Statička ekvivalencija

K. F.

dva sustava sila i momenata su **statički ekvivalentna** ako su njihovi doprinosi uvjetima ravnoteže jednaki

rezultirajuće djelovanje sustava sila i momenata u odnosu na točku A ili dinama u točki A:

- **rezultirajuća sila** ili glavni vektor sila (s hvatištem u točki A):

$$\vec{F}_{R/\mathsf{A}} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_R$$

- **rezultirajući moment** ili glavni vektor momenata u odnosu na točku A:

$$\overrightarrow{M}_{R/\mathsf{A}} = \sum_{i=1}^{n} \vec{r}_{\vec{F}_i/\mathsf{A}} \times \vec{F}_i + \sum_{j=1}^{m} \overrightarrow{M}_j$$

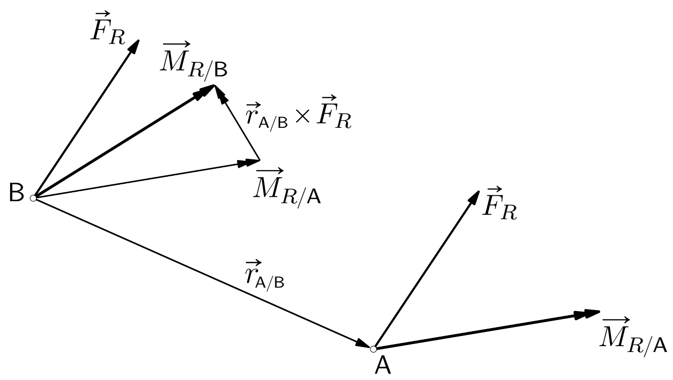
rezultirajuće djelovanje u odnosu na točku B (sl. a.):

- rezultirajuća sila (s hvatištem u točki B):

$$\vec{F}_{R/\mathsf{B}} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_R$$

- rezultirajući moment u odnosu na točku B:

$$\begin{aligned}
\overrightarrow{M}_{R/\mathsf{B}} &= \sum_{i=1}^{n} \vec{r}_{\vec{F}_i/\mathsf{B}} \times \vec{F}_i + \sum_{j=1}^{m} \overrightarrow{M}_j = \sum_{i=1}^{n} (\vec{r}_{\mathsf{A}/\mathsf{B}} + \vec{r}_{\vec{F}_i/\mathsf{A}}) \times \vec{F}_i + \sum_{j=1}^{m} \overrightarrow{M}_j \\
&= \vec{r}_{\mathsf{A}/\mathsf{B}} \times \sum_{i=1}^{n} \vec{F}_i + \left(\sum_{i=1}^{n} \vec{r}_{\vec{F}_i/\mathsf{A}} \times \vec{F}_i + \sum_{j=1}^{m} \overrightarrow{M}_j \right) = \vec{r}_{\mathsf{A}/\mathsf{B}} \times \vec{F}_R + \overrightarrow{M}_{R/\mathsf{A}}
\end{aligned}$$



a.

prva invarijanta sustava sila i momenata:

$$\vec{F}_R = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_{R/\mathsf{A}} = \vec{F}_{R/\mathsf{B}}$$

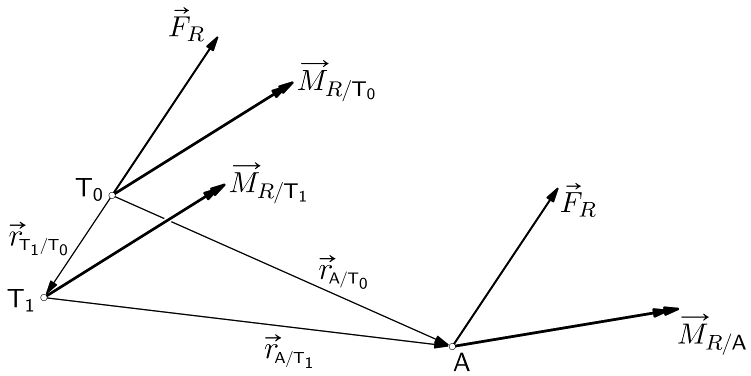
druga invarijanta sustava sila i momenata: (ortogonalna) projekcija $\overrightarrow{M}_R$ rezultirajućega momenta $\overrightarrow{M}_{R/\mathsf{T}}$ u odnosu na bilo koju točku na os paralelnu s pravcem sile $\vec{F}_R$, orijentiranu kao sila:

$$\overrightarrow{M}_R \;=\; \big(\overrightarrow{M}_{R/\mathsf{A}} \cdot \vec{e}_R\big)\,\vec{e}_R \;=\; \big(\overrightarrow{M}_{R/\mathsf{B}} \cdot \vec{e}_R\big)\,\vec{e}_R, \qquad \vec{e}_R \;=\; \frac{\vec{F}_R}{\|\vec{F}_R\|}$$

dokaz invarijantnosti vektora $\overrightarrow{M}_R$:

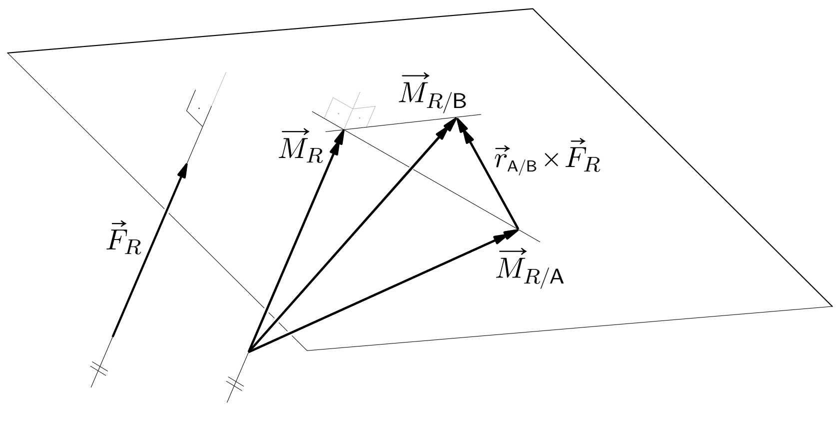
1. $\overrightarrow{M}_{R/\mathsf{T}}$ jednak za sve točke T_i na pravcu paralelnom sa $\vec{F}_R$ (sl. b.):

$$\begin{aligned}\overrightarrow{M}_{R/\mathsf{T}_0} &= \vec{r}_{\mathsf{A}/\mathsf{T}_0} \times \vec{F}_R + \overrightarrow{M}_{R/\mathsf{A}} \;=\; (\vec{r}_{\mathsf{T}_1/\mathsf{T}_0} + \vec{r}_{\mathsf{A}/\mathsf{T}_1}) \times \vec{F}_R + \overrightarrow{M}_{R/\mathsf{A}} \\ &= \underbrace{\vec{r}_{\mathsf{T}_1/\mathsf{T}_0} \times \vec{F}_R}_{\vec{0}} + \vec{r}_{\mathsf{A}/\mathsf{T}_1} \times \vec{F}_R + \overrightarrow{M}_{R/\mathsf{A}} \;=\; \overrightarrow{M}_{R/\mathsf{T}_1}\end{aligned}$$



b.

2. $\vec{r}_{\mathsf{A}/\mathsf{B}} \times \vec{F}_R \perp \vec{F}_R$ za bilo koje dvije točke A i B (sl. c.)



c.

centralna os: pravac paralelan s pravcem djelovanja $\vec{F}_R$ za čije sve točke C vrijedi $\overrightarrow{M}_{R/\mathsf{C}} || \vec{F}_R$

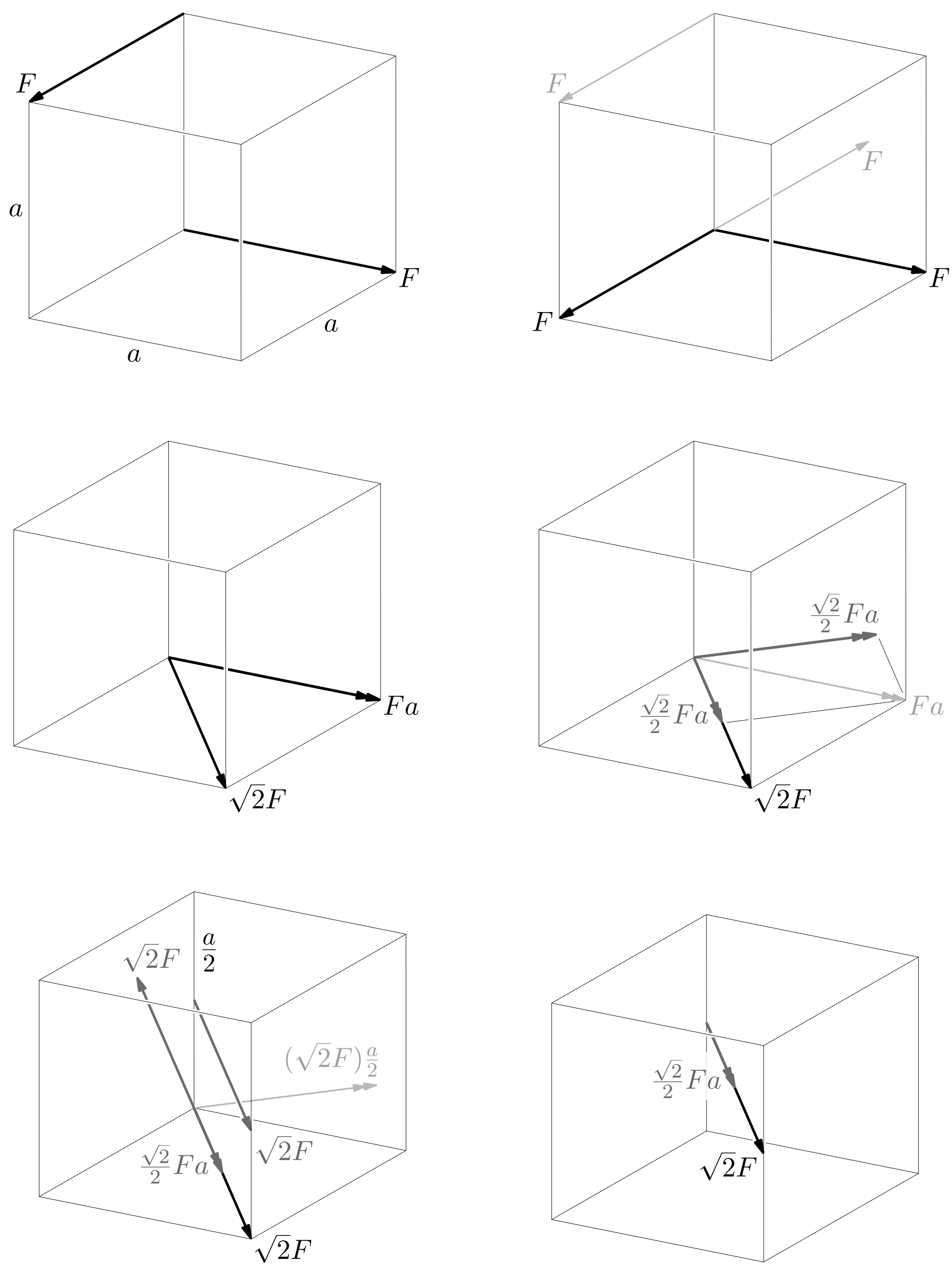
$\Longrightarrow$ $\overrightarrow{M}_{R/\mathsf{C}} = \overrightarrow{M}_R$ & $\overrightarrow{M}_R$ najmanji mogući rezultirajući moment

(centralna os postoji samo ako je $\vec{F}_R \neq \vec{0}$)

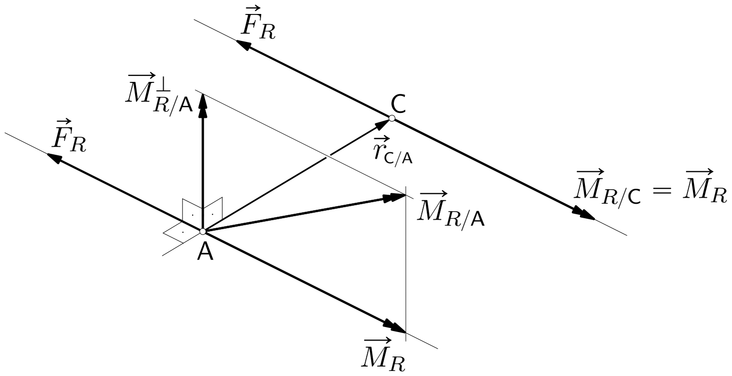
dinamički vijak: $\vec{F}_R$ na centralnoj osi & $\overrightarrow{M}_R$

$$\overrightarrow{M}_R = \left(\overrightarrow{M}_{R/\mathsf{A}} \cdot \vec{e}_R\right) \vec{e}_R = \left(\overrightarrow{M}_{R/\mathsf{A}} \cdot \frac{\vec{F}_R}{\|\vec{F}_R\|}\right) \frac{\vec{F}_R}{\|\vec{F}_R\|} = \frac{\vec{F}_R \cdot \overrightarrow{M}_{R/\mathsf{A}}}{\|\vec{F}_R\| \, \|\vec{F}_R\|} \, \vec{F}_R = \frac{\vec{F}_R \cdot \overrightarrow{M}_{R/\mathsf{A}}}{\vec{F}_R \cdot \vec{F}_R} \, \vec{F}_R$$

jednostavan primjer određivanja dinamičkoga vijka:



nalaženje centralne osi:

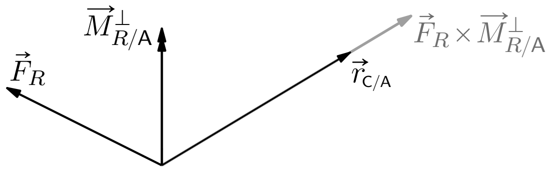


$$\overrightarrow{M}_{R/\mathsf{A}} = \overrightarrow{M}_R + \overrightarrow{M}^{\perp}_{R/\mathsf{A}} \qquad \Longrightarrow \qquad \overrightarrow{M}^{\perp}_{R/\mathsf{A}} = \overrightarrow{M}_{R/\mathsf{A}} - \overrightarrow{M}_R$$

$$\overrightarrow{M}^{\perp}_{R/\mathsf{A}} = \vec{r}_{\mathsf{C/A}} \times \vec{F}_R$$

$$\vec{r}_{\mathsf{C/A}} \perp \vec{F}_R \qquad \Longrightarrow \qquad \left\|\overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\| = \|\vec{r}_{\mathsf{C/A}}\| \, \|\vec{F}_R\| \qquad \Longrightarrow \qquad \|\vec{r}_{\mathsf{C/A}}\| = \frac{\left\|\overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\|}{\|\vec{F}_R\|}$$

$$\vec{r}_{\mathsf{C/A}} = \|\vec{r}_{\mathsf{C/A}}\| \, \frac{\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}}{\left\|\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\|}$$



$$\vec{F}_R \perp \overrightarrow{M}^{\perp}_{R/\mathsf{A}} \qquad \Longrightarrow \qquad \left\|\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\| = \|\vec{F}_R\| \, \left\|\overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\|$$

$$\vec{r}_{\mathsf{C/A}} = \frac{\left\|\overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\|}{\|\vec{F}_R\|} \, \frac{\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}}{\|\vec{F}_R\| \, \left\|\overrightarrow{M}^{\perp}_{R/\mathsf{A}}\right\|} = \frac{\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}}{\|\vec{F}_R\| \, \|\vec{F}_R\|} = \frac{\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}}}{\vec{F}_R \cdot \vec{F}_R}$$

$$\vec{F}_R \times \overrightarrow{M}^{\perp}_{R/\mathsf{A}} = \vec{F}_R \times \left(\overrightarrow{M}_{R/\mathsf{A}} - \overrightarrow{M}_R\right) = \vec{F}_R \times \overrightarrow{M}_{R/\mathsf{A}} - \underbrace{\vec{F}_R \times \overrightarrow{M}_R}_{\vec{0}}$$

$$\vec{r}_{\mathsf{C/A}} = \frac{\vec{F}_R \times \overrightarrow{M}_{R/\mathsf{A}}}{\vec{F}_R \cdot \vec{F}_R}$$

rezultanta: sila koja je sama statički ekvivalentna zadanom sustavu sila i momenata

$\vec{F}_R$ na centralnoj osi & $\overrightarrow{M}_R = \vec{0}$

ako je $\overrightarrow{M}_{R/\mathsf{T}} \perp \vec{F}_R$ za neku točku T izvan centralne osi, postoji rezultanta