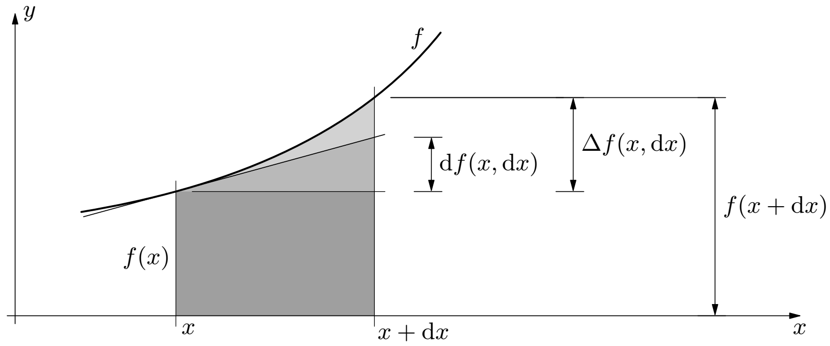
Štapna statika (2)

K. F.

Diferencijalne jednadžbe ravnoteže ravnoga štapa u ravnini

geometrijska interpretacija derivacije funkcije:



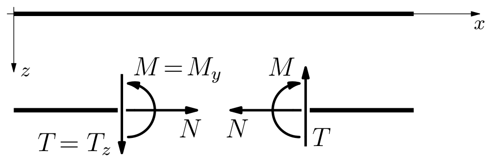
$$f(x+\mathrm{d}x) \;=\; f(x) \;+\; \Delta f(x,\mathrm{d}x) \;\approxeq\; f(x) \;+\; \mathrm{d}f(x,\mathrm{d}x)$$

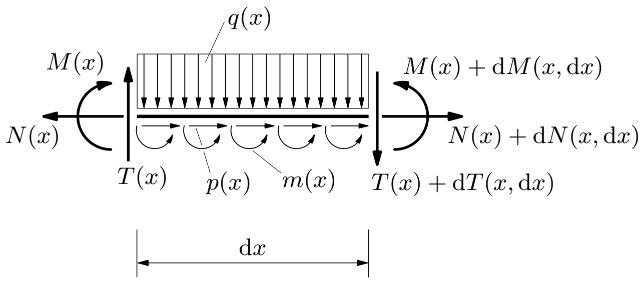
$$\frac{\mathrm{d}f(x,\mathrm{d}x)}{\mathrm{d}x} \;=\; f'(x) \qquad \text{nagib tangente na graf funkcije u točki } x$$

površina „ispod” funkcije f na odsječku duljine $\mathrm{d}x$:

$$\mathrm{d}F(x,\mathrm{d}x) \;\approxeq\; \frac{f(x) \;+\; [f(x)+\mathrm{d}f(x,\mathrm{d}x)]}{2}\,\mathrm{d}x \;=\; f(x)\,\mathrm{d}x \;+\; \underbrace{\tfrac{1}{2}\,\mathrm{d}f(x,\mathrm{d}x)\,\mathrm{d}x}_{\text{zanemarivo}} \;\approxeq\; f(x)\,\mathrm{d}x$$

štap u ravnini $\boldsymbol{xz}$





ravnoteža projekcija sila na os $\boldsymbol{x}$:

$$-N(x) \;+\; p(x)\,\mathrm{d}x \;+\; N(x) \;+\; \mathrm{d}N(x,\mathrm{d}x) \;=\; 0$$

$$\mathrm{d}N(x,\mathrm{d}x) \;+\; p(x)\,\mathrm{d}x \;=\; 0$$

$$\frac{\mathrm{d}N(x, \mathrm{d}x)}{\mathrm{d}x} + p(x) = 0$$

diferencijalna jednadžba ravnoteže: $\boxed{N'(x) + p(x) = 0}$

diferencijalni odnos: $N'(x) = -p(x)$

ravnoteža projekcija sila na os z:

$$-T(x) + q(x)\,\mathrm{d}x + T(x) + \mathrm{d}T(x, \mathrm{d}x) = 0$$

$$\mathrm{d}T(x, \mathrm{d}x) + q(x)\,\mathrm{d}x = 0$$

$$\frac{\mathrm{d}T(x, \mathrm{d}x)}{\mathrm{d}x} + q(x)x = 0$$

diferencijalna jednadžba ravnoteže: $\boxed{T'(x) + q(x) = 0}$

diferencijalni odnos: $T'(x) = -q(x)$

ravnoteža momenata (oko desnoga kraja odsječka):

$$-M(x) - T(x)\,\mathrm{d}x + m(x)\,\mathrm{d}x + \underbrace{\big[q(x)\,\mathrm{d}x\big]\frac{\mathrm{d}x}{2}}_{\text{zanemarivo}} + M(x) + \mathrm{d}M(x, \mathrm{d}x) = 0$$

$$\mathrm{d}M(x, \mathrm{d}x) - T(x)\,\mathrm{d}x + m(x)\,\mathrm{d}x = 0$$

$$\frac{\mathrm{d}M(x, \mathrm{d}x)}{\mathrm{d}x} + m(x) - T(x) = 0$$

diferencijalna jednadžba ravnoteže: $\boxed{M'(x) + m(x) - T(x) = 0}$

diferencijalni odnos: $M'(x) = -m(x) + T(x)$

ravnoteža momenata (oko lijevoga kraja):

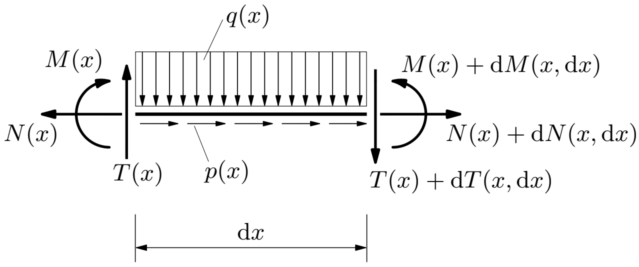
$$-M(x) + m(x)\,\mathrm{d}x - \underbrace{\big[q(x)\,\mathrm{d}x\big]\frac{\mathrm{d}x}{2}}_{\text{zanemarivo}} - T(x)\,\mathrm{d}x - \underbrace{\mathrm{d}T(x, \mathrm{d}x)\,\mathrm{d}x}_{\text{zanemarivo}} + M(x) + \mathrm{d}M(x, \mathrm{d}x) = 0$$

$$\mathrm{d}M(x, \mathrm{d}x) - T(x)\,\mathrm{d}x + m(x)\,\mathrm{d}x = 0$$

diferencijalna jednadžba ravnoteže: $M'(x) + m(x) - T(x) = 0$

veza momenta i distribuirane sile:

$m(x) \equiv 0$

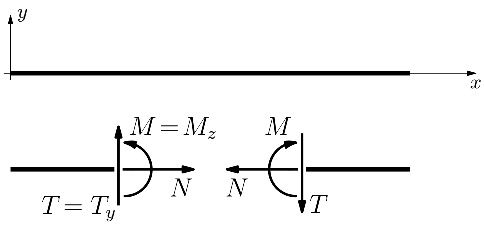


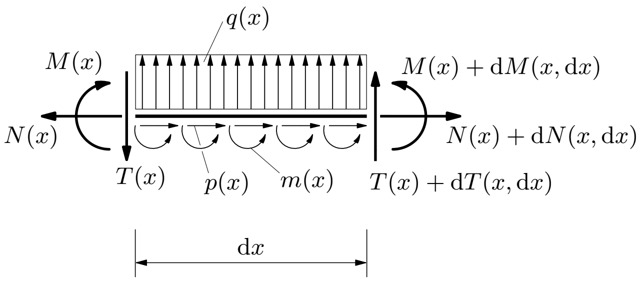
$$M'(x) - T(x) = 0 \quad \Rightarrow \quad M''(x) - T'(x) = 0$$

$$T'(x) = -q(x) \quad \Rightarrow \quad \boxed{M''(x) + q(x) = 0} \quad \text{(diferencijalna jednadžba ravnoteže)}$$

$$M''(x) = -q(x) \qquad \text{(diferencijalni odnos)}$$

štap u ravnini *xy*





$$N'(x) + p(x) = 0$$

$$T'(x) + q(x) = 0$$

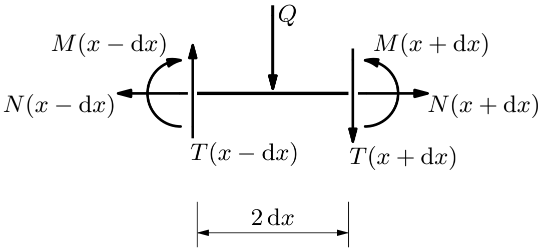
$$M'(x) + m(x) + T(x) = 0$$

$$\left[\text{ slijedi iz } \quad -M(x) + T(x)\,\mathrm{d}x + m(x)\,\mathrm{d}x - q(x)\,\frac{\mathrm{d}x^2}{2} + M(x) + \mathrm{d}M(x, \mathrm{d}x) = 0 \right]$$

$$M''(x) - q(x) = 0$$

Koncentrirana djelovanja (u ravnini xz)

djelovanje koncentrirane sile okomito na os štapa:



$$\mathrm{d}x \approxeq 0 \quad \Rightarrow \quad p(x)\cdot 2\,\mathrm{d}x \approxeq 0 \quad \& \quad q(x)\cdot 2\,\mathrm{d}x \approxeq 0$$

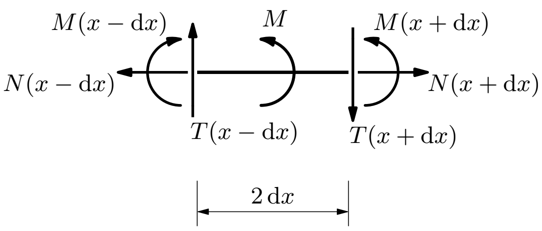
$$-N(x-\mathrm{d}x) + N(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{N(x-\mathrm{d}x) = N(x+\mathrm{d}x) = N(x)}$$

$$-T(x-\mathrm{d}x) + Q + T(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{T(x+\mathrm{d}x) - T(x-\mathrm{d}x) = -Q}$$

$$\mathrm{d}x \approxeq 0 \quad \Rightarrow \quad \mathrm{d}x\cdot Q \approxeq 0 \quad \& \quad 2\,\mathrm{d}x\cdot T(x-\mathrm{d}x) \approxeq 0$$

$$-M(x-\mathrm{d}x) + M(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{M(x-\mathrm{d}x) = M(x+\mathrm{d}x) = M(x)}$$

djelovanje koncentriranoga momenta:



$$-N(x-\mathrm{d}x) + N(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{N(x-\mathrm{d}x) = N(x+\mathrm{d}x) = N(x)}$$

$$-T(x-\mathrm{d}x) + T(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{T(x-\mathrm{d}x) = T(x+\mathrm{d}x) = T(x)}$$

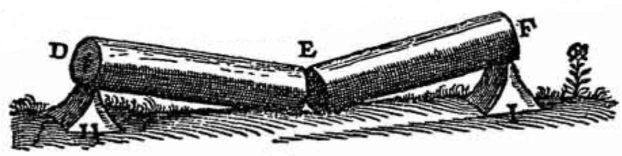
$$-M(x-\mathrm{d}x) + M + M(x+\mathrm{d}x) = 0 \quad \Rightarrow \quad \boxed{M(x+\mathrm{d}x) - M(x-\mathrm{d}x) = -M(x)}$$

djelovanje koncentrirane sile na osi štapa: [domaća zabava!]

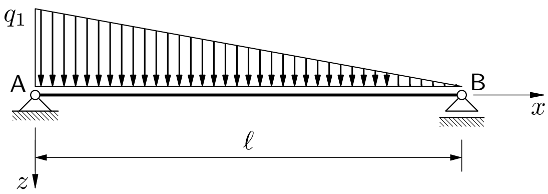
Jednostavno oslonjena greda kao primjer štapne konstrukcije

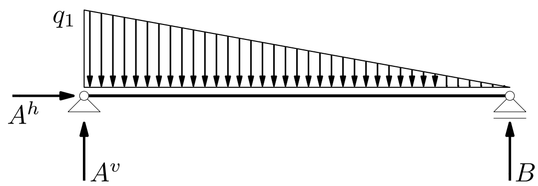
jednostavno oslonjena greda ili, tradicionalno, **prosta greda**: ravninska statički određena konstrukcija s jednim ravnim, ponajčešće horizontalno položenim štapnim elementom koji je s podlogom spojen u krajnjim točkama tako da je u jednoj točki nepomičan, a u drugoj pomičan zglobni ležaj, pri čemu je u pomičnom ležaju spriječen pomak po pravcu okomitom na os grede (ako je greda položena horizontalno, onda je taj ležaj horizontalno pomičan)

(slomljena) jednostavno oslonjena greda u Galilejevim *Razgovorima i matematičkim izlaganjima uz dvije nove znanosti* [1638.]:



sile u vanjskim vezama (reakcije):





integralni izrazi:

$$\sum F_x \;=\; 0 \;: \qquad A^h + \int_0^\ell p(x)\,\mathrm{d}x \;=\; 0$$

$$\boxed{A^h \;=\; -\int_0^\ell p(x)\,\mathrm{d}x}$$

$$\sum M_{/\mathsf{B}} \;=\; 0 \;: \qquad M_{\vec{A^v}/\mathsf{B}} + M_{\vec{q}/\mathsf{B}} \;=\; 0$$

$$-\,\ell\,A^v + \int_0^\ell (\ell - x)\,q(x)\,\mathrm{d}x \;=\; 0$$

$$\boxed{A^v \;=\; \frac{1}{\ell}\int_0^\ell (\ell - x)\,q(x)\,\mathrm{d}x}$$

$$\sum M_{/\mathsf{A}} = 0 : \qquad M_{\vec{q}/\mathsf{A}} + M_{\vec{B}/\mathsf{A}} = 0$$

$$-\int_0^\ell x\,q(x)\,\mathrm{d}x + \ell\,B = 0$$

$$\boxed{B = \frac{1}{\ell}\int_0^\ell x\,q(x)\,\mathrm{d}x}$$

$$p(x) = 0 \qquad \& \qquad q(x) = \frac{q_1}{\ell}\,(\ell - x)$$

$$A^h = -\int_0^\ell 0\,\mathrm{d}x = 0$$

$$A^v = \frac{1}{\ell}\int_0^\ell (\ell - x)\left[\frac{q_1}{\ell}\,(\ell - x)\right]\mathrm{d}x = \frac{q_1}{\ell^2}\int_0^\ell \left(\ell^2 - 2\ell x + x^2\right)\mathrm{d}x$$

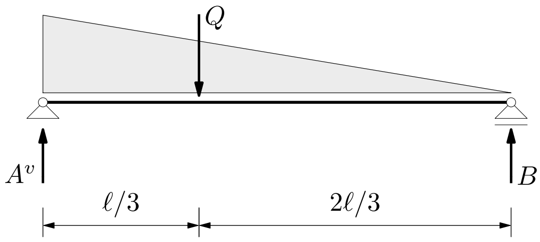
$$= \frac{q_1}{\ell^2}\left(\ell^2\,x - \ell\,x^2 + \frac{1}{3}\,x^3\right)\Bigg|_0^\ell = \frac{q_1\,\ell}{3}$$

$$B = \frac{1}{\ell}\int_0^\ell x\left[\frac{q_1}{\ell}\,(\ell - x)\right]\mathrm{d}x = \frac{q_1}{\ell^2}\int_0^\ell \left(\ell x - x^2\right)\mathrm{d}x = \frac{q_1}{\ell^2}\left(\frac{1}{2}\,\ell\,x^2 - \frac{1}{3}\,x^3\right)\Bigg|_0^\ell = \frac{q_1\,\ell}{6}$$

primjena rezultante distribuirane sile:

$$Q = \int_0^\ell q(x)\,\mathrm{d}x = \frac{q_1}{\ell}\int_0^\ell (\ell - x)\,\mathrm{d}x = \frac{q_1\,\ell}{2} \qquad \text{(geometrijski: površina trokuta)}$$

$$x_Q = \frac{M_{\vec{q}/0}}{Q} = \frac{M_{\vec{q}/\mathsf{A}}}{Q} = \frac{\displaystyle\int_0^\ell x\,q(x)\,\mathrm{d}x}{\displaystyle\int_0^\ell q(x)\,\mathrm{d}x} = \frac{q_1\,\ell^2/6}{q_1\,\ell/2} = \frac{\ell}{3} \qquad \text{(apscisa težišta trokuta)}$$



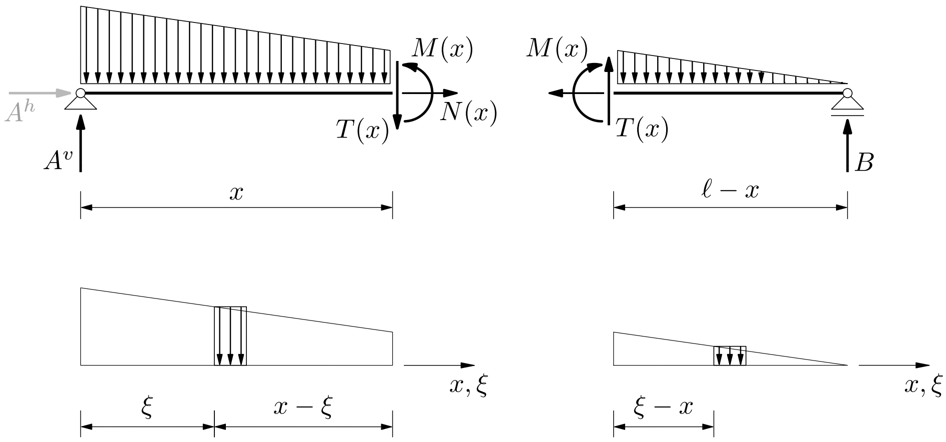
$$-\ell\,A^v + M_{\vec{Q}/\mathsf{B}} = 0 \quad \Rightarrow \quad A^v = \frac{1}{\ell}\,M_{\vec{Q}/\mathsf{B}} = \frac{1}{\ell}\,(\ell - x_Q)\,Q = \frac{1}{\ell}\left(\frac{2}{3}\,\ell\right)\left(\frac{1}{2}\,q_1\,\ell\right) = \frac{q_1\,\ell}{3}$$

$$-x_Q\,Q + \ell\,B = 0 \quad \Rightarrow \quad B = \frac{1}{\ell}\,x_Q\,Q = \frac{1}{\ell}\left(\frac{1}{3}\,\ell\right)\left(\frac{1}{2}\,q_1\,\ell\right) = \frac{q_1\,\ell}{6}$$

kontrola:

$$-A^v - B + Q = \left(-\frac{1}{3} - \frac{1}{6} + \frac{1}{2}\right)q_1\,\ell = 0$$

unutarnje sile:



integralni izrazi:

uzdužna sila:

$$\sum_{\text{lijevi dio}} F_x \;=\; 0 \;: \qquad A^h \,+\, \int_0^x p(\xi)\,\mathrm{d}\xi \,+\, N(x) \;=\; 0$$

$$\boxed{N(x) \;=\; -A^h \,-\, \int_0^x p(\xi)\,\mathrm{d}\xi}$$

$$N(x) \;=\; 0$$

$$\text{ili} \quad \sum_{\text{desni dio}} F_x \;=\; 0 \;: \qquad -N(x) \,+\, \int_x^\ell p(\xi)\,\mathrm{d}\xi \;=\; 0$$

$$\boxed{N(x) \;=\; \int_x^\ell p(\xi)\,\mathrm{d}\xi}$$

poprečna sila:

$$\sum_{\text{lijevi dio}} F_z \;=\; 0 \;: \qquad -A^v \,+\, \int_0^x q(\xi)\,\mathrm{d}\xi \,+\, T(x) \;=\; 0$$

$$\boxed{T(x) \;=\; A^v \,-\, \int_0^x q(\xi)\,\mathrm{d}\xi}$$

$$T(x) \;=\; A^v \,-\, \int_0^x \frac{q_1}{\ell}\,(\ell - \xi)\,\mathrm{d}\xi \;=\; \frac{q_1\,\ell}{3} \,-\, \frac{q_1}{\ell}\,\left(\ell\,\xi - \frac{1}{2}\,\xi^2\right)\Bigg|_0^x \;=\; \frac{q_1\,\ell}{3} \,-\, q_1\,x \,+\, \frac{q_1}{2\ell}\,x^2$$

ili $\displaystyle\sum_{\text{desni dio}} F_z = 0 : \qquad -T(x) + \int_x^\ell q(\xi)\,\mathrm{d}\xi - B = 0$

$$\boxed{T(x) = -B + \int_x^\ell q(\xi)\,\mathrm{d}\xi}$$

$$T(x) = -B + \frac{q_1}{\ell}\int_x^\ell (\ell-\xi)\,\mathrm{d}\xi = -\frac{q_1\,\ell}{6} + \frac{q_1}{\ell}\left(\ell\,\xi - \frac{1}{2}\,\xi^2\right)\bigg|_x^\ell = \frac{q_1\,\ell}{3} - q_1\,x + \frac{q_1}{2\ell}\,x^2$$

integralni odnos: distribuirana sila – linearna ⇒ poprečna sila – polinom drugog stupnja

općenito: poprečna sila – polinom za stupanj viši od stupnja polinoma distribuirane sile (ako je distribuirana sila zadana polinomom)

moment savijanja:

$$\sum_{\text{lijevi dio}} M_{/x} = 0 : \qquad -x\,A^v + \int_0^x (x-\xi)\,q(\xi)\,\mathrm{d}\xi + M(x) = 0$$

$$\boxed{M(x) = x\,A^v - \int_0^x (x-\xi)\,q(\xi)\,\mathrm{d}\xi}$$

$$M(x) = x\cdot A^v - \int_0^x (x-\xi)\left[\frac{q_1}{\ell}\,(\ell-\xi)\right]\mathrm{d}\xi = \frac{q_1\,\ell}{3}\,x - \frac{q_1}{\ell}\left[\ell\,x\,\xi - \frac{1}{2}\,(\ell+x)\,\xi^2 + \frac{1}{3}\,\xi^3\right]\bigg|_0^x$$

$$= \frac{q_1\,\ell}{3}\,x - \frac{q_1}{2}\,x^2 + \frac{q_1}{6\ell}\,x^3$$

ili $\displaystyle\sum_{\text{desni dio}} M_{/x} = 0 : \qquad -M(x) - \int_x^\ell (\xi-x)\,q(\xi)\,\mathrm{d}\xi + (\ell-x)\,B = 0$

$$\boxed{M(x) = (\ell-x)\,B - \int_x^\ell (\xi-x)\,q(\xi)\,\mathrm{d}\xi}$$

$$M(x) = (\ell-x)\,B - \frac{q_1}{\ell}\int_x^\ell (\xi-x)\,(\ell-\xi)\,\mathrm{d}\xi$$

$$= \frac{q_1\ell}{6}\,(\ell-x) - \frac{q_1}{\ell}\left[-\ell\,x\,\xi + \frac{1}{2}\,(\ell+x)\,\xi^2 - \frac{1}{3}\,\xi^3\right]\bigg|_x^\ell = \frac{q_1\,\ell}{3}\,x - \frac{q_1}{2}\,x^2 + \frac{q_1}{6\ell}\,x^3$$

integralni odnos: distribuirana sila – linearna ⇒ moment – polinom trećeg stupnja

općenito: moment – polinom za dva stupnja viši od stupnja polinoma distribuirane sile (ako je distribuirana sila zadana polinomom) *najveća vrijednost momenta*:

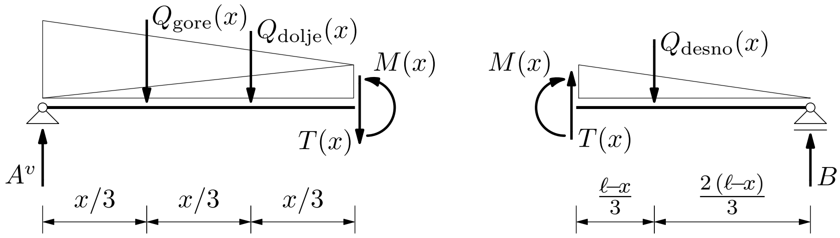
$$M'(x) \;=\; 0 \qquad\qquad \Big[\, M'(x) \;=\; T(x) \;=\; 0 \,\Big]$$

$$\left(\frac{q_1\,\ell}{3}\,x \;-\; \frac{q_1}{2}\,x^2 \;+\; \frac{q_1}{6\ell}\,x^3\right)' \;=\; \frac{q_1\,\ell}{3} \;-\; q_1\,x \;+\; \frac{q_1}{2\ell}\,x^2 \;=\; 0$$

$$\frac{\ell}{3} \;-\; x \;+\; \frac{1}{2\ell}\,x^2 \;=\; 0 \qquad \Longrightarrow \qquad x_{1,2} \;=\; \ell\left(1 \mp \frac{\sqrt{3}}{3}\right)$$

$$x_{\max} \;=\; \ell\left(1 \;-\; \frac{\sqrt{3}}{3}\right) \;=\; 0{,}423\,\ell \qquad \Longrightarrow \qquad M_{\max} \;=\; M(x_{\max}) \;=\; \frac{\sqrt{3}}{27}\,q_1\,\ell^2 \;=\; 0{,}06415\,q_1\,\ell^2$$

primjena rezultanata distribuirane sile:



dio lijevo od presjeka:

$$Q_{\text{gore}}(x) \;=\; \frac{q_1}{2}\,x \qquad \text{i} \qquad x_{Q_{\text{gore}}} \;=\; \frac{1}{3}\,x$$

$$Q_{\text{dolje}}(x) \;=\; \frac{1}{2}\,q(x)\,x = \frac{q_1}{2\ell}\,\big(\ell x - x^2\big) \qquad \text{i} \qquad x_{Q_{\text{dolje}}} \;=\; \frac{2}{3}\,x$$

$$-A^v \;+\; Q_{\text{gore}}(x) \;+\; Q_{\text{dolje}}(x) \;+\; T(x) \;=\; 0$$

$$\Rightarrow \qquad T(x) \;=\; A^v \;-\; Q_{\text{gore}}(x) \;-\; Q_{\text{dolje}}(x) \;=\; \frac{q_1\,\ell}{3} \;-\; q_1\,x \;+\; \frac{q_1}{2\ell}\,x^2$$

$$-x\,A^v \;+\; \tfrac{2}{3}\,x\,Q_{\text{gore}}(x) \;+\; \tfrac{1}{3}\,x\,Q_{\text{dolje}}(x) \;+\; M(x) = 0$$

$$\Rightarrow \qquad M(x) \;=\; x\,A^v \;-\; \tfrac{2}{3}\,x\,Q_{\text{gore}}(x) \;-\; \tfrac{1}{3}\,x\,Q_{\text{dolje}}(x) \;=\; \frac{q_1\,\ell}{3}\,x \;-\; \frac{q_1}{2}\,x^2 \;+\; \frac{q_1}{6\ell}\,x^3$$

dio desno od presjeka:

$$Q_{\text{desno}}(x) \;=\; \frac{1}{2}\,q(x)\,(\ell - x) \;=\; \frac{q_1}{2\ell}\,(\ell - x)^2 \;=\; \frac{q_1}{2\ell}\,\big(\ell^2 - 2\,\ell\,x + x^2\big)$$

$$-T(x) \;+\; Q_{\text{desno}}(x) \;-\; B \;=\; 0 \qquad \Rightarrow \qquad T(x) \;=\; Q_{\text{desno}}(x) \;-\; B$$

$$-M(x) \;-\; \frac{1}{3}\,(\ell - x)\,Q_{\text{desno}}(x) \;+\; (\ell - x)\,B \;=\; 0$$

$$\Rightarrow \qquad M(x) \;=\; (\ell - x)\,B \;-\; \frac{1}{3}\,(\ell - x)\,Q_{\text{desno}}(x)$$

kontrole:

$$M'(x) \;=\; T(x)$$

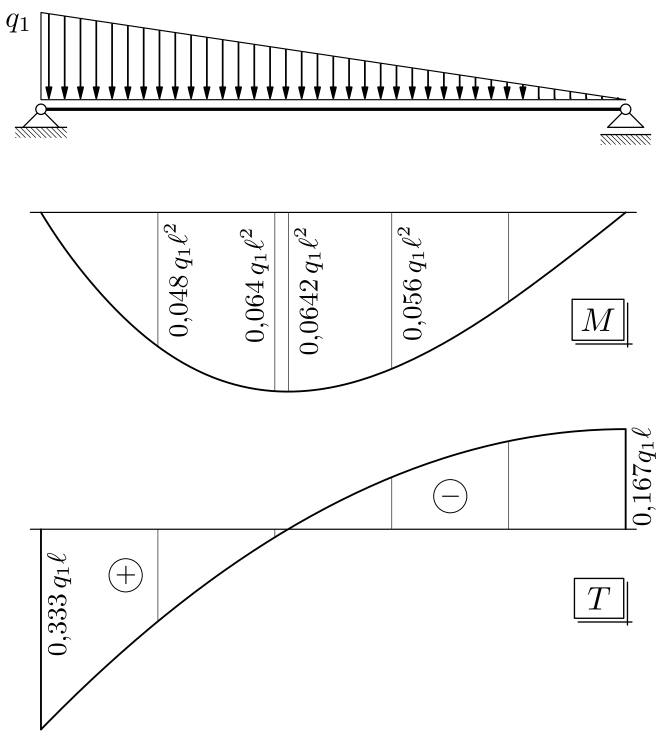
$$T'(x) \;=\; \left(\frac{q_1\,\ell}{3} - q_1\,x + \frac{q_1}{2\ell}\,x^2\right)' \;=\; -q_1 + \frac{q_1}{\ell}\,x \;=\; -\frac{q_1}{\ell}\,(\ell - x) \;=\; -q(x)$$

$$T(0+\mathrm{d}x) \;\approxeq\; T(0) \;=\; \frac{q_1\,\ell}{3} \;=\; A^v \qquad \mathrm{i} \qquad T(\ell-\mathrm{d}x) \;\approxeq\; T(\ell) \;=\; -\frac{q_1\,\ell}{6} \;=\; -B$$

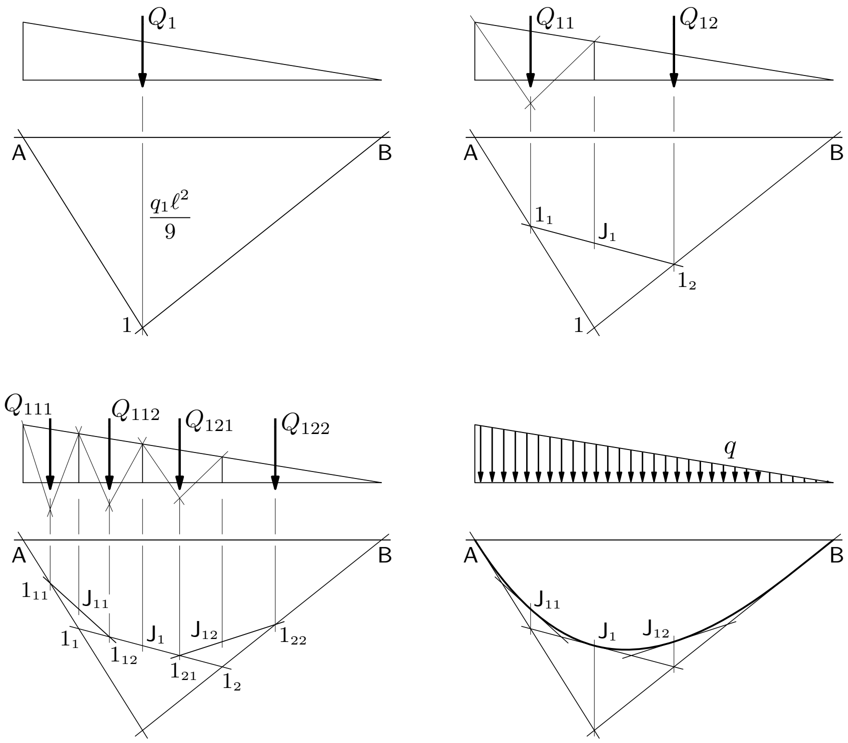
$$M(0+\mathrm{d}x) \;\approxeq\; M(0) \;=\; 0 \qquad \mathrm{i} \qquad M(\ell-\mathrm{d}x) \;\approxeq\; M(\ell) \;=\; 0$$

dijagrami unutarnjih sila:

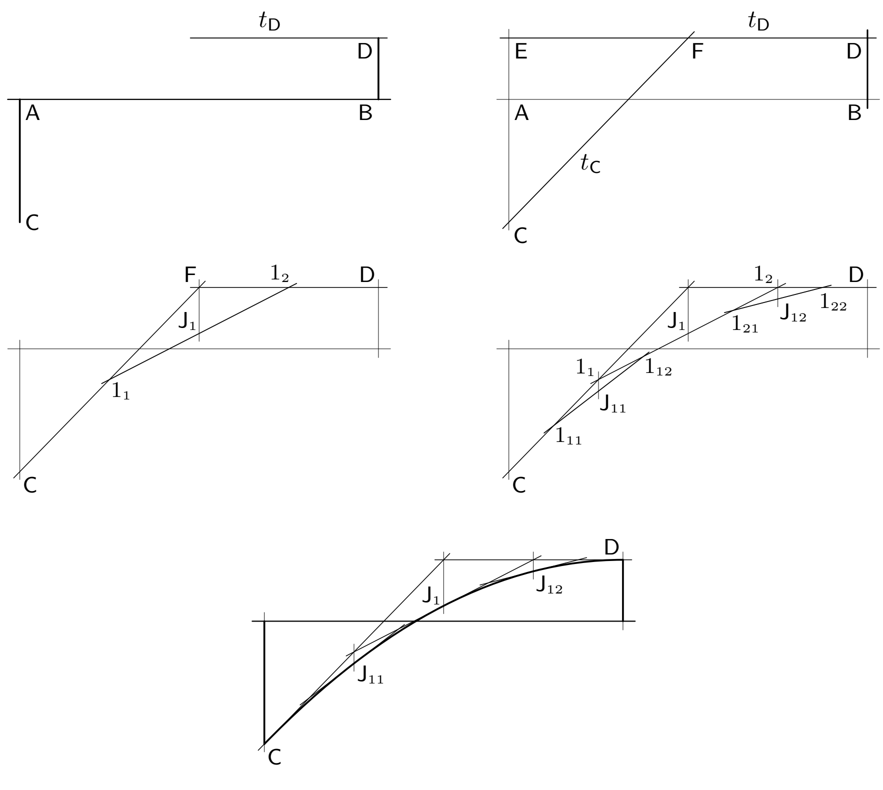
x	M(x)	T(x)
$0{,}0\,\ell$	$0{,}0$	$0{,}3333\ q_1\,\ell$
$0{,}2\,\ell$	$0{,}0480\ q_1\,\ell^2$	$0{,}1533\ q_1\,\ell$
$0{,}4\,\ell$	$0{,}0640\ q_1\,\ell^2$	$0{,}0133\ q_1\,\ell$
$0{,}423\,\ell$	$0{,}0642\ q_1\,\ell^2$	$0{,}0$
$0{,}6\,\ell$	$0{,}0560\ q_1\,\ell^2$	$-0{,}0867\ q_1\,\ell$
$0{,}8\,\ell$	$0{,}0320\ q_1\,\ell^2$	$-0{,}1467\ q_1\,\ell$
$1{,}0\,\ell$	$0{,}0$	$-0{,}1667\ q_1\,\ell$



rekurzivno crtanje momentnoga dijagrama:

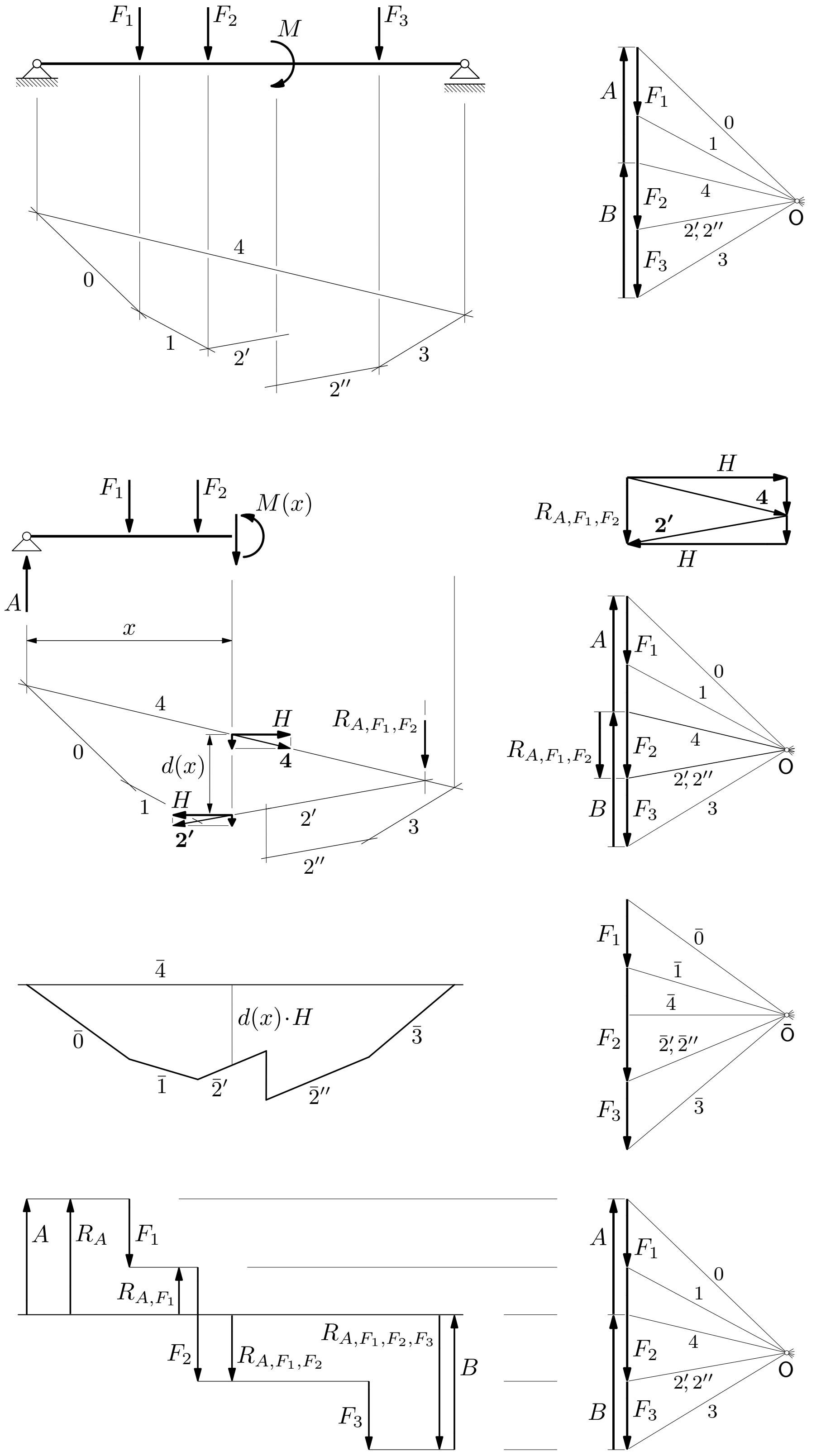


(rekurzivna) konstrukcija kvadratne parabole u dijagramu poprečnih sila:



Jednostavno oslonjena greda, još jednom

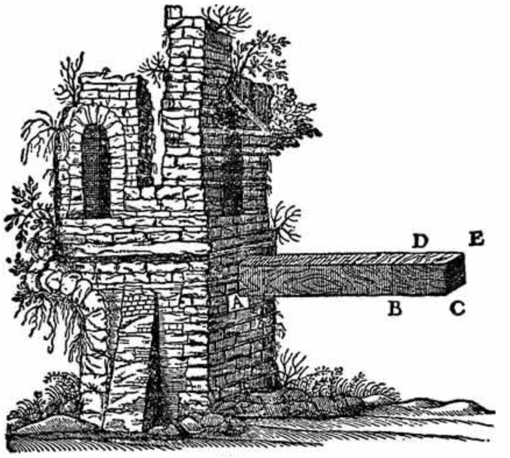
veza verižnoga poligona i momentnoga dijagrama te veza poligona sila i dijagrama poprečnih sila:



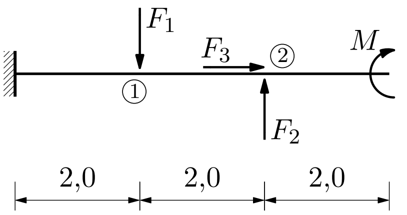
Konzola, kao treći primjer

konzola: statički određena konstrukcija s jednim ravnim štapnim elementom koji je na jednome kraju upet u podlogu, a na drugomu je slobodan

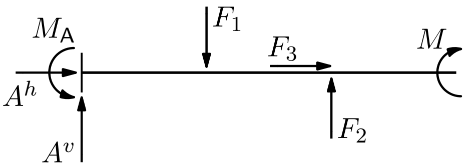
konzole u Galilejevim *Razgovorima . . .* :



izračunavanje **vrijednostî unutarnjih sila u karakterističnim točkama**:



$F_1 = 120{,}0$ kN
$F_2 = 50{,}0$ kN
$F_3 = 100{,}0$ kN (na osi grede)
$M = 50{,}0$ kNm



reakcije:

◇ $\sum F_x = 0$: $\quad A^h + F_3 = 0$

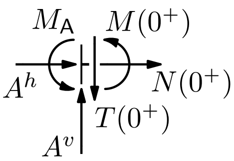
$A^h = -F_3 = -100{,}0$ kN

◇ $\sum F_z = 0$: $\quad -A^v + F_1 - F_2 = 0$

$A^v = F_1 - F_2 = 70{,}0$ kN

◇ $\sum M_{/\mathsf{A}} = 0$: $\quad M_\mathsf{A} - 2 \cdot F_1 + 4 \cdot F_2 - M = 0$

$M_\mathsf{A} = 2\,F_1 - 4\,F_2 + M = 90{,}0$ kNm



presjek $0^+ = 0 + \mathrm{d}x$:

◇ $\sum_{[0,\,0^+]} F_x = 0$: $\quad A^h + N(0^+) = 0$

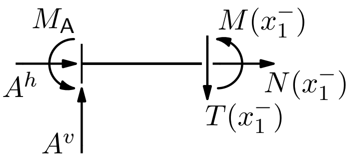
$N(0^+) = -A^h = -(-100{,}0) = 100{,}0$ kN

$$\diamond \sum_{[0,\,0^+]} F_z = 0 \;: \qquad -A^v + T(0^+) \;=\; 0$$

$$T(0^+) \;=\; A^v \;=\; 70{,}0\ \mathrm{kN}$$

$$\diamond \sum_{[0,\,0^+]} M_{/0^+} \;=\; 0 \;: \qquad M_{\mathsf{A}} + M(0^+) \;=\; 0$$

$$M(0^+) \;=\; -M_{\mathsf{A}} \;=\; -90{,}0\ \mathrm{kNm}$$



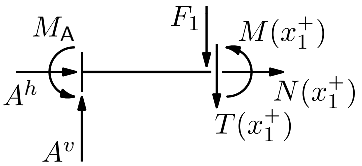
presjek $x_1^- = x_1 - \mathrm{d}x$:

$$\diamond\ N(x_1^-) \;=\; N(0^+) \;=\; 100{,}0\ \mathrm{kN}$$

$$\diamond\ T(x_1^-) \;=\; T(0^+) \;=\; 70{,}0\ \mathrm{kN}$$

$$\diamond \sum_{[0,\,x_1^-]} M_{/x_1^-} \;=\; 0 \;: \qquad M_{\mathsf{A}} - 2\cdot A^v + M(x_1^-) \;=\; 0$$

$$M(x_1^-) \;=\; -M_{\mathsf{A}} + 2\,A^v \;=\; 50{,}0\ \mathrm{kNm}$$



presjek $x_1^+ = x_1 + \mathrm{d}x$:

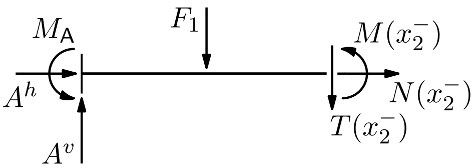
$$\diamond\ N(x_1^+) \;=\; N(x_1^-) \;=\; 100{,}0\ \mathrm{kN}$$

$$\diamond \sum_{[0,\,x_1^+]} F_z = 0 \;: \qquad -A^v + F_1 + T(x_1^+) \;=\; 0$$

$$T(x_1^+) \;=\; A^v - F_1 \;=\; -50{,}0\ \mathrm{kN}$$

$$\Big[\ T(x_1^+) - T(x_1^-) \;=\; -F_1\ \Big]$$

$$\diamond\ M(x_1^+) \;=\; M(x_1^-) \;=\; 50{,}0\ \mathrm{kNm}$$



presjek $x_2^- = x_2 - \mathrm{d}x$:

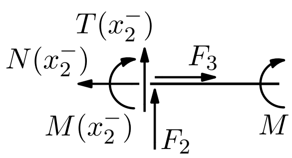
$$\diamond\ N(x_2^-) \;=\; N(x_1^+) \;=\; 100{,}0\ \mathrm{kN}$$

$$\diamond\ T(x_2^-) \;=\; T(x_1^+) \;=\; -50{,}0\ \mathrm{kN}$$

$$\diamond \sum_{[0,\,x_2^-]} M_{/x_2^-} \;=\; 0 \;:$$

$$M_{\mathsf{A}} - 4\cdot A^v + 2\cdot F_1 + M(x_2^-) \;=\; 0$$

$$M(x_2^-) = -M_{\mathsf{A}} + 4\,A^v - 2\,F_1 \;=\; -50{,}0\ \mathrm{kNm}$$



presjek x_2^-, još jednom:

$$\diamond \sum_{[x_2^-,\,\ell]} F_x = 0 \;: \qquad -N(x_2^-) + F_3 \;=\; 0$$

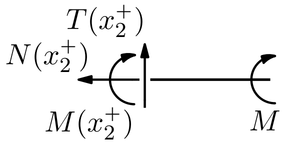
$$N(x_2^-) \;=\; F_3 \;=\; 100{,}0\ \mathrm{kN}$$

$$\diamond \sum_{[x_2^-,\,\ell]} F_z = 0 \;: \qquad -T(x_2^-) - F_2 + \;=\; 0$$

$$T(x_2^-) \;=\; -F_2 \;=\; -50{,}0\;\mathrm{kN}$$

$$\diamond \sum_{[x_2^-,\,\ell]} M_{/x_2^-} \;=\; 0 \;: \qquad -M(x_2^-) - M \;=\; 0$$

$$M(x_2^-) \;=\; -M \;=\; -50{,}0\;\mathrm{kNm}$$



presjek $x_2^+ = x_2 + \mathrm{d}x$:

$$\diamond \sum_{[x_2^+,\,\ell]} F_x = 0 \;: \qquad -N(x_2^+) \;=\; 0$$

$$N(x_2^+) \;=\; 0 \qquad \Big[\, N(x_2^+) - N(x_2^-) \;=\; -F_3 \,\Big]$$

$$\diamond \sum_{[x_2^+,\,\ell]} F_z = 0 \;: \qquad -T(x_2^+) \;=\; 0$$

$$T(x_2^+) \;=\; 0 \qquad \Big[\, T(x_2^+) - T(x_2^-) \;=\; F_2 \,\Big]$$

$$\diamond \; M(x_2^+) \;=\; M(x_2^-) \;=\; -50{,}0\;\mathrm{kNm}$$

dijagrami unutanjih sila: