} \\ & \quad -0,5 \cdot 0,55 = -0,3 \\ & \quad -0,5 \cdot 0,35 = -0,2 \\ & \quad -0,5 \cdot 0,05 = 0 \\ & \quad -0,5 \cdot 0,05 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad -0,5 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{5} \quad \Delta M_{\text{neurav.}} &= 0,4 \text{ kNm} \\ & \quad -0,4 \cdot 0,91 = -0,4 \\ & \quad -0,4 \cdot 0,09 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad -0,4 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{9} \quad \Delta M_{\text{neurav.}} &= 0,3 \text{ kNm} \\ & \quad -0,3 \cdot 0,87 = -0,3 \\ & \quad -0,3 \cdot 0,13 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad -0,3 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{6} \quad \Delta M_{\text{neurav.}} &= -0,2 - 0,1 = -0,3 \text{ kNm} \\ & \quad 0,3 \cdot 0,55 = 0,2 \\ & \quad 0,3 \cdot 0,35 = 0,1 \\ & \quad 0,3 \cdot 0,05 = 0 \\ & \quad 0,3 \cdot 0,05 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad 0,3 \end{aligned}$$

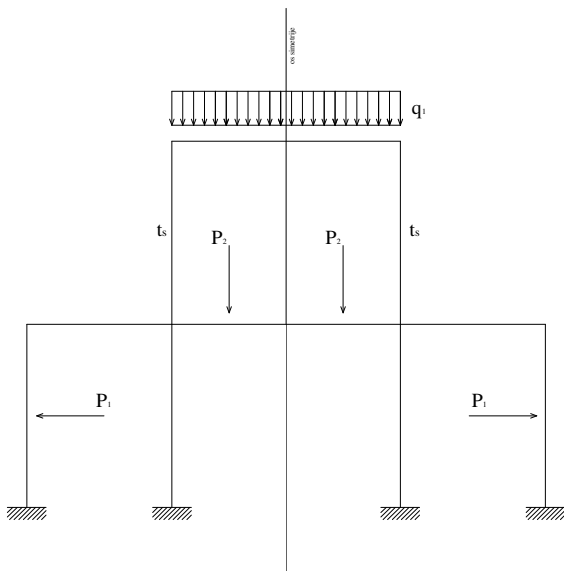
$$\begin{aligned} \text{čvor } \underline{5} \quad \Delta M_{\text{neurav.}} &= 0,1 \text{ kNm} \\ & \quad -0,1 \cdot 0,91 = -0,1 \\ & \quad -0,1 \cdot 0,09 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad -0,1 \end{aligned}$$

$$\begin{aligned} \text{čvor } \underline{8} \quad \Delta M_{\text{neurav.}} &= -0,1 \text{ kNm} \\ & \quad 0,1 \cdot 0,91 = 0,1 \\ & \quad 0,1 \cdot 0,09 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad 0,1 \end{aligned}$$

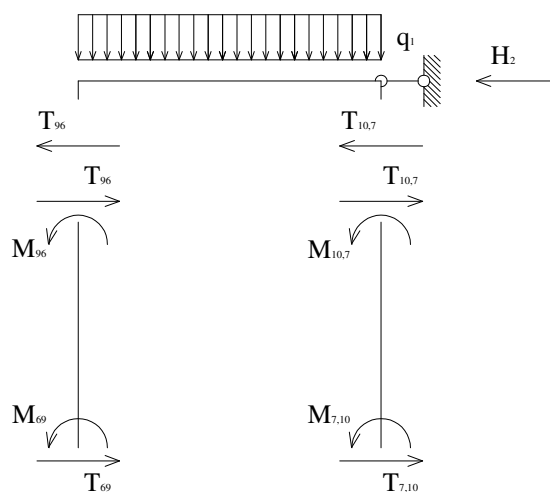
$$\begin{aligned} \text{čvor } \underline{10} \quad \Delta M_{\text{neurav.}} &= -0,1 \text{ kNm} \\ & \quad 0,1 \cdot 0,87 = 0,1 \\ & \quad 0,1 \cdot 0,13 = 0 \\ & \quad \underline{\hspace{1.5cm}} \\ & \quad \quad \quad 0,1 \end{aligned}$$



## SILE U PRIDRŽANJIMA:



Simetričan okvir opterećen je simetričnim opterećenjem.



$$\sum F_x = 0$$

$$-H_2 - T_{96} - T_{10,7} = 0$$

$$H_2 = -(T_{96} + T_{10,7})$$

$$M_{96} = -M_{10,7}$$

$$M_{69} = -M_{7,10}$$

$$h_{69} = h_{7,10}$$

$$T_{96} = -T_{10,7}$$

$$\sum M_{/6} = 0$$

$$-T_{96} \cdot h_{69} + M_{69} + M_{96} = 0$$

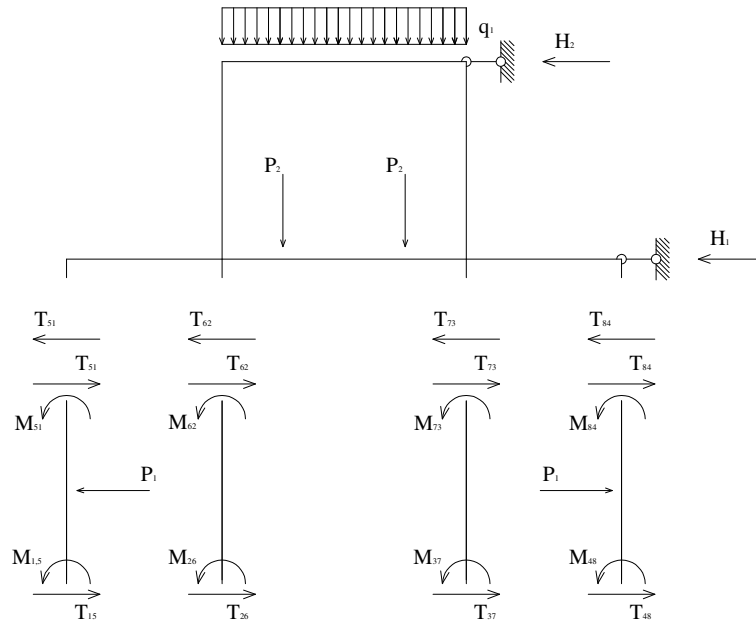
$$T_{96} = \frac{1}{h_{69}} (M_{69} + M_{96})$$

$$\sum M_{/7} = 0$$

$$-T_{10,7} \cdot h_{7,10} + M_{10,7} + M_{7,10} = 0$$

$$T_{10,7} = \frac{1}{h_{7,10}} (M_{7,10} + M_{10,7})$$

$$\rightarrow H_2 = 0 \text{ kN}$$



$$\sum F_x = 0$$

$$-H_1 - H_2 - T_{51} - T_{62} - T_{73} - T_{84} = 0$$

$$H_1 = -(T_{51} + T_{62} + T_{73} + T_{84}) - H_2$$

$$M_{15} = -M_{48}$$

$$M_{51} = -M_{84}$$

$$M_{26} = -M_{37}$$

$$M_{62} = -M_{73}$$

$$h_{48} = h_{26} = h_{73} = h_{84}$$

$$T_{51} = -T_{84}$$

$$T_{62} = -T_{73}$$

$$\sum M_{/1} = 0$$

$$-T_{51} \cdot h_{15} + M_{51} + M_{15} + P_1 \cdot \frac{h_{15}}{2} = 0$$

$$T_{51} = \frac{1}{h_{15}} (M_{15} + M_{51} + P_1 \cdot \frac{h_{15}}{2})$$

$$\sum M_{/3} = 0$$

$$-T_{73} \cdot h_{37} + M_{73} + M_{37} = 0$$

$$T_{73} = \frac{1}{h_{37}} (M_{73} + M_{37})$$

$$\sum M_{/4} = 0$$

$$-T_{84} \cdot h_{48} + M_{84} + M_{48} - P_1 \cdot \frac{h_{48}}{2} = 0$$

$$T_{84} = \frac{1}{h_{48}} (M_{48} + M_{84} - P_1 \cdot \frac{h_{48}}{2})$$

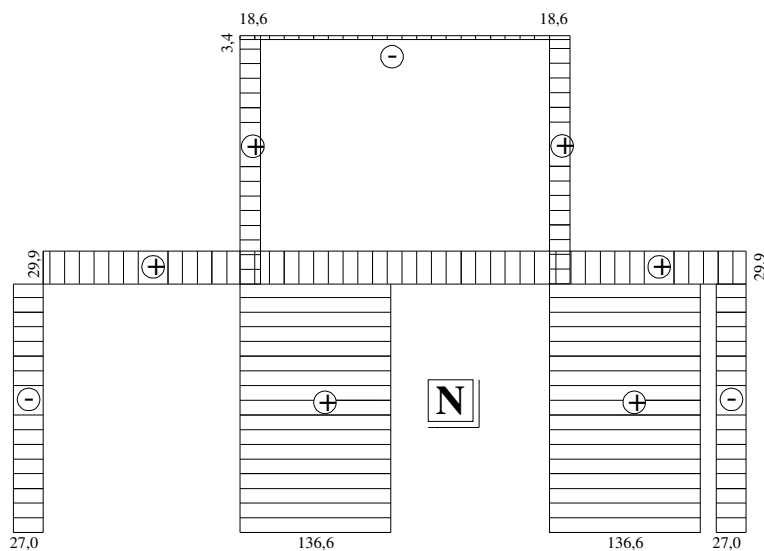
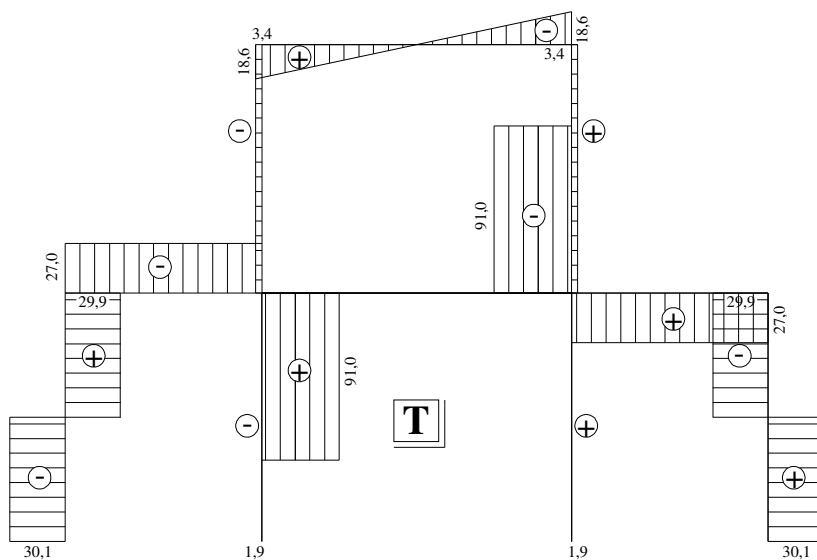
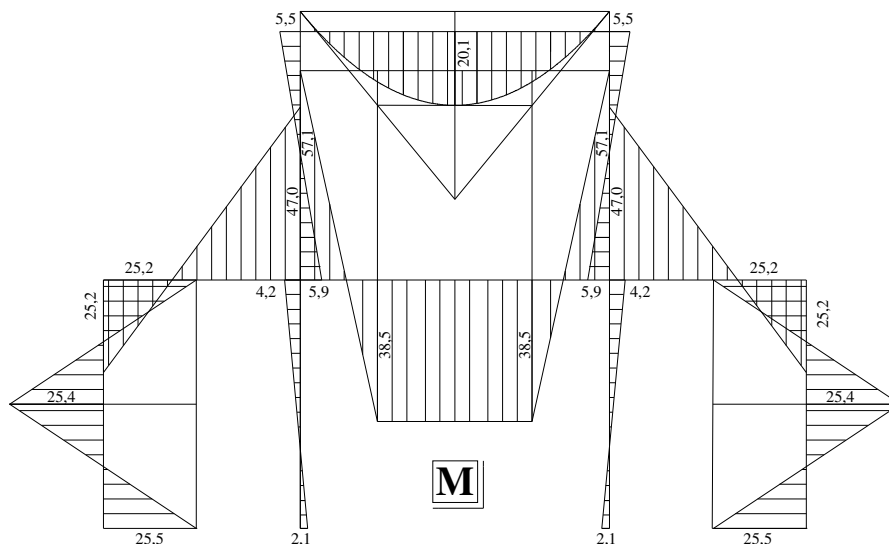
$$\rightarrow H_1 = 0 \text{ kN}$$

U proračunu Crossovim postupkom nastale su zanemarive razlike (veliĉine 0,1 kNm) kao posljedica zaokruŹivanja.

Sila u pridrŹanjima nema (tj. jednake su nuli).

Nije potrebno provoditi postupak Wernera i Csonke jer je  $\mathbf{M} = \mathbf{M}^{\text{Cr}}$ .

# DIJAGRAMI:



# REZULTATI – DIM:

Input data:

## Nodes:

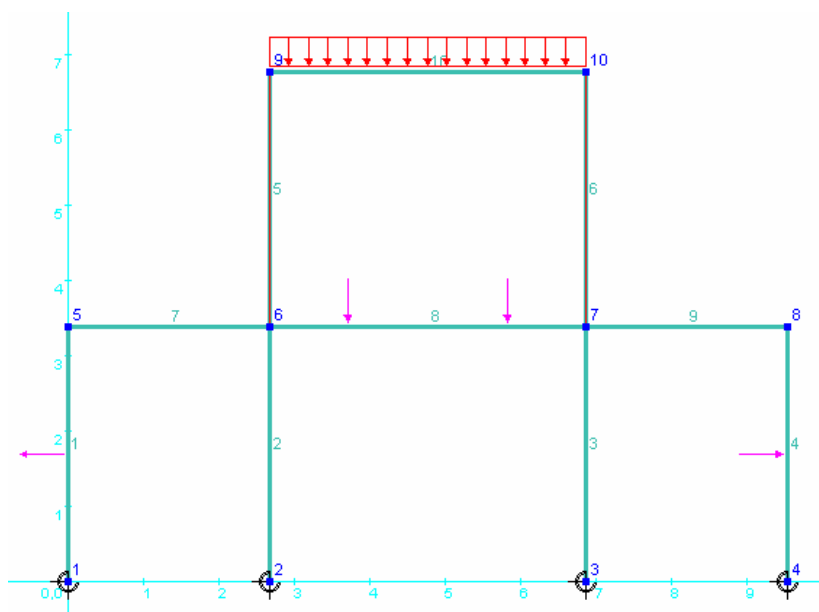
Label	x	y
1	0	0
2	2.67	0
3	6.87	0
4	9.54	0
5	0	3.38
6	2.67	3.38
7	6.87	3.38
8	9.54	3.38
9	2.67	6.76
10	6.87	6.76

## Elements:

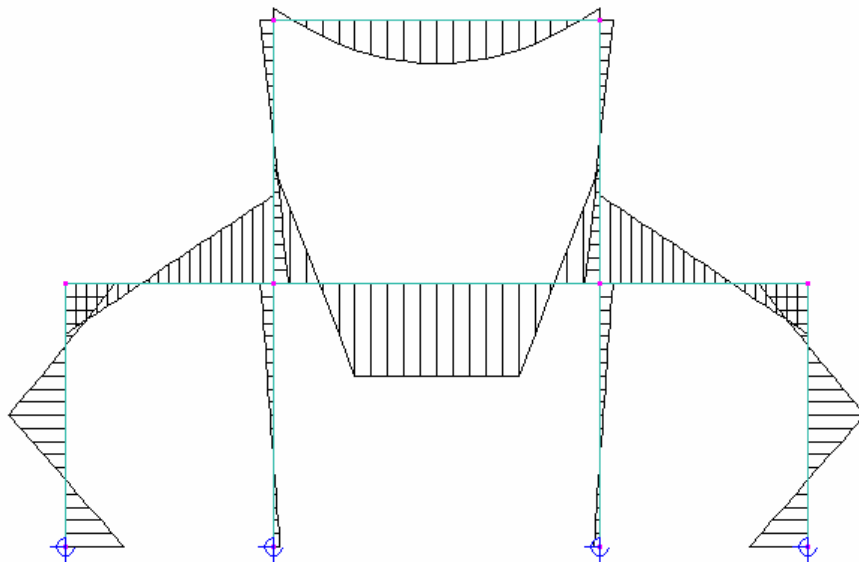
Label	1st node	2nd node	Characteristics
1	1	5	STUPOVI
2	2	6	STUPOVI
3	3	7	STUPOVI
4	4	8	STUPOVI
5	6	9	STUPOVI
6	7	10	STUPOVI
7	5	6	GREDE
8	6	7	GREDE
9	7	8	GREDE
10	9	10	GREDE

## Characteristics:

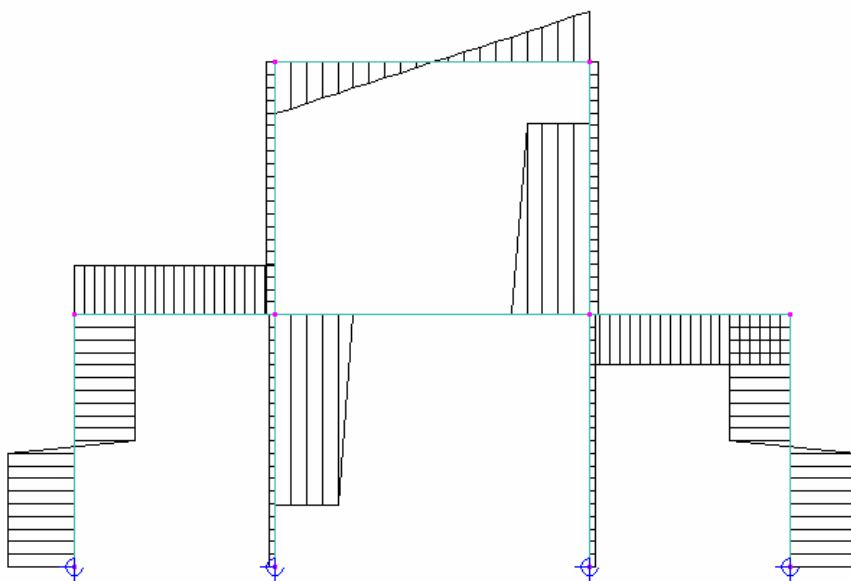
Label	E	rho	A	Iz
GREDE	3e+07	2.5	0.18	0.0054
STUPOVI	3e+07	2.5	0.09	0.000675



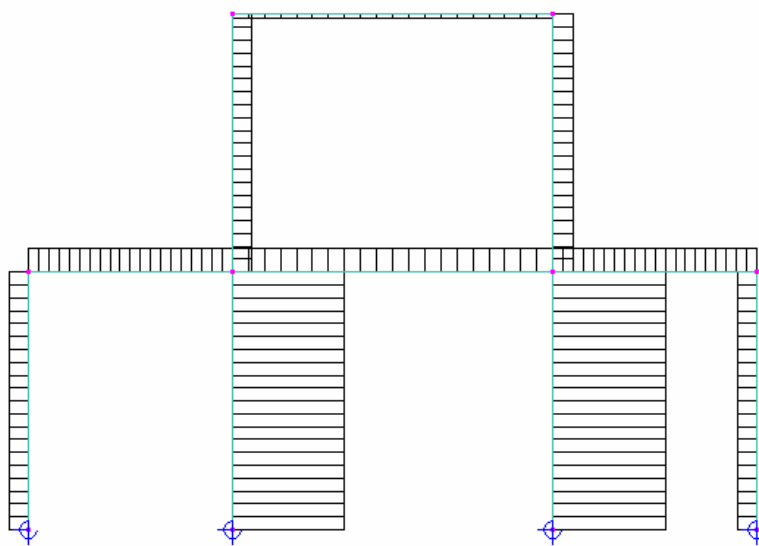
**M**



**T**



**N**



Degrees of Freedom: 18

Nodal displacements:

nd	u <sub>i</sub>	v <sub>i</sub>	phi <sub>i</sub>
1:	0	0	0
2:	0	0	0
3:	0	0	0
4:	0	0	0
5:	-2.60275e-05	2.9713e-05	-9.09285e-05
6:	-1.17533e-05	-0.000174127	-0.000235841
7:	1.17533e-05	-0.000174127	0.000235841
8:	2.60275e-05	2.9713e-05	9.09285e-05
9:	1.53061e-06	0.000167178	-0.0001421
10:	-1.53061e-06	0.000167178	0.0001421

Element end forces:

el	N <sub>ij</sub>	T <sub>ij</sub>	M <sub>ij</sub>	N <sub>ji</sub>	T <sub>ji</sub>	M <sub>ji</sub>
1:	-23.7353	-31.1308	-26.7163	23.7353	-28.8692	22.8941
2:	139.095	-2.58216	-2.9509	-139.095	2.58216	-5.77681
3:	139.095	2.58216	2.9509	-139.095	-2.58216	5.77681
4:	-23.7353	31.1308	26.7163	23.7353	28.8692	-22.8941
5:	24.36	-3.93586	-7.21321	-24.36	3.93586	-6.08999
6:	24.36	3.93586	7.21321	-24.36	-3.93586	6.08999
7:	-28.8692	-23.7353	-22.8941	28.8692	23.7353	-40.479
8:	-30.2229	91	53.4691	30.2229	91	-53.4691
9:	-28.8692	23.7353	40.479	28.8692	-23.7353	22.8941
10:	3.93586	24.36	6.08999	-3.93586	24.36	-6.08999

Reactions:

nd	R <sub>x</sub>	R <sub>y</sub>	M
1:	31.1308	-23.7353	-26.7163
2:	2.58216	139.095	-2.9509
3:	-2.58216	139.095	2.9509
4:	-31.1308	-23.7353	26.7163

## Usporedba:

momenti na krajevima elemenata	CROSS	DIM (OMP)
M <sub>15</sub>	-25,5	-26,7
M <sub>51</sub>	25,2	22,8
M <sub>26</sub>	-2,1	-2,9
M <sub>62</sub>	-4,2	-5,7
M <sub>37</sub>	2,1	2,9
M <sub>73</sub>	4,2	5,7
M <sub>48</sub>	25,5	26,7
M <sub>84</sub>	-25,2	-22,8
M <sub>56</sub>	-25,2	-22,8
M <sub>65</sub>	-47,0	-40,4
M <sub>67</sub>	57,1	53,4
M <sub>76</sub>	-57,0	-53,4
M <sub>78</sub>	47,0	40,4
M <sub>87</sub>	25,2	22,8
M <sub>69</sub>	-5,9	-7,2
M <sub>96</sub>	-5,5	-6,0
M <sub>7,10</sub>	5,8	7,2
M <sub>10,7</sub>	5,5	6,0
M <sub>9,10</sub>	5,5	6,0
M <sub>10,9</sub>	-5,5	-6,0

