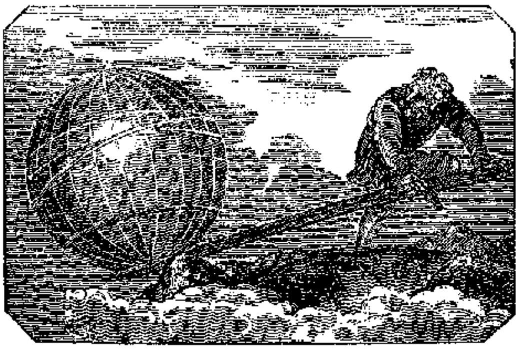
Pojam momenta

K. F.

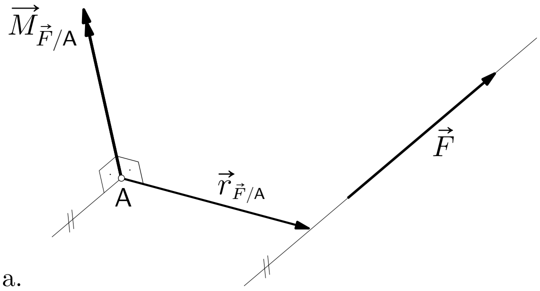
moment sile: fizikalna veličina kojom se izražava utjecaj položaja sile na uvjete ravnoteže

Arhimedova poluga & zakon poluge u *Provedbi umijeća vaganja* Simona Stevina [1586.]:

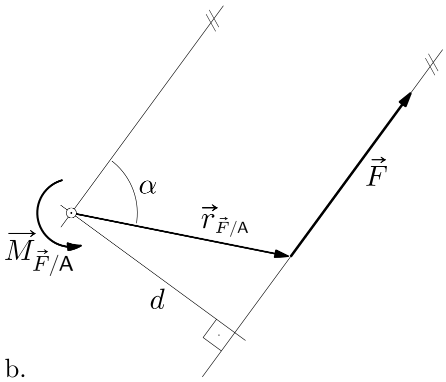


moment sile $\vec{F}$ u odnosu na točku A (sl. a.): vektor dobiven kao vektorski produkt položajnoga vektora pravca djelovanja sile u odnosu na točku A i vektora sile, pri čemu je početak položajnog vektora u točki A, a njegov vrh u bilo kojoj točki pravca djelovanja sile:

$$\overrightarrow{M}_{\vec{F}/\mathsf{A}} = \vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ x_{\vec{F}/\mathsf{A}} & y_{\vec{F}/\mathsf{A}} & z_{\vec{F}/\mathsf{A}} \\ F_x & F_y & F_z \end{vmatrix}$$



a.



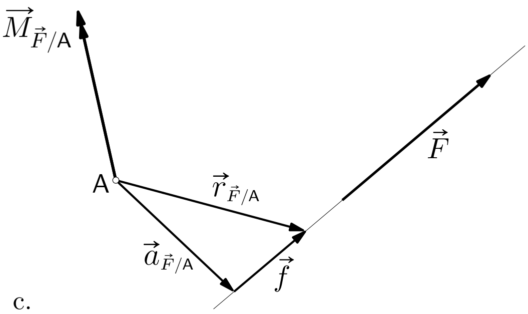
b.

(neka) svojstva vektora $\overrightarrow{M}_{\vec{F}/\mathsf{A}}$:

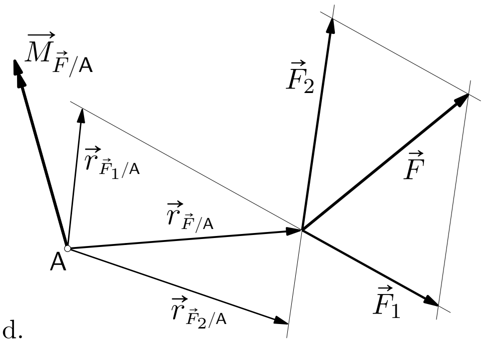
- $\overrightarrow{M}_{\vec{F}/\mathsf{A}} \perp \vec{r}_{\vec{F}/\mathsf{A}}$ & $\overrightarrow{M}_{\vec{F}/\mathsf{A}} \perp \vec{F}$ (sl. a.)
- $\|\overrightarrow{M}_{\vec{F}/\mathsf{A}}\| = \|\vec{r}_{\vec{F}/\mathsf{A}}\| \, \|\vec{F}\| \sin\alpha = \big(\|\vec{r}_{\vec{F}/\mathsf{A}}\| \sin\alpha\big) \, \|\vec{F}\| = d\,|F|$ uz $\alpha \leqslant \pi$ (sl. b.)

- „vrh u bilo kojoj točki pravca djelovanja sile” (sl. c.):

$$\overrightarrow{M}_{\vec{F}/\mathsf{A}} = \vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F} = \left(\vec{a}_{\vec{F}/\mathsf{A}} + \vec{f}\right) \times \vec{F} = \vec{a}_{\vec{F}/\mathsf{A}} \times \vec{F} + \underbrace{\vec{f} \times \vec{F}}_{\vec{0}} = \vec{a}_{\vec{F}/\mathsf{A}} \times \vec{F}$$



c.



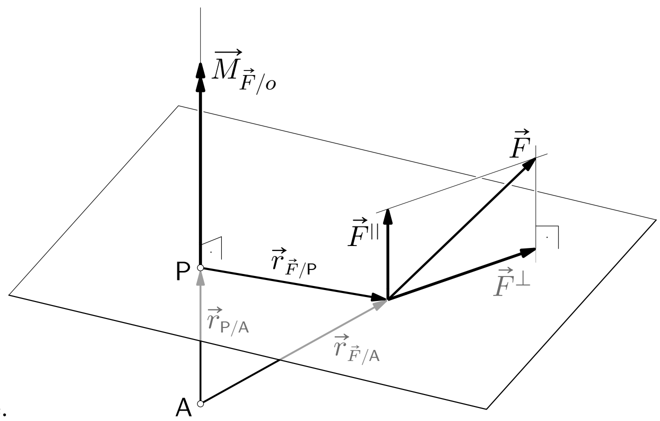
d.

Varignonov teorem (sl. d.): moment sile $\vec{F}$ u odnosu na točku A jednak je zbroju momenata njezinih komponenata $\vec{F}_i$ u odnosu na tu točku:

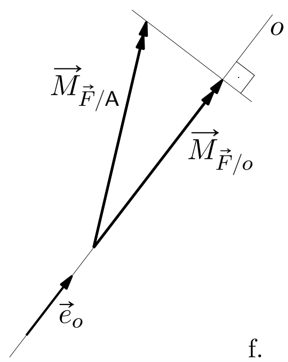
$$\overrightarrow{M}_{\vec{F}/\mathsf{A}} = \vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F} = \vec{r}_{\vec{F}/\mathsf{A}} \times \left(\sum_{i=1}^{n} \vec{F}_i\right) = \sum_{i=1}^{n} \left(\vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F}_i\right) = \sum_{i=1}^{n} \left(\vec{r}_{\vec{F}_i/\mathsf{A}} \times \vec{F}_i\right) = \sum_{i=1}^{n} \overrightarrow{M}_{\vec{F}_i/\mathsf{A}}$$

moment sile $\vec{F}$ oko osi p (sl. e.): moment komponente $\vec{F}^{\perp}$ sile $\vec{F}$ u ravnini okomitoj na os o u odnosu na probodište P osi s tom ravninom:

$$\overrightarrow{M}_{\vec{F}/o} = \overrightarrow{M}_{\vec{F}^{\perp}/\mathsf{P}} = \vec{r}_{\vec{F}/\mathsf{P}} \times \vec{F}^{\perp} \qquad \left[\vec{F}^{\perp} \perp o \;\;\&\;\; \vec{r}_{\vec{F}/\mathsf{P}} \perp o \implies \overrightarrow{M}_{\vec{F}/o} \parallel o\right]$$



e.



f.

$\overrightarrow{M}_{\vec{F}/o}$ se može izraziti kao ortogonalna projekcija na os o vektora $\overrightarrow{M}_{\vec{F}/\mathsf{A}}$ momenta sile $\vec{F}$ u odnosu na bilo koju točku A te osi (sl. e. i f.):

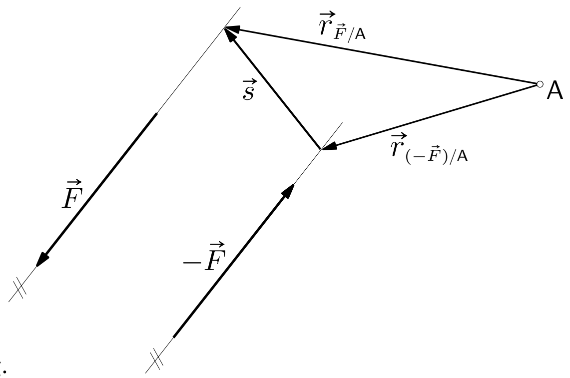
$$\begin{aligned}\overrightarrow{M}_{\vec{F}/\mathsf{A}} &= \vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F} = \left(\vec{r}_{\mathsf{P}/\mathsf{A}} + \vec{r}_{\vec{F}/\mathsf{P}}\right) \times \left(\vec{F}^{\parallel} + \vec{F}^{\perp}\right) \\ &= \underbrace{\vec{r}_{\mathsf{P}/\mathsf{A}} \times \vec{F}^{\parallel}}_{\vec{0}} + \underbrace{\vec{r}_{\mathsf{P}/\mathsf{A}} \times \vec{F}^{\perp}}_{\perp\, o} + \underbrace{\vec{r}_{\vec{F}/\mathsf{P}} \times \vec{F}^{\parallel}}_{\perp\, o} + \underbrace{\vec{r}_{\vec{F}/\mathsf{P}} \times \vec{F}^{\perp}}_{\parallel\, o}\end{aligned}$$

$$\overrightarrow{M}_{\vec{F}/\mathsf{A}} \cdot \vec{e}_o \;=\; \underbrace{\big(\vec{r}_{\mathsf{P}/\mathsf{A}} \times \vec{F}^{\perp}\big) \cdot \vec{e}_o}_{0} \;+\; \underbrace{\big(\vec{r}_{\vec{F}/\mathsf{P}} \times \vec{F}^{||}\big) \cdot \vec{e}_o}_{0} \;+\big(\vec{r}_{\vec{F}/\mathsf{P}} \times \vec{F}^{\perp}\big) \cdot \vec{e}_o \;=\; M_{\vec{F}^{\perp}/\mathsf{P}} \;=\; M_{\vec{F}/o}$$

$$\begin{aligned}\overrightarrow{M}_{\vec{F}/o} \;&=\; M_{\vec{F}/o}\,\vec{e}_o \;=\; \big(\overrightarrow{M}_{\vec{F}/\mathsf{A}} \cdot \vec{e}_o\big)\,\vec{e}_o \;=\; \big(\vec{e}_o \cdot \overrightarrow{M}_{\vec{F}/\mathsf{A}}\big)\,\vec{e}_o \\ &=\; \big[\vec{e}_o \cdot \big(\vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F}\big)\big]\,\vec{e}_o \;=\; \begin{vmatrix} e_{o,x} & e_{o,y} & e_{o,z} \\ x_{\vec{F}/\mathsf{A}} & y_{\vec{F}/\mathsf{A}} & z_{\vec{F}/\mathsf{A}} \\ F_x & F_y & F_z \end{vmatrix}\,\vec{e}_o\end{aligned}$$

moment sprega (sl. g.): vektor dobiven zbrajanjem vektorâ momenata sila sprega (dviju suprotno orijentiranih sila jednakih intenziteta koje djeluju na usporednim pravcima) u odnosu na istu točku A:

$$\overrightarrow{M}_{\vec{F}} \;=\; \vec{r}_{\vec{F}/\mathsf{A}} \times \vec{F} \;+\; \vec{r}_{(-\vec{F})/\mathsf{A}} \times (-\vec{F}) \;=\; \big[\vec{r}_{(-\vec{F})/\mathsf{A}} + \vec{s}\big] \times \vec{F} \;-\; \vec{r}_{(-\vec{F})/\mathsf{A}} \times \vec{F} \;=\; \vec{s} \times \vec{F}$$



g.

$\Longrightarrow$ $\overrightarrow{M}_{\vec{F}}$ jednak vektorskom produktu položajnoga vektora i vektora jedne od sila sprega, pri čemu je vrh položajnog vektora bilo gdje na pravcu te sile, a početak bilo gdje na pravcu druge sile; neovisan, dakle, o izboru točke A

koncentrirani moment: moment sprega neizmjerno velikih sila na pravcima čija je udaljenost neizmjerno malena (infinitezimalna)

uvjeti ravnoteže tijela u vektorskom obliku:

$$\sum_{i=1}^{n} \vec{F}_i \;=\; \vec{0}$$

$$\sum_{i=1}^{n} \vec{r}_{\vec{F}_i/0} \times \vec{F}_i \;+\; \sum_{j=1}^{m} \overrightarrow{M}_j \;=\; \vec{0} \qquad \left[\;\text{sažetije:}\quad \sum_{k=1}^{n+m} \overrightarrow{M}_k \;=\; \vec{0}\;\right]$$

uvjeti ravnoteže tijela u skalarnom obliku (osnovna formulacija / kanonski oblik):

$$\sum_{i=1}^{n} F_{i,x} \;=\; 0, \qquad \sum_{i=1}^{n} F_{i,y} \;=\; 0, \qquad \sum_{i=1}^{n} F_{i,z} \;=\; 0$$

$$\sum_{k=1}^{n+m} M_{k,x} \;=\; 0, \qquad \sum_{k=1}^{n+m} M_{k,y} \;=\; 0, \qquad \sum_{k=1}^{n+m} M_{k,z} \;=\; 0$$