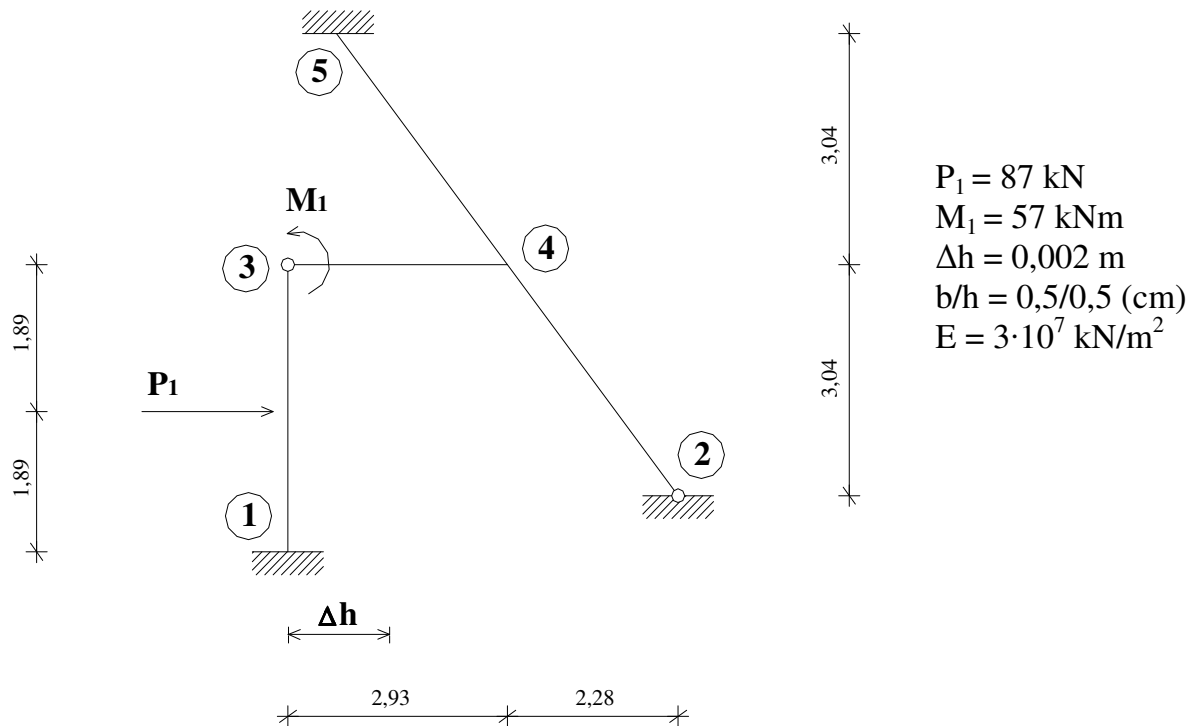


INŽENJERSKA METODA POMAKA

Zadatak C3



GEOMETRIJSKE I MATERIJALNE KARAKTERISTIKE:

$$I = \frac{b \cdot h^3}{12} = \frac{0,5 \cdot 0,5^3}{12} = 5\,208,33 \cdot 10^{-6} \text{ m}^4$$

$$EI = 156\,250 \text{ kNm}^2$$

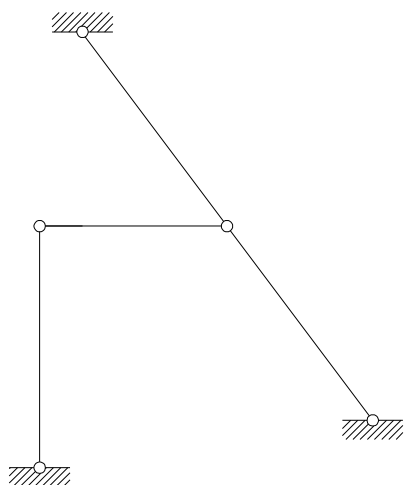
KOEFICIJENTI KRUTOSTI:

$$k_{13} = \frac{EI}{l_{13}} = \frac{156250}{3,78} = 41\,335,98 \text{ kNm}$$

$$k_{34} = \frac{EI}{l_{34}} = \frac{156250}{2,93} = 53\,327,65 \text{ kNm}$$

$$k_{24} = k_{45} = \frac{EI}{l_{24}} = \frac{156250}{\sqrt{2,28^2 + 3,04^2}} = 41\,118,42 \text{ kNm}$$

ZGLOBNA SHEMA:



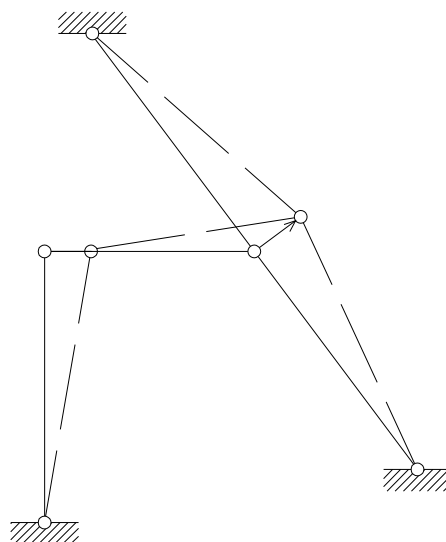
Minimalni broj stupnjeva slobode zglobne sheme:

$$S = 2 \cdot n_{\check{c}} - n_{\xi}$$

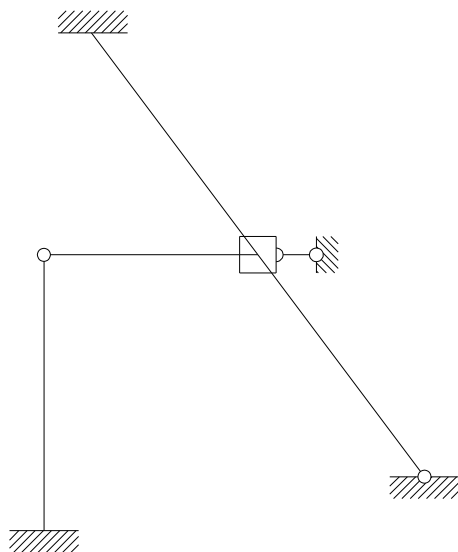
$$S = 2 \cdot 2 - 4$$

$$S = 0$$

Zglobna shema je geometrijski promjenjiva →
3 zgloba na istom pravcu.



OSNOVNI SISTEM:



NEPOZNANICE:

ρ_4, u_{34}

MOMENTI NA KRAJEVIMA ELEMENATA:

- element 1-3, („jednostrano upeta greda“):

$$M_{13} = M_{13}^C = m_{13}^C + \overline{M_{13}^C}$$

$$M_{13} = 3k_{13}\rho_1 - 3k_{13}\psi_{13} + \overline{M_{13}^C}(P_1) + \overline{M_{13}^C}(\Delta h)$$

$$M_{13} = 0 - 3k_{13}\psi_{13} + \left(\overline{M_{13}^C} - \frac{1}{2} \overline{M_{13}^C} \right) - 3k_{13} \overline{\psi_{13}}$$

$$M_{13} = -3k_{13}\psi_{13} + \left(\frac{P_1 \cdot l_{13}}{8} - \frac{1}{2} \left[-\frac{P_1 \cdot l_{13}}{8} \right] \right) - 3k_{13} \overline{\psi_{13}}$$

$$M_{13} = -3k_{13}\psi_{13} + \frac{3}{2} \frac{P_1 \cdot l_{13}}{8} - 3k_{13} \overline{\psi_{13}}$$

- element 3-4, („jednostrano upeta greda“):

$$M_{43} = M_{43}^C = m_{43}^C + \overline{M_{43}^C}$$

$$M_{43} = 3k_{34}\rho_4 - 3k_{34}\psi_{34} + \frac{1}{2} M_1$$

- element 4-2, („jednostrano upeta greda“):

$$M_{42} = M_{42}^C = m_{42}^C + \overline{M_{42}^C}$$

$$M_{42} = 3k_{24}\rho_4 - 3k_{24}\psi_{24} + 0$$

$$M_{42} = 3k_{24}\rho_4 - 3k_{24}\psi_{24}$$

- element 5-4, („obostrano upeta greda“):

$$M_{45} = m_{45} + \overline{M_{45}}$$

$$M_{45} = 2k_{45}\rho_5 + 4k_{45}\rho_4 - 6k_{45}\psi_{45} + 0$$

$$M_{45} = 0 + 4k_{45}\rho_4 - 6k_{45}\psi_{45}$$

$$M_{45} = 4k_{45}\rho_4 - 6k_{45}\psi_{45}$$

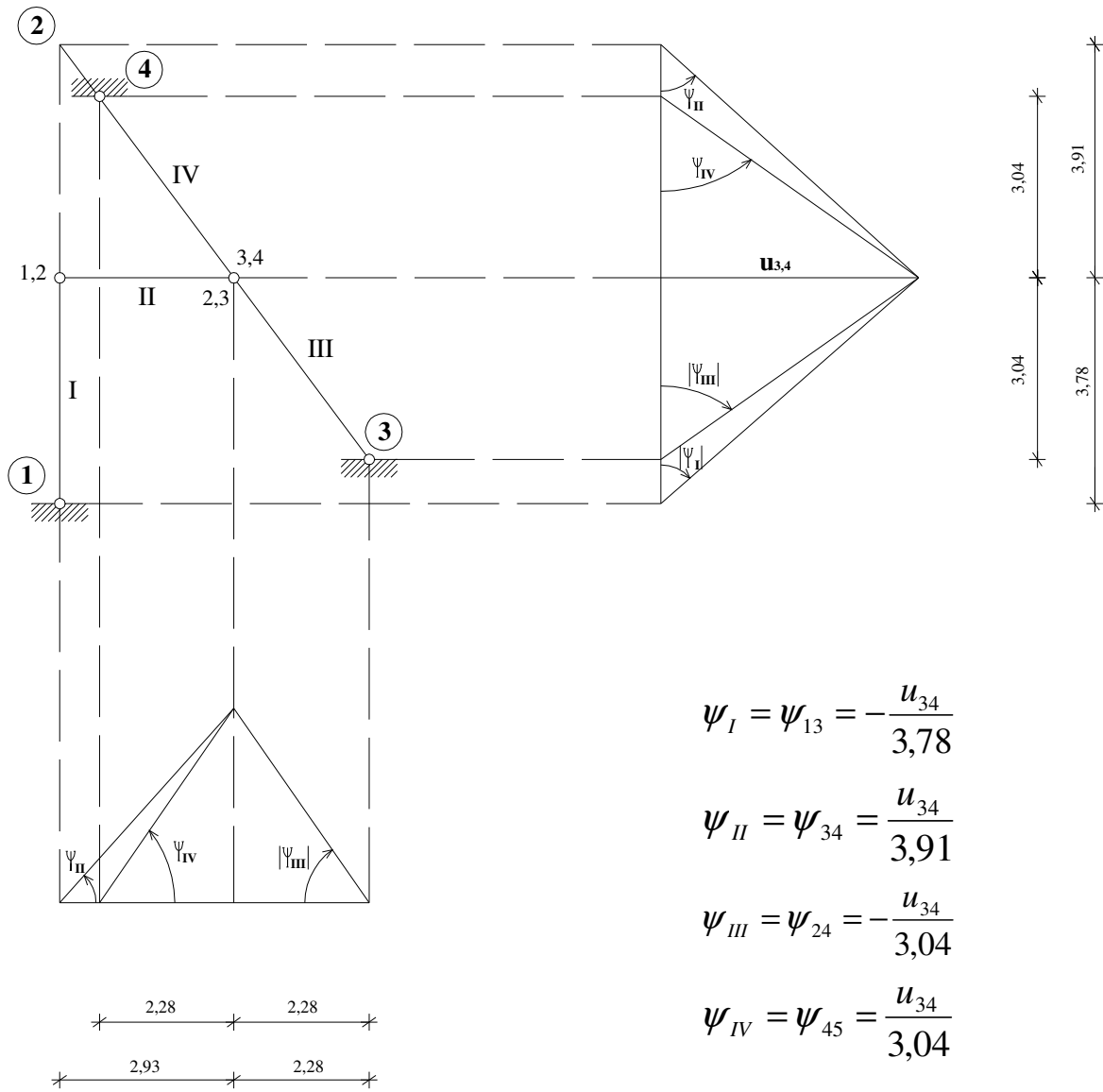
$$M_{54} = m_{54} + \overline{M_{54}}$$

$$M_{54} = 4k_{45}\rho_5 + 2k_{45}\rho_4 - 6k_{45}\psi_{45} + 0$$

$$M_{54} = 0 + 2k_{45}\rho_4 - 6k_{45}\psi_{45}$$

$$M_{54} = 2k_{45}\rho_4 - 6k_{45}\psi_{45}$$

PLAN POMAKA:



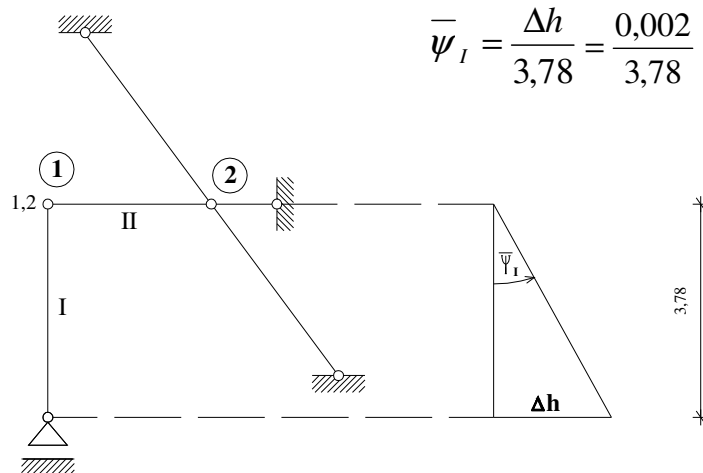
$$\psi_I = \psi_{13} = -\frac{u_{34}}{3,78}$$

$$\psi_{II} = \psi_{34} = \frac{u_{34}}{3,91}$$

$$\psi_{III} = \psi_{24} = -\frac{u_{34}}{3,04}$$

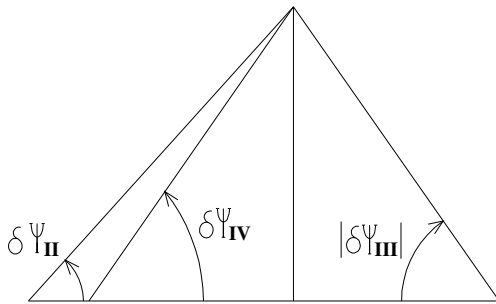
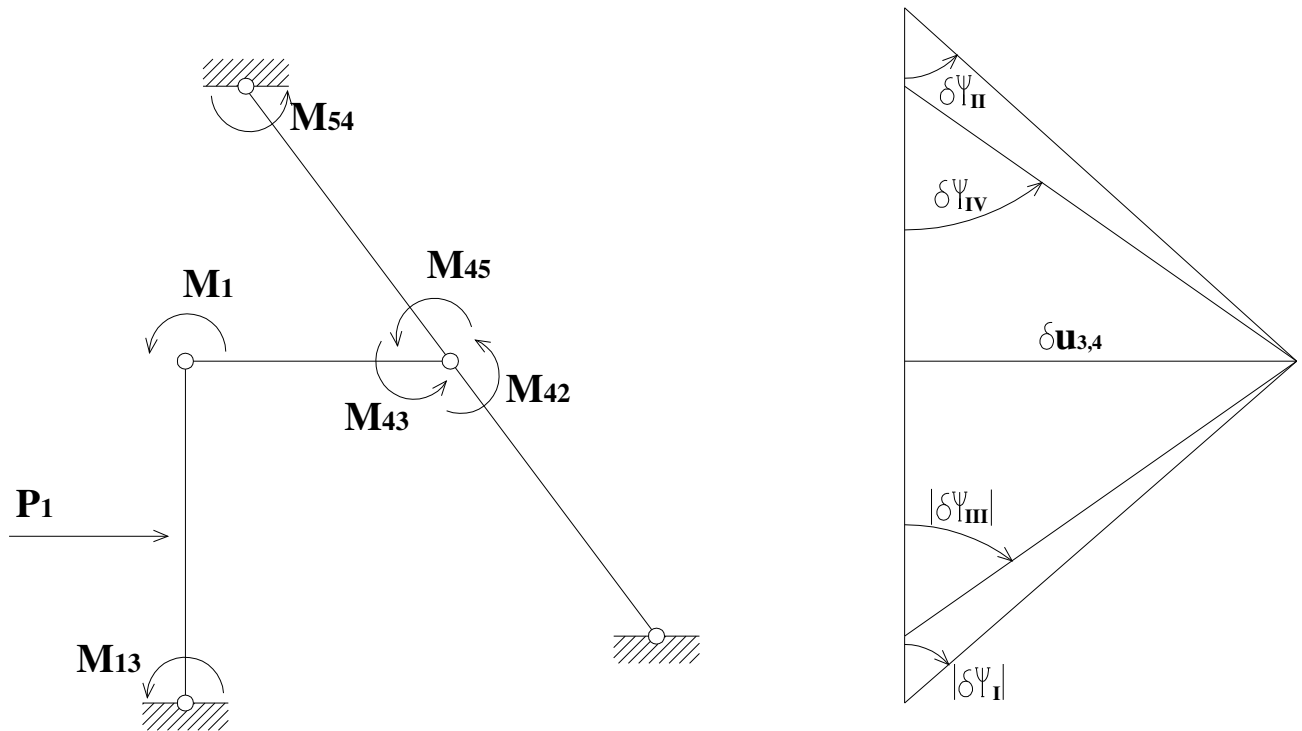
$$\psi_{IV} = \psi_{45} = \frac{u_{34}}{3,04}$$

PLAN POMAKA za prisilni pomak:



$$\bar{\psi}_I = \frac{\Delta h}{3,78} = \frac{0,002}{3,78}$$

1. JEDNADŽBA:



$$\delta \psi_I = \delta \psi_{13} = -\frac{\delta u_{34}}{3,78}$$

$$\delta \psi_{II} = \delta \psi_{34} = \frac{\delta u_{34}}{3,91}$$

$$\delta \psi_{III} = \delta \psi_{24} = -\frac{\delta u_{34}}{3,04}$$

$$\delta \psi_{IV} = \delta \psi_{45} = \frac{\delta u_{34}}{3,04}$$

$$\sum W_{\delta u_{34}} = 0$$

$$M_{13} \cdot \delta \psi_I + M_{43} \cdot \delta \psi_{II} + M_{42} \cdot \delta \psi_{III} + (M_{45} + M_{54}) \cdot \delta \psi_{IV} + P_1 \cdot \frac{\delta u_{34}}{2} + M_1 \cdot \delta \psi_{II} = 0$$

$$\left(-3k_{13}\psi_{13} + \frac{3}{2} \frac{P_1 \cdot l_{13}}{8} - 3k_{13}\overline{\psi}_{13}\right) \cdot \delta\psi_I + \left(3k_{34}\rho_4 - 3k_{34}\psi_{34} + \frac{1}{2}M_1\right) \cdot \delta\psi_{II} +$$

$$(3k_{24}\rho_4 - 3k_{24}\psi_{24}) \cdot \delta\psi_{III} + [(4k_{45}\rho_4 - 6k_{45}\psi_{45}) + (2k_{45}\rho_4 - 6k_{45}\psi_{45})] \cdot \delta\psi_{IV} +$$

$$P_1 \cdot \frac{\delta u_{34}}{2} + M_1 \cdot \delta\psi_{II} = 0$$

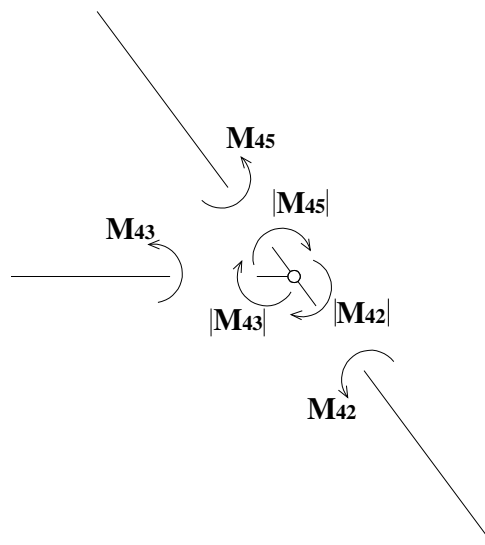
$$\left(-3k_{13}\psi_{13} + \frac{3}{2} \frac{P_1 \cdot l_{13}}{8} - 3k_{13}\overline{\psi}_{13}\right) \cdot \left(-\frac{\delta u_{34}}{3,78}\right) + \left(3k_{34}\rho_4 - 3k_{34}\psi_{34} + \frac{1}{2}M_1\right) \cdot \frac{\delta u_{34}}{3,91} +$$

$$(3k_{24}\rho_4 - 3k_{24}\psi_{24}) \cdot \left(-\frac{\delta u_{34}}{3,04}\right) + [(4k_{45}\rho_4 - 6k_{45}\psi_{45}) + (2k_{45}\rho_4 - 6k_{45}\psi_{45})] \cdot \frac{\delta u_{34}}{3,04} +$$

$$P_1 \cdot \frac{\delta u_{34}}{2} + M_1 \cdot \frac{\delta u_{34}}{3,91} = 0 \quad / : \delta u_{34}$$

$$-85\,882,64 \cdot u_{34} + 81\,493,87 \cdot \rho_4 = -66,41$$

2. JEDNADŽBA:



$$-M_{43} - M_{42} - M_{45} = 0$$

$$M_{43} + M_{42} + M_{45} = 0$$

$$3k_{34}\rho_4 - 3k_{34}\psi_{34} + \frac{1}{2}M_1 + 3k_{24}\rho_4 - 3k_{24}\psi_{24} + 4k_{45}\rho_4 - 6k_{45}\psi_{45} = 0$$

$$-81\,493,74 \cdot u_{34} + 447\,811,88 \cdot \rho_4 = -\frac{57}{2} \quad / \cdot (-1)$$

$$81\,493,74 \cdot u_{34} - 447\,811,88 \cdot \rho_4 = \frac{57}{2}$$

SUSTAV JEDNADŽBI:

$$\begin{bmatrix} 81493,74 & -447811,88 \\ -85882,64 & 81493,74 \end{bmatrix} \cdot \begin{bmatrix} u_{34} \\ \rho_4 \end{bmatrix} = \begin{bmatrix} -\frac{57}{2} \\ -66,41 \end{bmatrix}$$

RJEŠENJE SUSTAVA JEDNADŽBI:

$$u_{34} = 861,702 \cdot 10^{-6}$$

$$\rho_4 = 93,172 \cdot 10^{-6}$$

Usporedba:

poopćeni pomaci	IMP	DIM (OMP)
ρ_4	$93,172 \cdot 10^{-6}$	$94,076 \cdot 10^{-6}$
u_{34}	$861,702 \cdot 10^{-6}$	$860,125 \cdot 10^{-6}$

IZNOSI MOMENATA NA KRAJEVIMA ELEMENATA:

$$M_{13} = -3k_{13}\psi_{13} + \frac{3}{2} \frac{P_1 \cdot l_{13}}{8} - 3k_{13} \overline{\psi}_{13}$$

$$M_{13} = 32806,33 \cdot u_{34} - 3,95$$

$$M_{13} = 24,32kNm$$

$$M_{43} = 3k_{34}\rho_4 - 3k_{34}\psi_{34} + \frac{1}{2}M_1$$

$$M_{43} = -40916,35 \cdot u_{34} + 159982,94\rho_4 + \frac{57}{2}$$

$$M_{43} = 8,14kNm$$

$$M_{42} = 3k_{24}\rho_4 - 3k_{24}\psi_{24}$$

$$M_{42} = 40577,39 \cdot u_{34} + 123355,26 \cdot \rho_4$$

$$M_{42} = 46,46kNm$$

$$M_{45} = 4k_{45}\rho_4 - 6k_{45}\psi_{45}$$

$$M_{45} = -81154,78 \cdot u_{34} + 164473,68 \cdot \rho_4$$

$$M_{45} = -54,61kNm$$

$$M_{54} = 2k_{45}\rho_4 - 6k_{45}\psi_{45}$$

$$M_{54} = -81154,78 \cdot u_{34} + 82236,84 \cdot \rho_4$$

$$M_{54} = -62,27kNm$$

PRORAČUN U MATHEMATICI:

$$EI = 156250$$

$$P = 87;$$

$$M = 57;$$

$$\Delta h = 0.002$$

$$k_{13} = 156250 \cdot 3.78$$

$$k_{34} = 156250 \cdot 3.91$$

$$k_{24} = 156250 \cdot 3.04$$

$$k_{45} = k_{24}$$

$$\psi_{13} = -u \cdot 3.78;$$

$$\psi_{34} = u \cdot 3.91;$$

$$\psi_{24} = -u \cdot 3.04;$$

$$\psi_{45} = u \cdot 3.04;$$

$$\psi = \Delta h \cdot 3.78;$$

$$M_{13} = -3k_{13}\psi_{13} + 3 \cdot 2P \cdot 3.78 - 3k_{13}\psi;$$

$$M_{43} = 3k_{34}\psi_{34} - 3k_{34}\psi_{34} + 1 \cdot 2 \cdot M;$$

$$M_{42} = 3k_{24}\psi_{24} - 3k_{24}\psi_{24};$$

$$M_{54} = 2k_{45}\psi_{45} - 6k_{45}\psi_{45};$$

$$M_{45} = 4k_{45}\psi_{45} - 6k_{45}\psi_{45};$$

Solve[{M13 ψ_{13} + P*u/2+M* ψ_{34} +M43 ψ_{34} +M42* ψ_{24} +(M54 + M45)
)* ψ_{45} 0,

M43+M42+M45 0}]

{{ $\rho \rightarrow -0.0000636428$, $u \rightarrow 0.$ }, { $\rho \rightarrow 0.0000931717$, $u \rightarrow 0.000861703$ }}

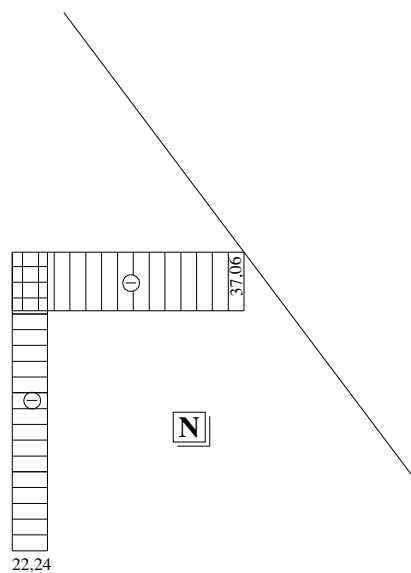
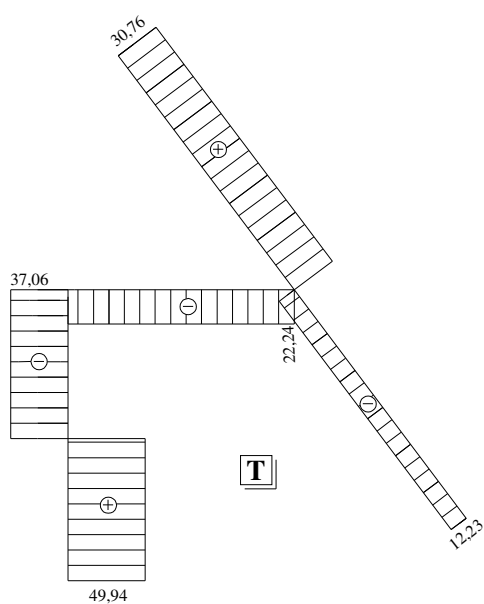
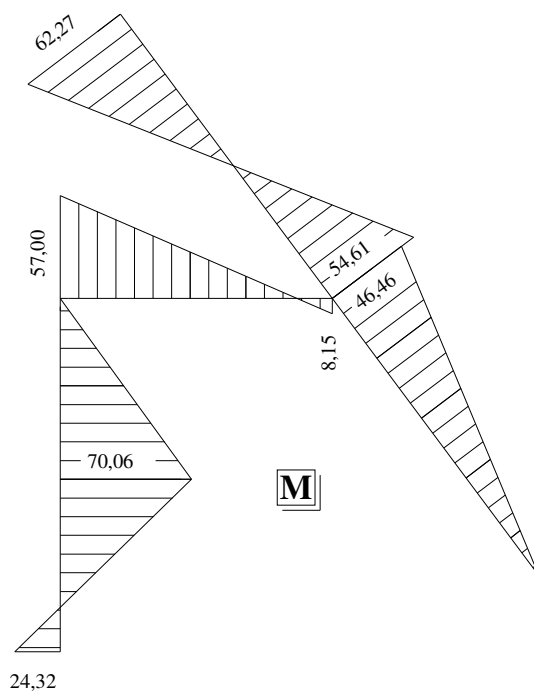
$$\rho = 0.00009317166568572969;$$

$$u = 0.0008617027299804683;$$

{M13, M43, M42, M45, M54}

{24.3179, 8.14814, 46.4589, -54.607, -62.2692}

DIJAGRAMI:



uzdužne sile na elementima 5-4 i
4-2 ne mogu se izracunati iz uvjeta
ravnoteže

REZULTATI – DIM:

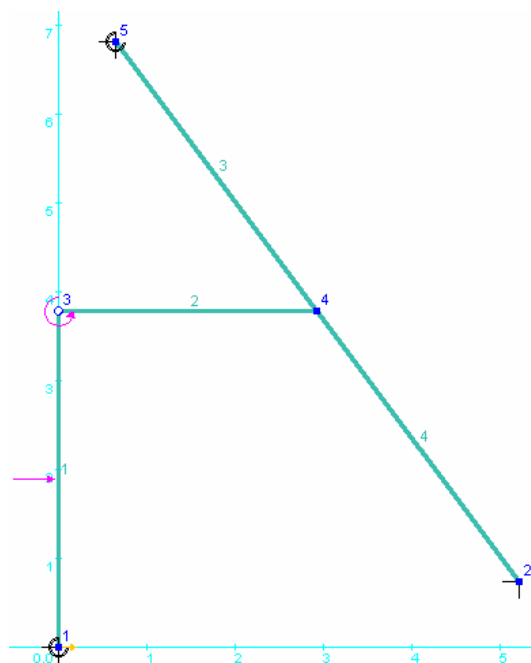
Input data:

Nodes:		
Label	x	y
1	0	0
2	5.21	0.74
3	0	3.78
4	2.93	3.78
5	0.65	6.82

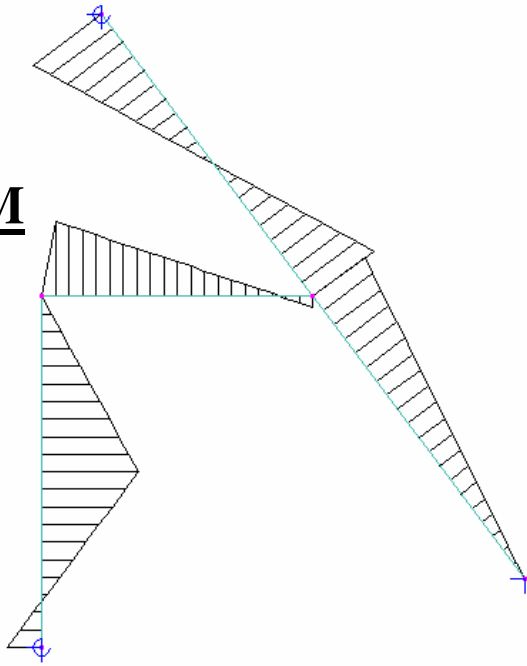
Elements:			
Label	1st node	2nd node	Characteristics
1	1	3	1
2	3	4	1
3	5	4	1
4	4	2	1

Characteristics:				
Label	E	rho	A	Iz
1	3e+07	2.5	0.25	0.00520833

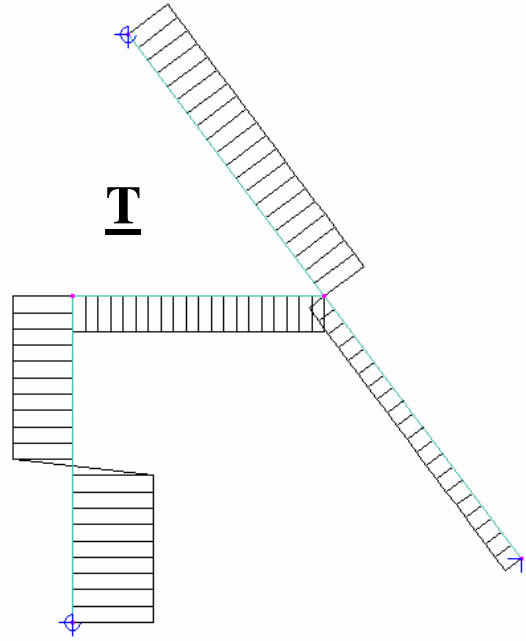
sustav:



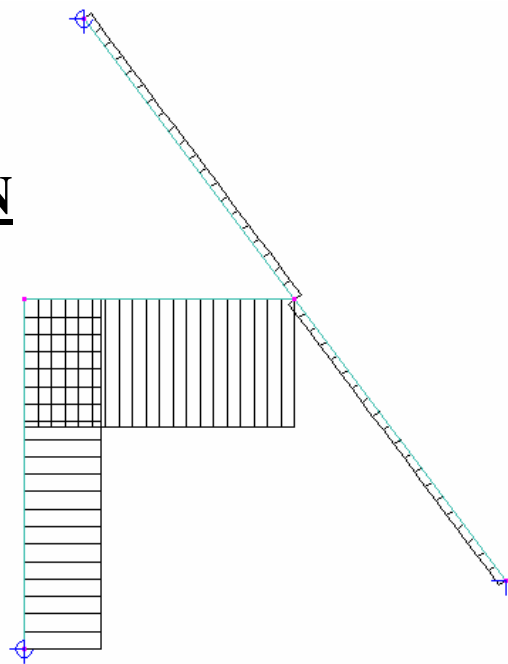
M



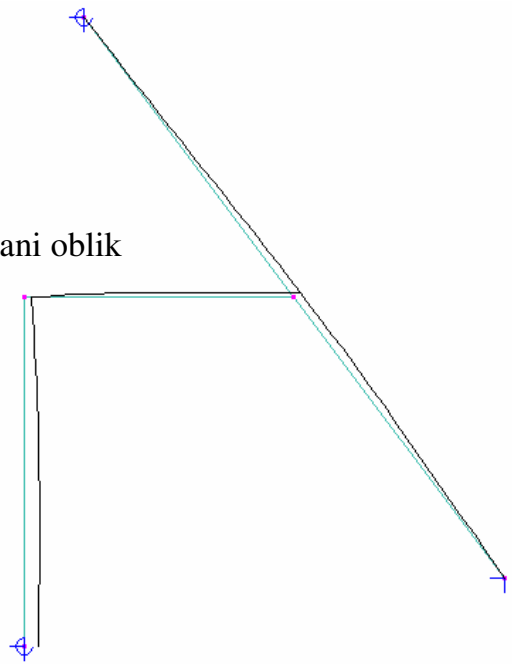
T



N



deformirani oblik



Degrees of Freedom: 6

Nodal displacements:

nd	u _j	v _j	phi _i
1:	0.002	0	0
2:	0	0	-0.000471106
3:	0.000874562	-1.11458e-05	0
4:	0.000860125	0.000643675	9.40758e-05
5:	0	0	0

Element end forces:

el	H _{ij}	T _{ij}	M _{ij}	H _{ji}	T _{ji}	M _{ji}
1:	22.1148	50.0449	24.7398	-22.1148	36.9551	0
2:	36.9551	22.1148	0	-36.9551	-22.1148	7.79624
3:	-2.24062	-30.6017	-62.0114	2.24062	30.6017	-54.2749
4:	2.24062	12.2312	46.4787	-2.24062	-12.2312	0

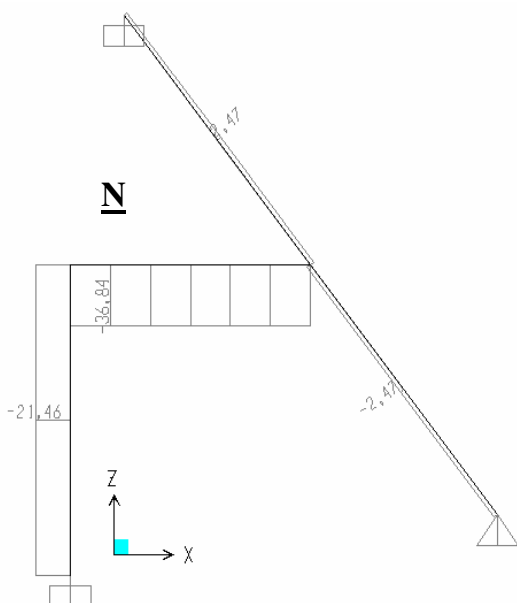
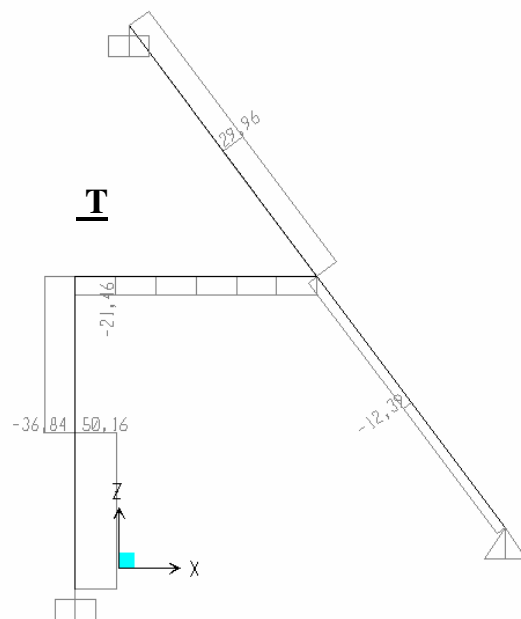
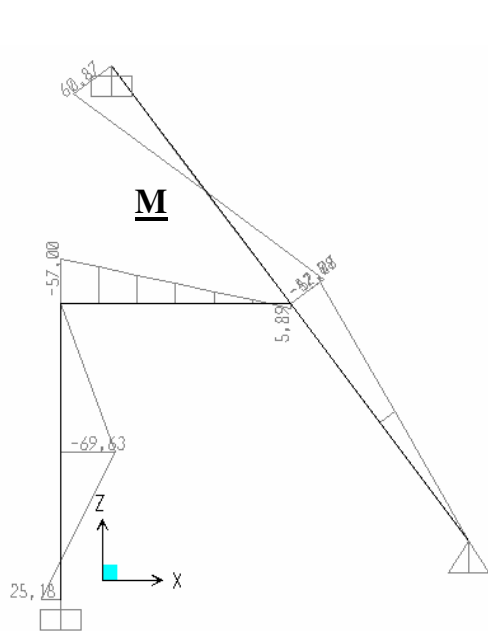
Reactions:

nd	R _x	R _y	M
1:	-50.0449	22.1148	24.7398
2:	-11.1294	-5.54625	
5:	-25.8257	-16.5685	-62.0114

Usporedba:

momenti na krajevima elemenata	IMP	DIM (OMP)	SAP
M ₁₃	24,32	24,74	25,18
M ₄₃	8,15	7,78	5,89
M ₄₂	46,46	46,48	47,08
M ₄₅	-54,61	-54,27	-52,80
M ₅₄	-62,27	-62,01	-60,87

DIJAGRAMI – SAP:



deformirani oblik

