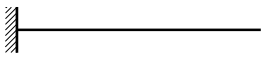
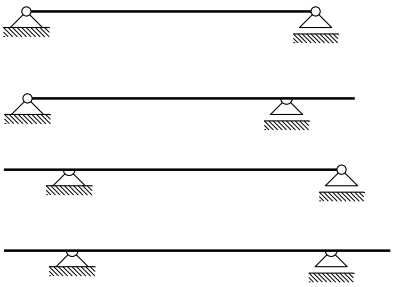
# Složeniji statički određeni štapni sistemi u ravnini

K. F.

### Elementarni statički određeni sistemi

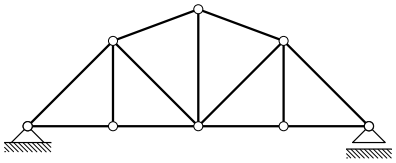
jedno tijelo & podloga:

- (punostjena) konzola 
- (punostjena) jednostavno oslonjena greda (s varijacijama)



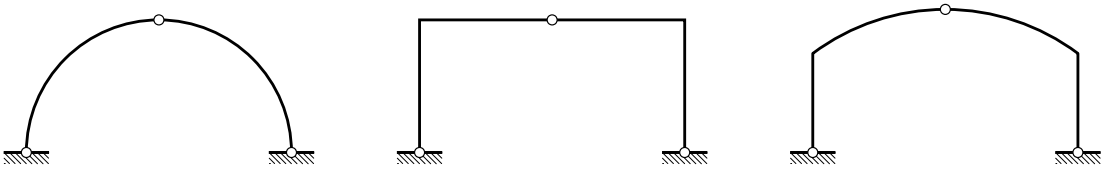
greda s prepustom

greda s prepustima

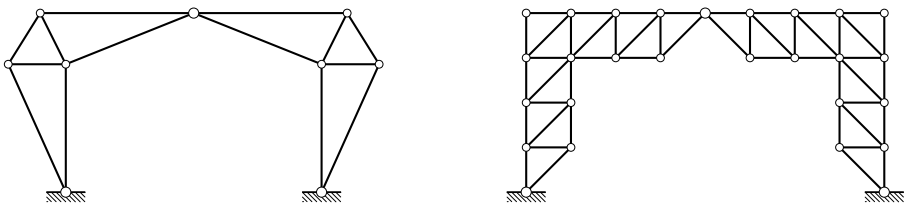
- (punostjena) greda spojena s pomoću tri zglobna štapa 
- rešetkasta jednostavno oslonjena greda 

dva tijela & podloga:

- (punostjeni) trozglobni lukovi & okviri:



- rešetkasti trozglobni okviri:

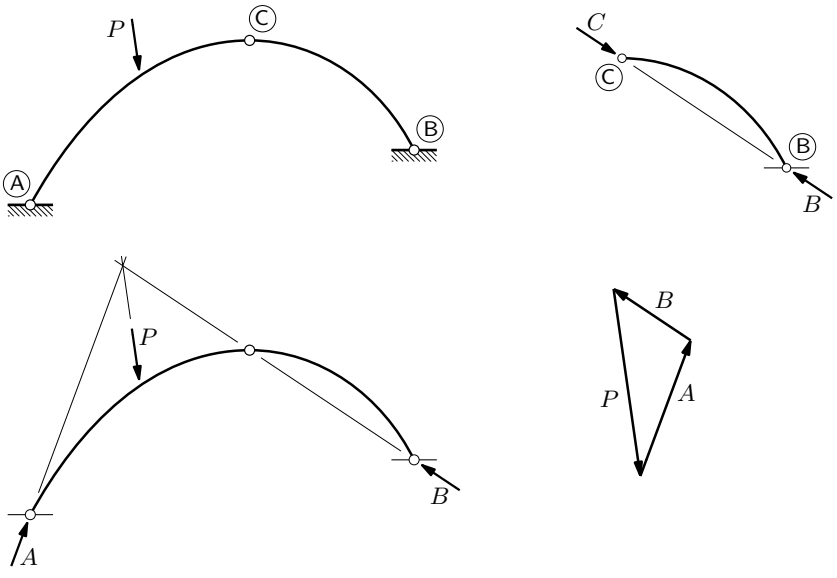


## Trozglobni lukovi & okviri

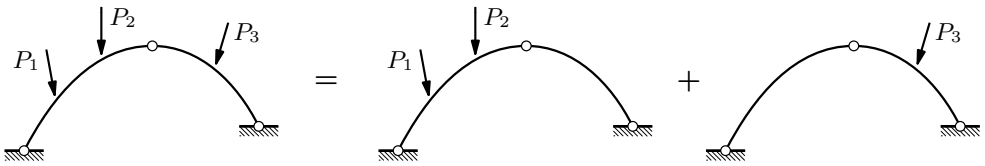
Pont Alexandre III, Pariz, 1896.–1900.

grafički način određivanja sila u spojevima:

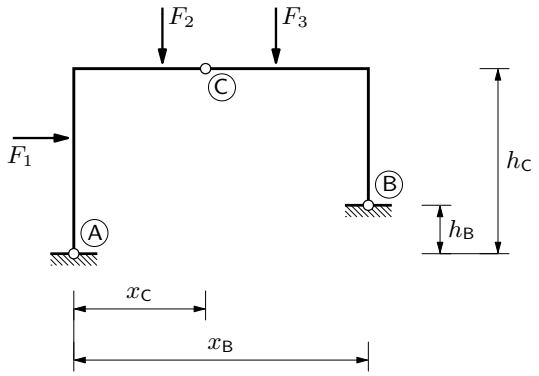
opterećeno jedno tijelo:

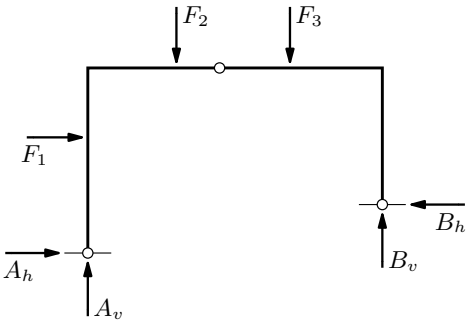


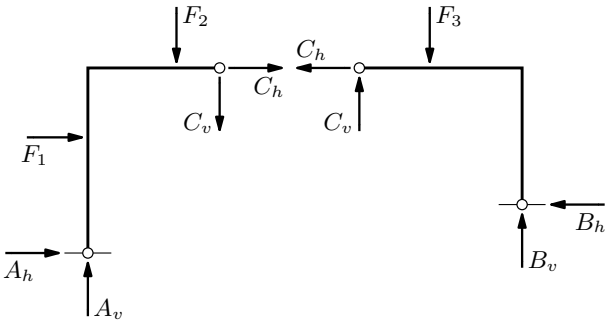
opterećena oba tijela:




analitički način izračunavanja sila u spojevima:




$$\sum_{[A,C]} F_x = \underline{A_h} + F_1 + \underline{C_h} = 0$$

$$\sum_{[C,B]} F_x = -\underline{C_h} - \underline{B_h} = 0$$

$$\sum_{[A,B]} F_x = \underline{A_h} + F_1 - \underline{B_h} = 0 \qquad \Longrightarrow \qquad \sum_{[A,B]} F_x = \sum_{[A,C]} F_x + \sum_{[C,B]} F_x$$

$$\sum_{[A,C]} F_z = -\underline{A_v} + F_2 + \underline{C_v} = 0$$

$$\sum_{[C,B]} F_z = -\underline{C_v} + F_3 - \underline{B_v} = 0$$

$$\sum_{[A,B]} F_z = -\underline{A_v} + F_2 + F_3 - \underline{B_v} = 0 \qquad \Longrightarrow \qquad \sum_{[A,B]} F_z = \sum_{[A,C]} F_z + \sum_{[C,B]} F_z$$

$$\sum_{[\mathsf{A},\mathsf{C}]} M_{/\mathsf{A}} \;=\; -h_1\,F_1 \;-\; x_2\,F_2 \;-\; x_\mathsf{C}\,\underline{C_v} \;-\; h_\mathsf{C}\,\underline{C_h} \;=\; 0$$

$$\sum_{[\mathsf{C},\mathsf{B}]} M_{/\mathsf{B}} \;=\; -(x_\mathsf{B} - x_\mathsf{C})\,\underline{C_v} \;+\; (h_\mathsf{C} - h_\mathsf{B})\,\underline{C_h} \;+\; (x_\mathsf{B} - x_3)\,F_3 \;=\; 0$$

$$\sum_{[\mathsf{A},\mathsf{B}]} M_{/\mathsf{A}} \;=\; -h_1\,F_1 \;-\; x_2\,F_2 \;-\; x_3\,F_3 \;+\; x_\mathsf{B}\,\underline{B_v} \;+\; h_\mathsf{B}\,\underline{B_h} \;=\; 0$$

$$-h_\mathsf{B} \sum_{[\mathsf{C},\mathsf{B}]} F_x \;=\; h_\mathsf{B}\,\underline{C_h} \;+\; h_\mathsf{B}\,\underline{B_h} \;=\; 0$$

$$-x_\mathsf{B} \sum_{[\mathsf{C},\mathsf{B}]} F_z \;=\; x_\mathsf{B}\,\underline{C_v} \;-\; x_\mathsf{B}\,F_3 \;+\; x_\mathsf{B}\,\underline{B_v} \;=\; 0$$

$$\Longrightarrow \qquad \sum_{[\mathsf{A},\mathsf{C}]} M_{/\mathsf{A}} \;+\; \sum_{[\mathsf{C},\mathsf{B}]} M_{/\mathsf{B}} \;-\; h_\mathsf{B} \sum_{[\mathsf{C},\mathsf{B}]} F_x \;-\; x_\mathsf{B} \sum_{[\mathsf{C},\mathsf{B}]} F_z$$

$$= \; -h_1\,F_1 \;-\; x_2\,F_2 \;+\; x_\mathsf{B}\,\underline{B_v} \;+\; h_\mathsf{B}\,\underline{B_h} \;-\; x_3\,F_3 \;=\; \sum_{[\mathsf{A},\mathsf{B}]} M_{/\mathsf{A}}$$

zaključak: od mogućih devet jednadžbi ravnoteže samo ih je šest (ispravno odabranih) linearno nezavisno

uobičajeni redoslijed rješavanja:

$$\sum_{[\mathsf{A},\mathsf{B}]} M_{/\mathsf{B}} = 0 \;:\quad \underline{A_h}\,h_\mathsf{B} - \underline{A_v}\,x_\mathsf{B} - F_1\,(h_1 - h_\mathsf{B}) + F_2\,(x_\mathsf{B} - x_2) + F_3\,(x_\mathsf{B} - x_3) \;=\; 0$$

$$\sum_{[\mathsf{A},\mathsf{C}]} M_{/\mathsf{C}} = 0 \;:\quad \underline{A_h}\,h_\mathsf{C} - \underline{A_v}\,x_\mathsf{C} + F_1\,(h_\mathsf{C} - h_1) + F_2\,(x_\mathsf{c} - x_2) \;=\; 0$$

$$\Rightarrow \qquad A_h \;\&\; A_v$$

$$\sum_{[\mathsf{A},\mathsf{B}]} M_{/\mathsf{A}} = 0 \;:\quad -F_1\,h_1 - F_2\,x_2 - F_3\,x_3 + \underline{B_v}\,x_\mathsf{B} + \underline{B_h}\,h_\mathsf{B} \;=\; 0$$

$$\sum_{[\mathsf{C},\mathsf{B}]} M_{/\mathsf{C}} = 0 \;:\quad -F_3\,(x_3 - x_\mathsf{c}) - \underline{B_h}\,(h_\mathsf{C} - h_\mathsf{B}) + \underline{B_v}\,(x_\mathsf{C} - x_\mathsf{C}) \;=\; 0$$

$$\Rightarrow \qquad B_h \;\&\; B_v$$

provjera $A_h$ & $B_h$: $\sum_{[\mathsf{A},\mathsf{B}]} F_x$

provjera $A_v$ & $B_v$: $\sum_{[\mathsf{A},\mathsf{B}]} F_z$

$$\sum_{[\mathsf{A},\mathsf{C}]} F_x = 0 \;:\quad \underline{A_h} + F_1 + \underline{C_h} \;=\; 0 \qquad \Rightarrow \qquad C_h$$

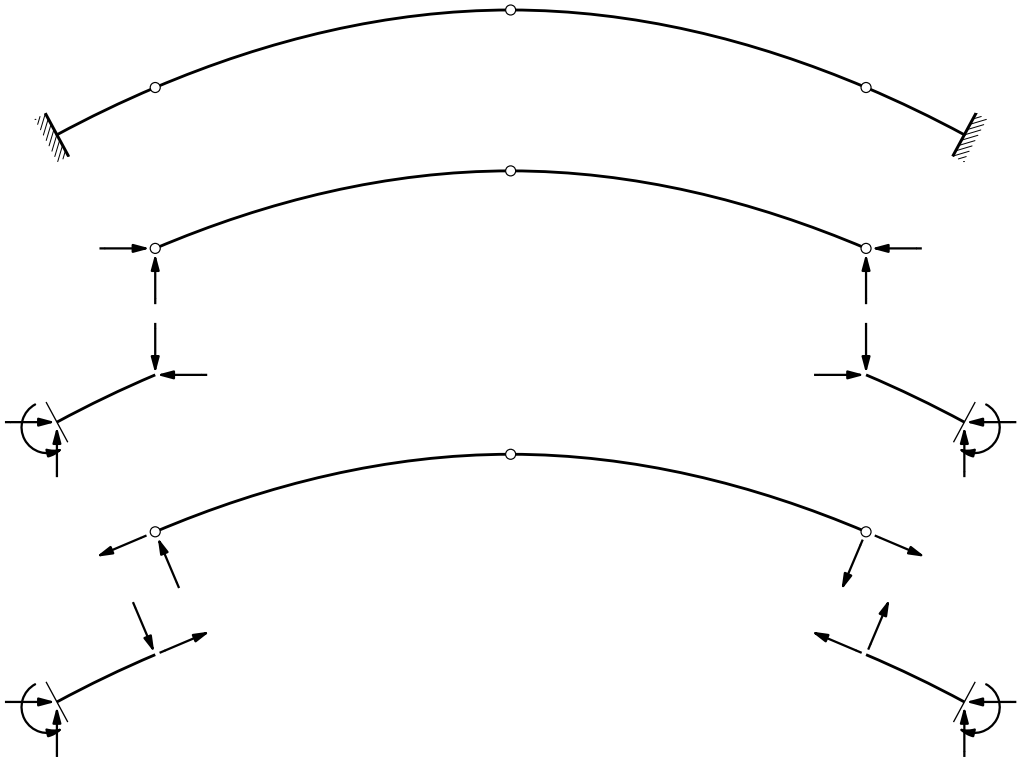
$$\sum_{[\mathsf{A},\mathsf{C}]} F_z = 0 \;:\quad -\underline{A_v} + F_2 + \underline{C_v} \;=\; 0 \qquad \Rightarrow \qquad C_v$$

provjera $C_h$ & $C_v$: $\sum_{[\mathsf{C},\mathsf{B}]} F_x$ & $\sum_{[\mathsf{C},\mathsf{B}]} F_z$

## Ulančeni sistem koji sadrži trozglobni luk

Viaduc d'Austerlitz, Pariz, 1904.

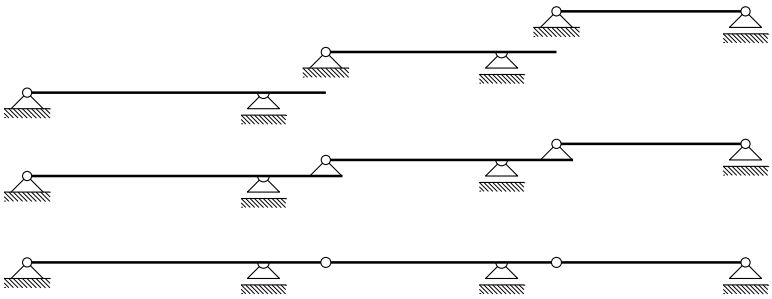
proračunska shema:



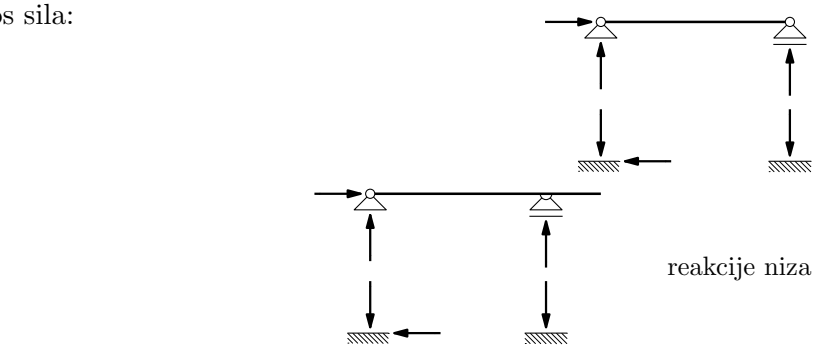
## Ulančeni Gerberovi nosači

1. primjer:

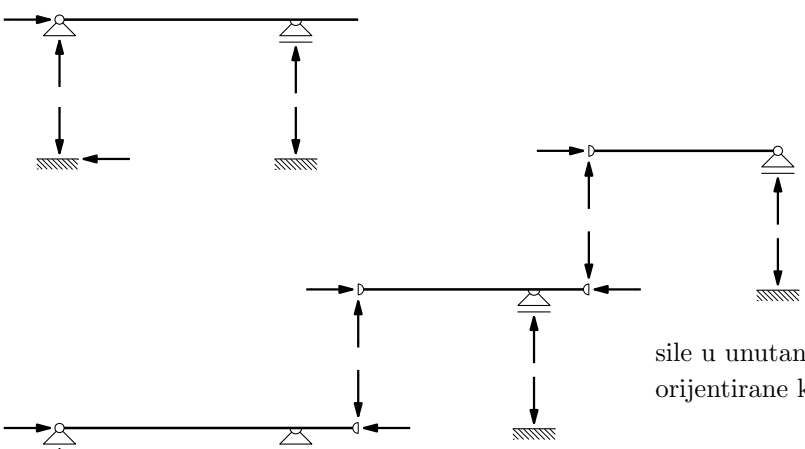
sastavljanje:



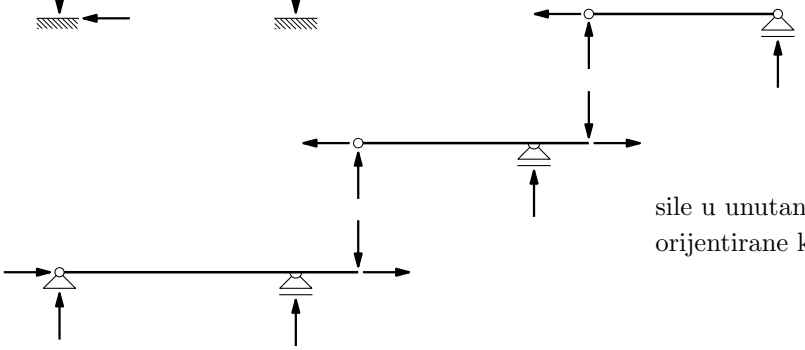
prijenos sila:



reakcije niza greda

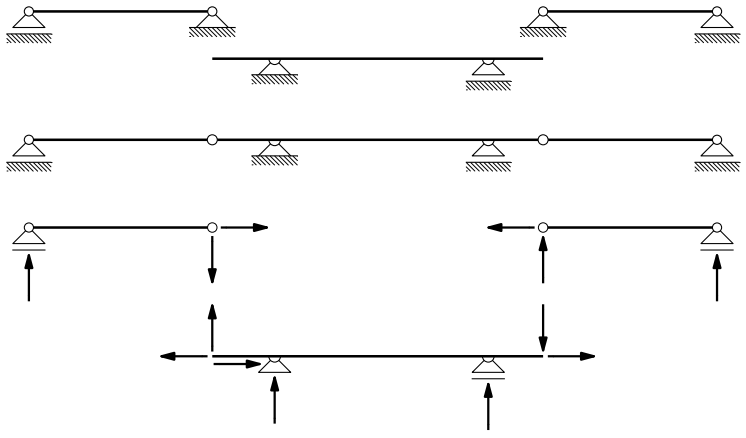


sile u unutanjim spojevima
orijentirane kao reakcije



sile u unutanjim spojevima
orijentirane kao unutanje sile

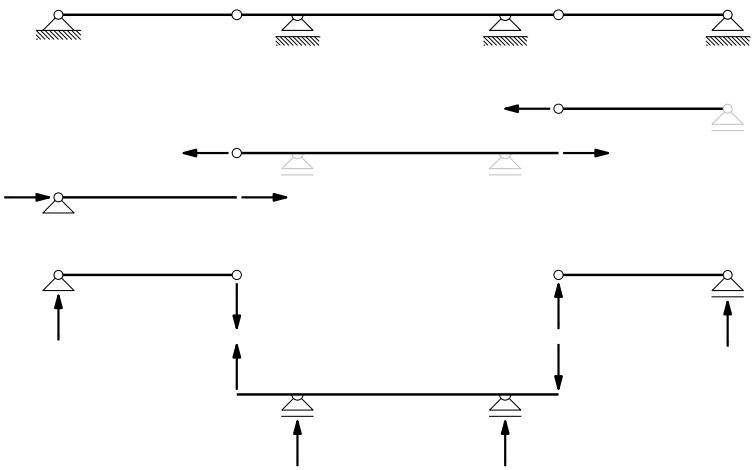
2. primjer:



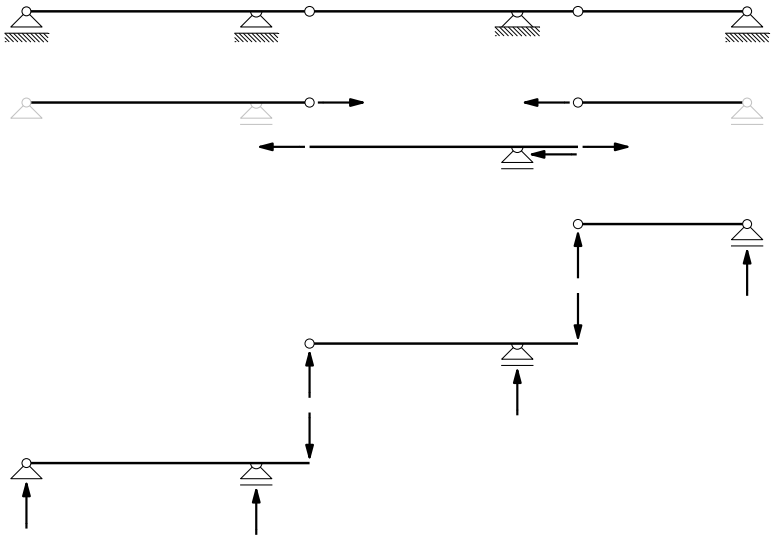
## Gerberovi nosači koji nisu ulančeni

- načini prijenosa sila koje djeluju na osi nosača i sila koje djeluju okomito na os razlikuju se

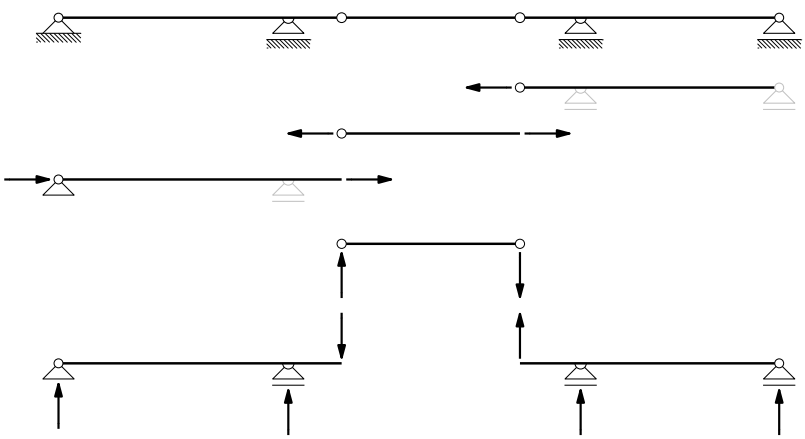
1. primjer:

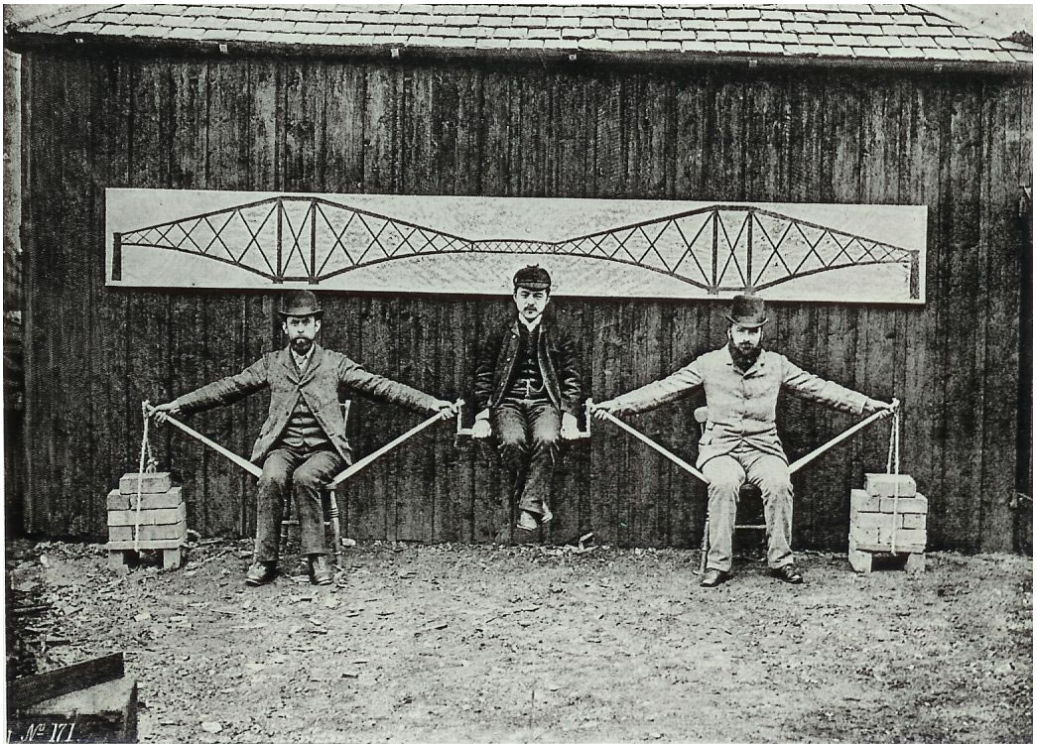


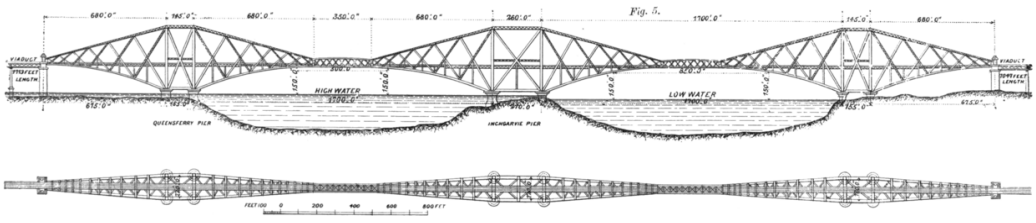
2. primjer:

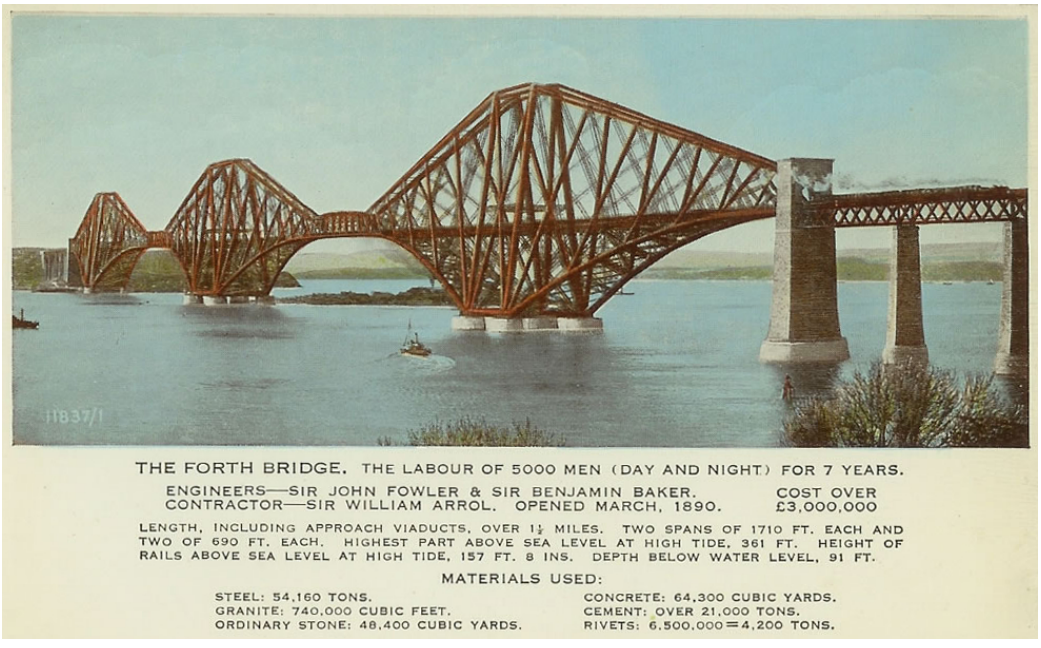


3. primjer:






Forth Railway Bridge, Firth of Forth, 1882.–1890.