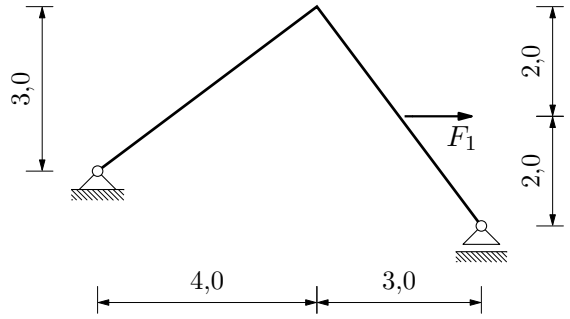
Štapna statika (3)

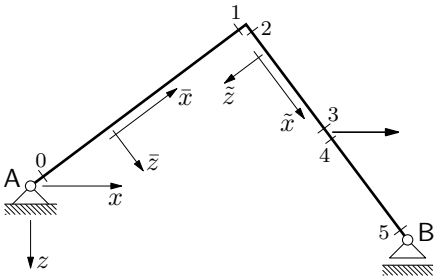
K. F.

Poligonalni štap u ravnini

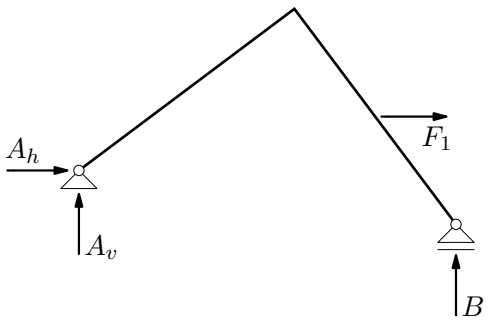


$F_1 \;=\; 100{,}0\,\mathrm{kN}$

globalni i lokalni koordinatni sustavi *&* karakteristični presjeci:



reakcije (sile u vanjskim vezama):



◇ $\sum\limits_{[\mathsf{A,B}]} F_x = 0 \;:\qquad A_h + F_1 \;=\; 0$

$A_h \;=\; -F_1 \;=\; -100{,}0\,\mathrm{kN}$

◇ $\sum\limits_{[\mathsf{A,B}]} M_{/\mathsf{A}} = 0 \;:\qquad -1{,}0\,F_1 \;+\; 7{,}0\,B \;=\; 0$

$B \;=\; \frac{1}{7{,}0}\,F_1 \;=\; 14{,}29\,\mathrm{kN}$

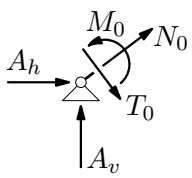
◇ $\sum\limits_{[\mathsf{A,B}]} M_{/(7{,}0)} = 0 \;:\qquad -7{,}0\,A_v - 1{,}0\,F_1 \;=\; 0$

$A_v \;=\; -\frac{1}{7{,}0}\,F_1 \;=\; -14{,}29\,\mathrm{kN}$

◇ kontrola: $\sum\limits_{[\mathsf{A,B}]} M_{/(4,\,-1)} \;=\; 1{,}0\,A_h \;-\; 4{,}0\,A_v \;+\; 3{,}0\,B$

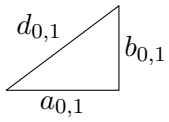
$\qquad\qquad = \; 1{,}0\cdot(-100{,}0) \;-\; 4{,}0\cdot(-14{,}29) \;+\; 3{,}0\cdot 14{,}29 \;=\; 0{,}03 \;\approx\; 0$

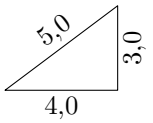
sile u presjeku 0 (neposredno uz ležaj A):



$$d_{0,1} = \sqrt{a_{0,1}^2 + b_{0,1}^2} = \sqrt{3^2 + 4^2} = 5$$

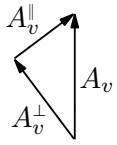
$$c_{0,1} = \frac{a_{0,1}}{d_{0,1}} = \frac{4}{5}, \qquad s_{0,1} = \frac{b_{0,1}}{d_{0,1}} = \frac{3}{5}$$



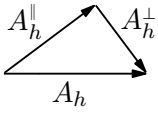


$$\frac{A_v^{\perp}}{A_v} = \frac{a_{0,1}}{d_{0,1}} \quad \Rightarrow \quad A_v^{\perp} = \frac{a_{0,1}}{d_{0,1}} A_v = c_{0,1}\, A_v$$

$$\frac{A_v^{\parallel}}{A_v} = \frac{b_{0,1}}{d_{0,1}} \quad \Rightarrow \quad A_v^{\parallel} = \frac{b_{0,1}}{d_{0,1}} A_v = s_{0,1}\, A_v$$



$$\frac{A_h^{\perp}}{A_h} = \frac{b_{0,1}}{d_{0,1}} \quad \Rightarrow \quad A_h^{\perp} = \frac{b_{0,1}}{d_{0,1}} A_h = s_{0,1}\, A_h$$



$$\frac{A_h^{\parallel}}{A_h} = \frac{a_{0,1}}{d_{0,1}} \quad \Rightarrow \quad A_h^{\parallel} = \frac{a_{0,1}}{d_{0,1}} A_h = c_{0,1}\, A_h$$

$\diamond \sum_{[A,0]} F_{\bar{x}} = 0$: $\qquad A_h^{\parallel} + A_v^{\parallel} + N_0 = 0$

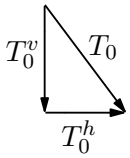
$$N_0 = -A_h^{\parallel} - A_v^{\parallel} = -c_{0,1}\, A_h - s_{0,1}\, A_v = -\tfrac{4}{5}\,(-100{,}0) - \tfrac{3}{5}\,(-14{,}29) = 88{,}57\,\text{kN}$$

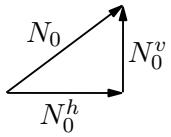
$\diamond \sum_{[A,0]} F_{\bar{z}} = 0$: $\qquad A_h^{\perp} - A_v^{\perp} + T_0 = 0$

$$T_0 = -A_h^{\perp} + A_v^{\perp} = -s_{0,1}\, A_h + c_{0,1}\, A_v = -\tfrac{3}{5}\,(-100{,}0) + \tfrac{4}{5}\,(-14{,}29) = 48{,}57\,\text{kN}$$

$\diamond \sum_{[A,0]} M_{/0} = 0$: $\qquad M_0 = 0$

drugi način izračunavanja T_0 i N_0:





$$\frac{T_0^v}{T_0} = \frac{a_{0,1}}{d_{0,1}} \quad \Rightarrow \quad T_0^v = \frac{a_{0,1}}{d_{0,1}} T_0 = c_{0,1}\, T_0$$

$$\frac{T_0^h}{T_0} = \frac{b_{0,1}}{d_{0,1}} \quad \Rightarrow \quad T_0^h = \frac{b_{0,1}}{d_{0,1}} T_0 = s_{0,1}\, T_0$$

$$\frac{N_0^v}{N_0} = \frac{b_{0,1}}{d_{0,1}} \quad \Rightarrow \quad N_0^v = \frac{b_{0,1}}{d_{0,1}} N_0 = s_{0,1}\, N_0$$

$$\frac{N_0^h}{N_0} = \frac{a_{0,1}}{d_{0,1}} \quad \Rightarrow \quad N_0^h = \frac{a_{0,1}}{d_{0,1}} N_0 = c_{0,1}\, N_0$$

$\diamond \sum_{[A,0]} F_x = 0$: $\qquad A_h + T_0^h + N_0^h = 0$

$$s_{0,1}\, T_0 + c_{0,1}\, N_0 = -A_h$$

$$0{,}60\, T_0 + 0{,}80\, N_0 = 100{,}0$$

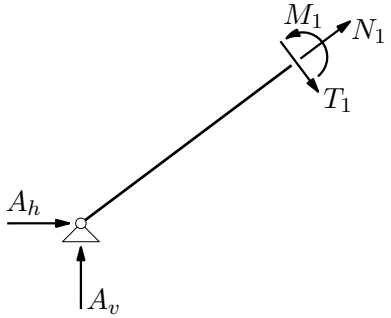
$\diamond \sum_{[A,0]} F_z = 0 :$ $\quad -A_v + T_0^v - N_0^v = 0$

$$c_{0,1}\, T_0 - s_{0,1}\, N_0 = A_v$$

$$0{,}80\, T_0 - 0{,}60\, N_0 = -14{,}29$$

$$\Longrightarrow \quad N_0 = 88{,}57\,\text{kN} \qquad \& \qquad T_0 = 48{,}57\,\text{kN}$$

sile u presjeku 1 (neposredno ispod „loma" osi, lijevo):



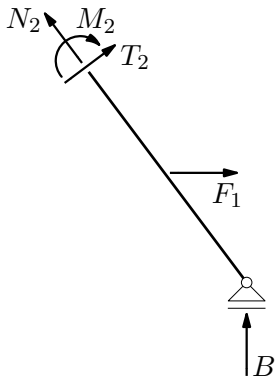
$\diamond \quad N_1 = N_0 = 88{,}57\,\text{kN}$

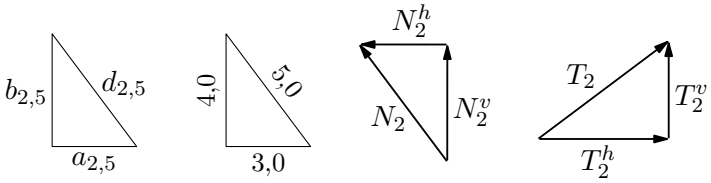
$\diamond \quad T_1 = T_0 = 48{,}57\,\text{kN}$

$\diamond \sum_{[A,1]} M_{/1} = 0 :$ $\quad 3{,}0\, A_h - 4{,}0\, A_v + M_1 = 0$

$$M_1 = -3{,}0\, A_h + 4{,}0\, A_v$$
$$= -3{,}0\,(-100{,}0) + 4{,}0\,(-14{,}29) = 242{,}84\,\text{kNm}$$

sile u presjeku 2 (neposredno ispod „loma" osi, desno):





$$d_{2,5} = \sqrt{a_{2,5}^2 + b_{2,5}^2} = \sqrt{4^2 + 2^2} = 5$$

$$c_{2,5} = \frac{a_{2,5}}{d_{2,5}} = \frac{3}{5} = 0{,}60, \qquad s_{2,5} = \frac{b_{2,5}}{d_{2,5}} = \frac{4}{5} = 0{,}80$$

$$\frac{N_2^v}{N_2} = \frac{b_{2,5}}{d_{2,5}} \quad \Rightarrow \quad N_2^v = \frac{b_{2,5}}{d_{2,5}}\, N_2 = s_{2,5}\, N_2$$

$$\frac{N_2^h}{N_2} = \frac{a_{2,5}}{d_{2,5}} \quad \Rightarrow \quad N_2^h = \frac{a_{2,5}}{d_{2,5}}\, N_2 = c_{2,5}\, N_2$$

$$\frac{T_2^v}{T_2} = \frac{a_{2,5}}{d_{2,5}} \quad \Rightarrow \quad T_2^v = \frac{a_{2,5}}{d_{2,5}}\, T_2 = c_{2,5}\, T_2$$

$$\frac{T_2^h}{T_2} = \frac{b_{2,5}}{d_{2,5}} \quad \Rightarrow \quad T_2^h = \frac{b_{2,5}}{d_{2,5}}\, T_2 = s_{2,5}\, T_2$$

$\diamond \sum_{[2,B]} F_x = 0 :$ $\quad -N_2^h + T_2^h + F_1 = 0$

$$-c_{2,5}\, N_2 + s_{2,5}\, T_2 = -F_1$$

$$-0{,}60\, N_2 + 0{,}80\, T_2 = -100{,}0$$

$\diamond \sum_{[2,\mathrm{B}]} F_z = 0 :$ $\quad -N_2^v - T_2^v - B = 0$

$$-s_{2,5}\, N_2 - c_{2,5}\, T_2 = B$$

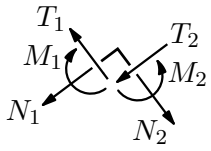
$$-0{,}80\, N_2 - 0{,}60\, T_2 = 14{,}286$$

$$\Longrightarrow \quad N_2 = 48{,}57\,\mathrm{kN} \qquad \& \qquad T_2 = -88{,}57\,\mathrm{kN}$$

$\diamond \sum_{[2,\mathrm{B}]} M_{/2} = 0 :$ $\quad -M_2 + 2{,}0\, F_1 + 3{,}0\, B = 0$

$$M_2 = 2{,}0\, F_1 + 3{,}0\, B = 2{,}0 \cdot 100{,}0 + 3{,}0 \cdot 14{,}29 = 242{,}87\,\mathrm{kNm}$$

kontrola ravnoteže odsječka 1–2:



$$\sum_{[1,2]} F_x = -N_1^h - T_1^h + N_2^h - T_2^h$$

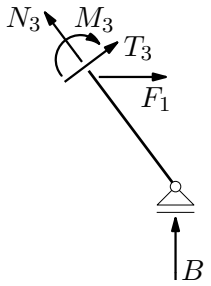
$$= -c_{0,1}\, N_1 - s_{0,1}\, T_1 + c_{2,5}\, N_2 - s_{2,5}\, T_2$$

$$= -0{,}80 \cdot 88{,}57 - 0{,}60 \cdot 48{,}57 + 0{,}60 \cdot 48{,}57 - 0{,}80 \cdot (-88{,}57)$$

$$= 0{,}00$$

$$\sum_{[1,2]} M_{/(4,\,-3)} = -M_1 + M_2 = -242{,}84 + 242{,}87 = 0{,}03 \approx 0$$

sile u presjeku 3 (neposredno iznad hvatišta sile F_1):



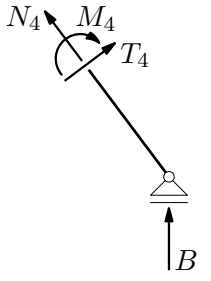
$\diamond \quad N_3 = N_2 = 48{,}57\,\mathrm{kN}$

$\diamond \quad T_3 = T_2 = -88{,}57\,\mathrm{kN}$

$\diamond \sum_{[3,\mathrm{B}]} M_{/3} = 0 :$ $\quad -M_3 + 1{,}5\, B = 0$

$$M_3 = 1{,}5\, B = 1{,}5 \cdot 14{,}29 = 21{,}44\,\mathrm{kNm}$$

sile u presjeku 4 (neposredno ispod hvatišta sile F_1):



$\diamond \sum_{[4,\mathrm{B}]} F_x = 0 :$ $\quad -N_4^h + T_4^h = 0$

$$-c_{2,5}\, N_4 + s_{2,5}\, T_4 = 0$$

$$-0{,}60\, N_4 + 0{,}80\, T_4 = 0$$

$\diamond \sum_{[4,\mathrm{B}]} F_z = 0 :$ $\quad -N_4^v - T_4^v - B = 0$

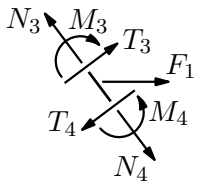
$$-s_{2,5}\, N_4 - c_{2,5}\, T_4 = B$$

$$-0{,}80\, N_4 - 0{,}60\, T_4 = 14{,}29$$

$$\Longrightarrow \quad N_4 = -11{,}43\,\mathrm{kN} \qquad \& \qquad T_4 = -8{,}57\,\mathrm{kN}$$

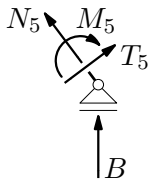
$\diamond \quad M_4 = M_3 = 21{,}44\,\mathrm{kNm}$

kontrola ravnoteže odsječka 3–4:



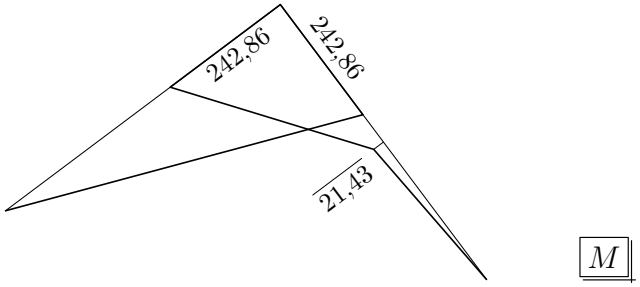
$$\begin{aligned}\sum_{[3,4]} F_x &= -N_3^h + T_3^h + N_4^h - T_4^h + F_1 \\ &= -c_{2,5}\,N_3 + s_{2,5}\,T_3 + c_{2,5}\,N_4 - s_{2,5}\,T_4 + F_1 \\ &= -0{,}60 \cdot 48{,}57 + 0{,}80\,(-88{,}57) \\ &\quad + 0{,}60\,(-11{,}43) - 0{,}80\,(-8{,}57) + 100{,}0 = 0{,}00\end{aligned}$$

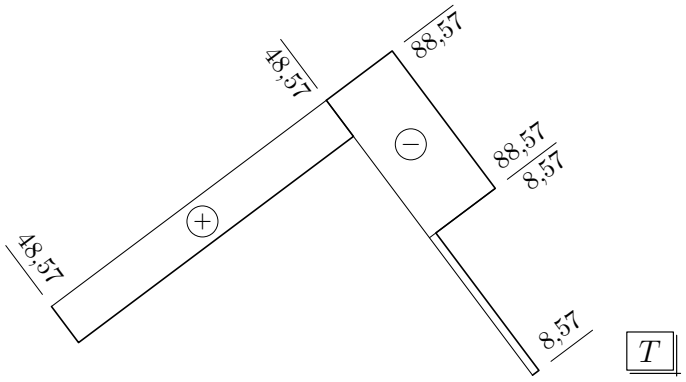
sile u presjeku 5 (neposredno iznad ležaja B):

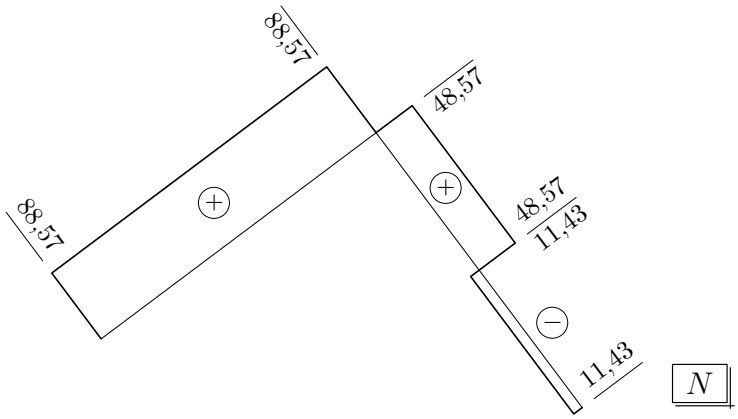


- $N_5 = N_4 = -11{,}43\,\text{kN}$
- $T_5 = T_4 = -8{,}57\,\text{kN}$
- $\sum_{[5,\mathsf{B}]} M_{/5} = 0: \quad -M_5 = 0 \quad \Rightarrow \quad M_5 = 0$

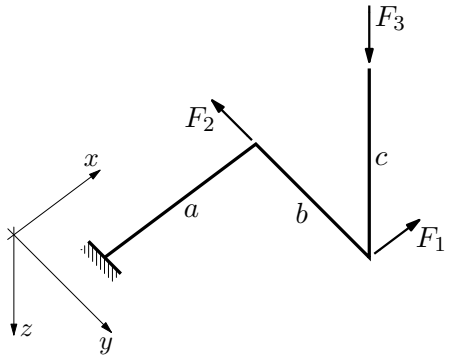
dijagrami unutarnjih sila:



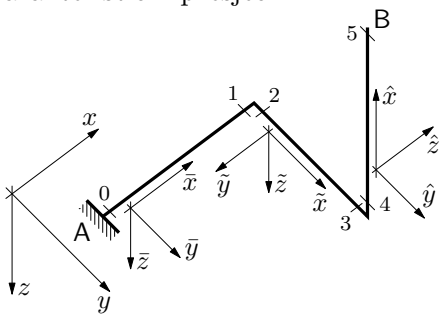




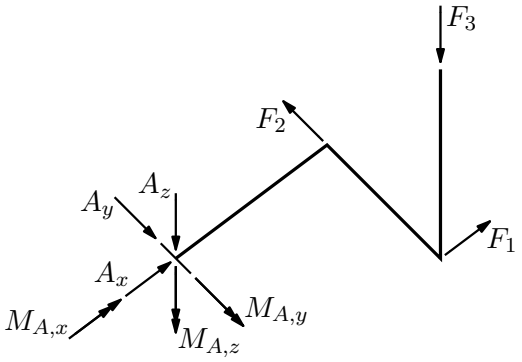
Poligonalni štap u prostoru



lokalni koordinatni sustavi
& karakteristični presjeci:



reakcije:



$\diamond \sum_{[A,B]} F_x = 0 : \qquad A_x + F_1 = 0$

$A_x = -F_1$

$\diamond \sum_{[A,B]} F_y = 0 : \qquad A_y - F_2 = 0$

$A_y = F_2$

$\diamond \sum_{[A,B]} F_z = 0 : \qquad A_z + F_3 = 0$

$A_z = -F_3$

$\diamond \sum_{[A,B]} M_{/x} = 0 : \qquad M_{A,x} + b\,F_3 = 0$

$M_{A,x} = -b\,F_3$

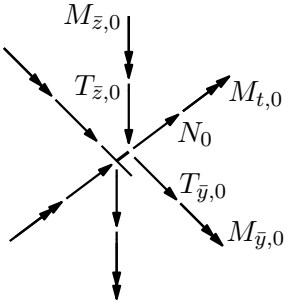
$\diamond \sum_{[A,B]} M_{/y} = 0 : \qquad M_{A,y} - a\,F_3 = 0$

$M_{A,y} = a\,F_3$

$\diamond \sum_{[A,B]} M_{/z} = 0 : \qquad M_{A,z} - a\,F_2 - b\,F_1 = 0$

$M_{A,z} = a\,F_2 + b\,F_1$

sile u presjeku 0:



$\diamond \sum_{[A,0]} F_{\bar{x}} = 0 : \qquad A_x + N_0 = 0$

$N_0 = -A_x = -(-F_1) = F_1$

$\diamond \sum_{[A,0]} F_{\bar{y}} = 0 : \qquad A_y + T_{\bar{y},0} = 0$

$T_{\bar{y},0} = -A_y = -F_2$

◇ $\sum_{[A,0]} F_{\bar{z}} = 0 : \qquad A_z + T_{\bar{z},0} = 0$

$T_{\bar{z},0} = -A_z = F_3$

◇ $\sum_{[A,0]} M_{/\bar{x}} = 0 : \qquad M_{A,x} + M_{t,0} = 0$

$M_{t,0} = -M_{A,x} = b\,F_3$

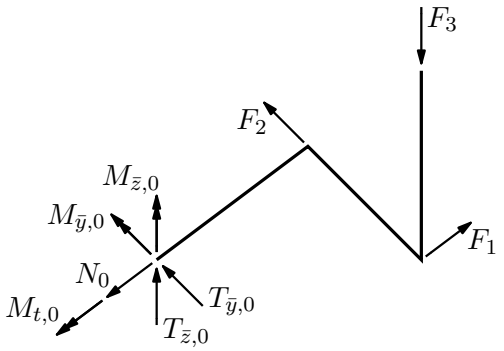
◇ $\sum_{[A,0]} M_{/\bar{y}_0} = 0 : \qquad M_{A,y} + M_{\bar{y},0} = 0$

$M_{\bar{y},0} = -M_{A,y} = -a\,F_3$

◇ $\sum_{[A,0]} M_{/\bar{z}_0} = 0 : \qquad M_{A,z} + M_{\bar{z},0} = 0$

$M_{\bar{z},0} = -M_{A,z} = -a\,F_2 - b\,F_1$

sile u presjeku 0, još jednom:



◇ $\sum_{[0,B]} F_{\bar{x}} = 0 : \qquad -N_0 + F_1 = 0$

$N_0 = F_1$

◇ $\sum_{[0,B]} F_{\bar{y}} = 0 : \qquad -T_{\bar{y},0} - F_2 = 0$

$T_{\bar{y},0} = -F_2$

◇ $\sum_{[0,B]} F_{\bar{z}} = 0 : \qquad -T_{\bar{z},0} + F_3 = 0$

$T_{\bar{z},0} = F_3$

◇ $\sum_{[0,B]} M_{/\bar{x}} = 0 : \qquad -M_{t,0} + b\,F_3 = 0$

$M_{t,0} = b\,F_3$

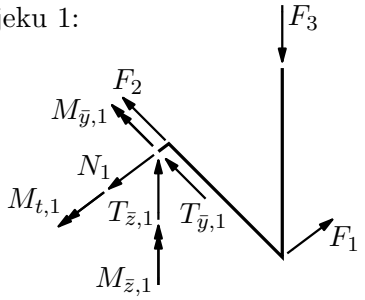
◇ $\sum_{[0,B]} M_{/\bar{y}_0} = 0 : \qquad -M_{\bar{y},0} - a\,F_3 = 0$

$M_{\bar{y},0} = -a\,F_3$

◇ $\sum_{[0,B]} M_{/\bar{z}_0} = 0 : \qquad -M_{\bar{z},0} - a\,F_2 - b\,F_1 = 0$

$M_{\bar{z},0} = -a\,F_2 - b\,F_1$

sile u presjeku 1:



◇ $N_1 = N_0 = F_1$

◇ $T_{\bar{y},1} = T_{\bar{y},0} = -F_2$

◇ $T_{\bar{z},1} = T_{\bar{z},0} = F_3$

◇ $M_{t,1} = M_{t,0} = b\,F_3$

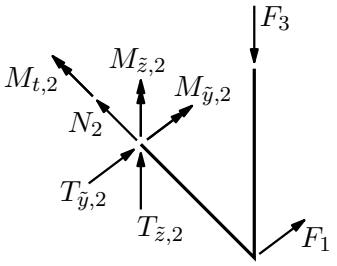
$\diamond \ \sum_{[1,\mathrm{B}]} M_{/\bar{y}_1} = 0 \ : \qquad -M_{\bar{y},1} \ = \ 0$

$M_{\bar{y},1} \ = \ 0$

$\diamond \ \sum_{[1,\mathrm{B}]} M_{/\bar{z}_1} = 0 \ : \qquad -M_{\bar{z},1} \ - \ b\,F_1 \ = \ 0$

$M_{\bar{z},1} \ = \ -b\,F_1$

sile u presjeku 2:



$\diamond \ \sum_{[2,\mathrm{B}]} F_{\tilde{x}} = 0 \ : \qquad -N_2 \ = \ 0$

$N_2 \ = \ 0$

$\diamond \ \sum_{[2,\mathrm{B}]} F_{\tilde{y}} = 0 \ : \qquad -T_{\tilde{y},2} - F_1 \ = \ 0$

$T_{\tilde{y},2} \ = \ -F_1$

$\diamond \ \sum_{[2,\mathrm{B}]} F_{\tilde{z}} = 0 \ : \qquad -T_{\tilde{z},2} + F_3 \ = \ 0$

$T_{\tilde{z},2} \ = \ F_3$

$\diamond \ \sum_{[2,\mathrm{B}]} M_{/\tilde{x}} = 0 \ : \qquad -M_{t,2} \ = \ 0$

$M_{t,2} \ = \ 0$

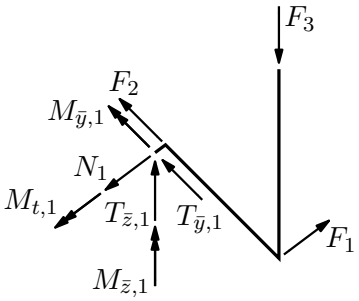
$\diamond \ \sum_{[2,\mathrm{B}]} M_{/\tilde{y}_2} = 0 \ : \qquad -M_{\tilde{y},2} \ - \ b\,F_3 \ = \ 0$

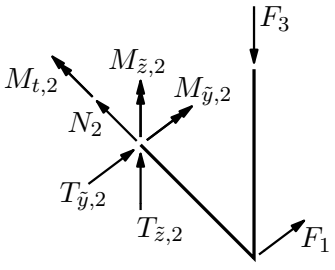
$M_{\tilde{y},2} \ = \ -b\,F_3$

$\diamond \ \sum_{[2,\mathrm{B}]} M_{/\tilde{z}_2} = 0 \ : \qquad -M_{\tilde{z},2} \ - \ b\,F_1 \ = \ 0$

$M_{\tilde{z},2} \ = \ -b\,F_1$

ili, sile u presjeku 2:



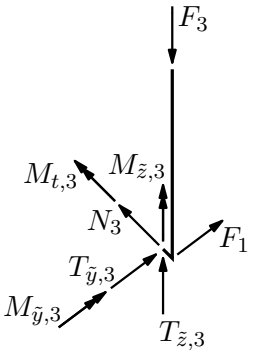


$\diamond \quad \tilde{x} \parallel \bar{y} \ : \qquad N_2 \ = \ T_{\bar{y},1} + F_2 \ = \ -F_2 + F_2 \ = \ 0, \qquad M_{t,2} \ = \ M_{\bar{y},1} \ = \ 0$

$\diamond \quad \tilde{y} \parallel\!\downarrow \bar{x} \ : \qquad T_{\tilde{y},2} \ = \ -N_1 \ = \ -F_1, \qquad M_{\tilde{y},2} \ = \ -M_{t,1} \ = \ -b\,F_3$

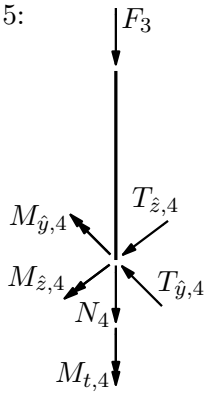
$\diamond \quad \tilde{z} \parallel \bar{z} \ : \qquad T_{\tilde{z},2} \ = \ T_{\bar{z},1} \ = \ F_3, \qquad M_{\tilde{z},2} \ = \ M_{\bar{y},1} \ = \ -b\,F_1$

sile u presjeku 3:



- $N_3 = N_1 = 0$
- $T_{\tilde{y},3} = T_{\tilde{y},2} = -F_1$
- $T_{\tilde{z},3} = T_{\tilde{z},2} = F_3$
- $\sum_{[3,\mathsf{B}]} M_{/\tilde{x}} = 0 : \qquad M_{t,3} = 0$
- $\sum_{[3,\mathsf{B}]} M_{/\tilde{y}_3} = 0 : \qquad M_{\tilde{y},3} = 0$
- $\sum_{[3,\mathsf{B}]} M_{/\tilde{z}_3} = 0 : \qquad M_{\tilde{z},3} = 0$

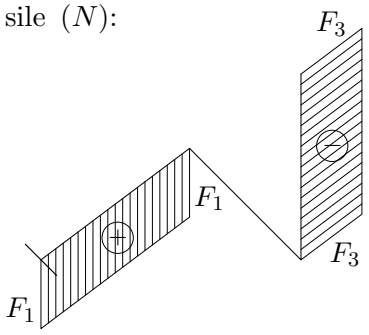
sile u presjecima 4 i 5:



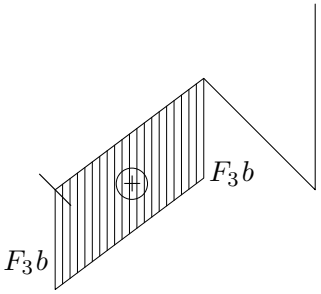
- $\sum_{[4,\mathsf{B}]} F_{\hat{x}} = 0 : \quad -N_4 - F_3 = 0, \quad N_4 = N_5 = -F_3$
- $T_{\hat{y},4} = T_{\hat{y},5} = 0$
- $T_{\hat{z},4} = T_{\hat{z},5} = 0$
- $M_{t,4} = M_{t,5} = 0$
- $M_{\hat{y},4} = M_{\hat{y},5} = 0$
- $M_{\hat{z},4} = M_{\hat{z},5} = 0$

dijagrami unutarnjih sila:

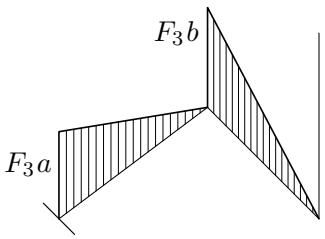
uzdužne sile (N):



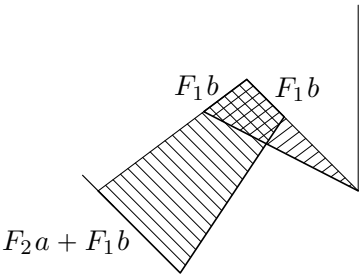
momenti torzije (M_t):



momenti savijanja oko osî u ravnini xy $(M_{\tilde{y}}$ & $M_{\tilde{y}})$:



momenti savijanja oko osî usporednih s osi z $(M_{\tilde{z}}$ & $M_{\tilde{z}})$:



[domaća zadaća: nacrtajte dijagrame poprečnih sila!]