

Utjecajne funkcije i utjecajne linije

I. Statički određeni sistemi

Funkcija $\eta_{\tilde{\mathfrak{F}}_{x_t}}$ položaja jedinične sile, čija je vrijednost vrijednost statičke ili kinematičke veličine $\tilde{\mathfrak{F}}$ u odabranoj točki x_t na nosaču, prouzročena djelovanjem te jedinične sile, naziva se *utjecajnom funkcijom* za veličinu $\tilde{\mathfrak{F}}$ u točki x_t .

Utjecajna funkcija, drugim riječima, preslikava položaj jedinične sile, koja se „kreće” po nosaču, u vrijednost tom silom izazvane sile ili pomaka u prethodno odabranoj točki nosača:

$$\eta_{\tilde{\mathfrak{F}}_{x_t}} : x \mapsto \eta_{\tilde{\mathfrak{F}}_{x_t}}(x).$$

[Iz predavanja *Građevna statika 1.*]

Grafički prikaz utjecajne funkcije na štapnim nosačima naziva se *utjecajnom linijom*.

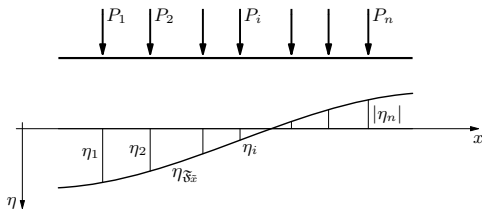
Utjecajna linija za neku veličinu je, prema tome, linija čije ordinate daju vrijednosti te veličine ako jedinična sila djeluje u točkama za koje očitavamo te ordinate.

Formalno, utjecajna je linija graf funkcije $\eta_{\mathfrak{F}_{x_t}}$:

$$\left\{ (x, \eta_{\mathfrak{F}_{x_t}}(x)) \in \mathbb{R}^2 : x \in [0, \ell] \right\}.$$

[Iz predavanja *Građevna statika 1.*]

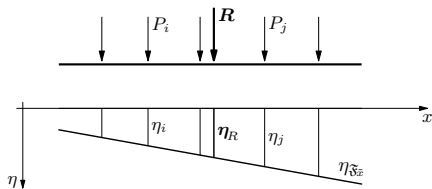
utjecaj niza koncentriranih sila:



$$\bar{\eta}_x(\{\vec{P}_i\}_{i=1}^n) = P_1 \eta_1 + P_2 \eta_2 + \dots + P_n \eta_n = \sum_{i=1}^n P_i \eta_i$$

[Iz predavanja *Grđevna statika 1.*]

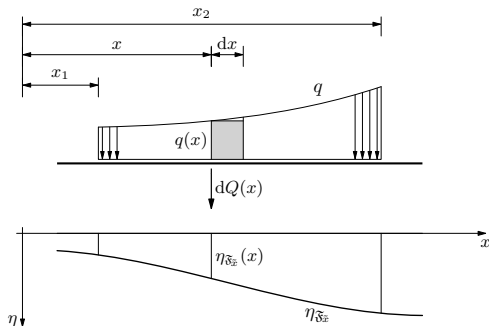
utjecaj niza koncentriranih sila:



$$\tilde{\mathfrak{F}}_{\tilde{x}}(\{\vec{P}_i\}_{i=1}^n) = R \eta_R = \tilde{\mathfrak{F}}_{\tilde{x}}(R)$$

[Iz predavanja *Gradjevna statika 1.*]

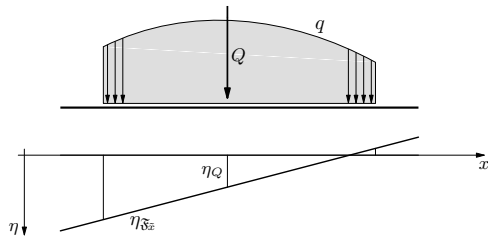
utjecaj distribuirane sile:



$$\mathfrak{F}_{\bar{x}}(\vec{q}) = \int_{x_1}^{x_2} q(x) \eta_{\delta_{x_t}}(x) dx$$

[Iz predavanja *Grđevna statika 1.*]

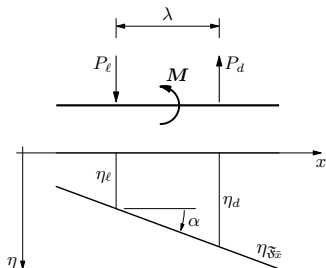
utjecaj distribuirane sile:



$$\mathfrak{F}_{\bar{x}}(\vec{q}) = Q \eta_Q = \mathfrak{F}_{\bar{x}}(\vec{Q})$$

[Iz predavanja *Gradjevna statika 1.*]

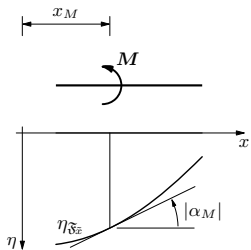
utjecaj koncentriranoga momenta:



$$\mathfrak{F}_{\tilde{x}}(\vec{M}) = -M \operatorname{tg} \alpha$$

[Iz predavanja *Gradjevna statika 1.*]

utjecaj koncentriranoga momenta:

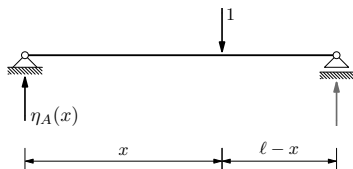


$$\vec{s}_x(\vec{M}) = -M \operatorname{tg} \alpha$$

[Iz predavanja *Gradska statika 1.*]

statički postupak — jednostavno oslonjena gređa:

UF & UL za reakcije:

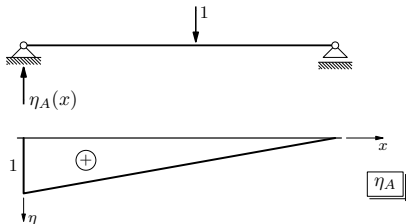


$$\sum M_{[A,B]/B} = 0 : \quad -\eta_A(x) \cdot l + 1 \cdot (l-x) = 0$$

$$\Rightarrow \quad \eta_A(x) = \frac{l-x}{l} \quad \text{za } 0 \leq x \leq l$$

statički postupak — jednostavno oslonjena gređa:

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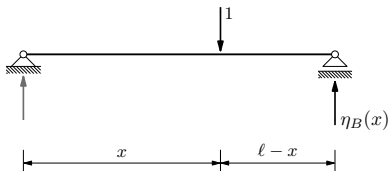
$$\eta_A(x) = \frac{l-x}{l} \quad \text{za } 0 \leq x \leq l$$

$$\eta_A(0) = \frac{l-0}{l} = \frac{l}{l} = 1 \quad (\text{greška u skriptama})$$

$$\eta_A(l) = \frac{l-l}{l} = \frac{0}{l} = 0$$

statički postupak — jednostavno oslonjena gređa:

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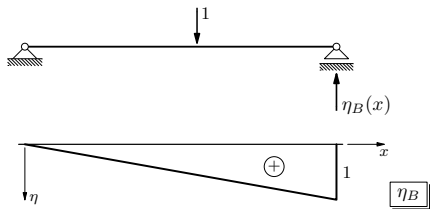


$$\sum M_{[A,B]/A} = 0 : \quad -1 \cdot x + \eta_B(x) \cdot l = 0$$

$$\Rightarrow \quad \eta_B(x) = \frac{x}{l} \quad \text{za } 0 \leq x \leq l$$

statički postupak — jednostavno oslonjena gređa:

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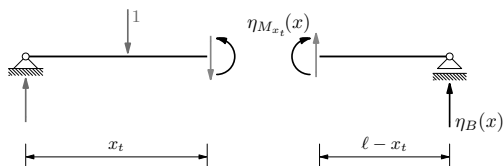
$$\eta_B(x) = \frac{x}{\ell} \quad \text{za } 0 \leq x \leq \ell$$

$$\eta_B(0) = \frac{0}{\ell} = 0$$

$$\eta_B(\ell) = \frac{\ell}{\ell} = 1 \quad (\text{greška u skriptama})$$

statički postupak — jednostavno oslonjena gređa:

UF & UL za moment savijanju u presjeku x_t :

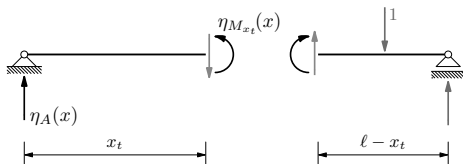


$$\sum M_{[x_t, B]/x_t} = 0 : \quad -\eta_{M_{x_t}}(x) + \eta_B(x) (\ell - x_t) = 0$$

$$\Rightarrow \eta_{M_{x_t}}(x) = \eta_B(x) (\ell - x_t) = \frac{x}{\ell} (\ell - x_t) \quad \text{za } 0 \leq x \leq x_t$$

statički postupak — jednostavno oslonjena gređa:

UF & UL za moment savijanju u presjeku x_t :



$$\sum M_{[A, x_t]/x_t} = 0 : \quad -\eta_A(x) x_t + \eta_{M_{x_t}}(x) = 0$$

$$\Rightarrow \eta_{M_{x_t}}(x) = \eta_A(x) x_t = \frac{l-x}{l} x_t \quad \text{za } x_t \leq x \leq l$$

statički postupak — jednostavno oslonjena greda:

UF & UL za moment savijanju u presjeku x_t :

$$\eta_{M_{x_t}}(x) = \begin{cases} \frac{\ell - x_t}{\ell} x & \text{za } 0 \leq x \leq x_t \\ \frac{x_t}{\ell} (\ell - x) & \text{za } x_t \leq x \leq \ell \end{cases}$$

statički postupak — jednostavno oslonjena gređa:

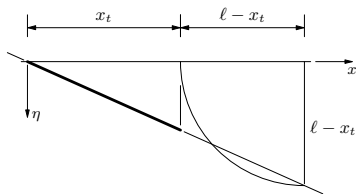
UF & UL za moment savijanju u presjeku x_t :

$$\eta_{M_{x_t}}(x) = \begin{cases} \frac{l-x_t}{l} x & \text{za } 0 \leq x \leq x_t \\ \frac{x_t}{l} (l-x) & \text{za } x_t \leq x \leq l \end{cases}$$

$$\eta_{M_{x_t}}(x_t) = \begin{cases} \frac{l-x_t}{l} x_t = \frac{x_t(l-x_t)}{l} \\ \frac{x_t}{l} (l-x_t) = \frac{x_t(l-x_t)}{l} \end{cases}$$

statički postupak — jednostavno oslonjena gređa:

UF & UL za moment savijanju u presjeku x_t :



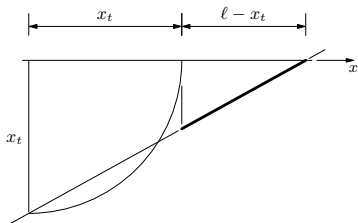
$$\eta_{M_{x_t}}(x) = \frac{l - x_t}{l} x \quad \text{za } 0 \leq x \leq x_t$$

$$\eta_{M_{x_t}}(0) = \frac{l - x_t}{l} \cdot 0 = 0$$

$$\eta_{M_{x_t}}(l) = \frac{l - x_t}{l} l = l - x_t$$

statički postupak — jednostavno oslonjena greda:

UF & UL za moment savijanju u presjeku x_t :



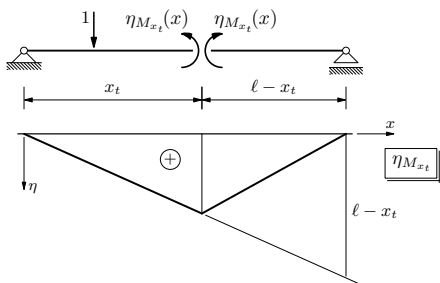
$$\eta_{M_{x_t}}(x) = \frac{x_t}{l} (l - x) \quad \text{za } x_t \leq x \leq l$$

$$\eta_{M_{x_t}}(0) = \frac{x_t}{l} (l - 0) = x_t$$

$$\eta_{M_{x_t}}(l) = \frac{x_t}{l} (l - l) = \frac{x_t}{l} \cdot 0 = 0$$

statički postupak — jednostavno oslonjena greda:

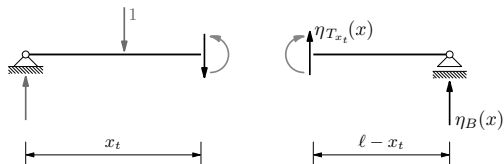
UF & UL za moment savijanju u presjeku x_t :



$$\eta_{M_{x_t}}(x) = \begin{cases} \frac{l - x_t}{l} x & \text{za } 0 \leq x \leq x_t \\ \frac{x_t}{l} (l - x) & \text{za } x_t \leq x \leq l \end{cases}$$

statički postupak — jednostavno oslonjena gređa:

UF & UL za poprečnu silu u presjeku x_t :

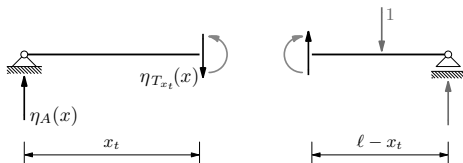


$$\sum F_{z, \langle x_t, B \rangle} = 0 : \quad -\eta_{T_{x_t}}(x) - \eta_B(x) = 0$$

$$\Rightarrow \quad \eta_{T_{x_t}}(x) = -\eta_B(x) = -\frac{x}{l} \quad \text{za } 0 \leq x < x_t$$

statički postupak — jednostavno oslonjena gređa:

UF & UL za poprečnu silu u presjeku x_t :



$$\sum F_{z,[A,x_t]} = 0 : \quad -\eta_A(x) + \eta_{T_{x_t}}(x) = 0$$

$$\Rightarrow \quad \eta_{T_{x_t}}(x) = \eta_A(x) = \frac{\ell - x}{\ell} \quad \text{za } x_t < x \leq \ell$$

statički postupak — jednostavno oslonjena greda:

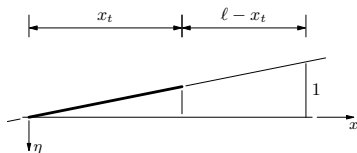
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$$\eta_{T_{x_t}}(x) = \begin{cases} -\frac{x}{l} & \text{za } 0 \leq x < x_t \\ \frac{l-x}{l} & \text{za } x_t < x \leq l \end{cases}$$

$$\frac{l-x_t}{l} - \left(-\frac{x_t}{l}\right) = \frac{l-x_t+x_t}{l} = 1$$

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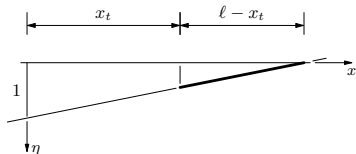
$$\eta_{T_{x_t}}(x) = -\frac{x}{l} \quad \text{za } 0 \leq x < x_t$$

$$\eta_{T_{x_t}}(0) = -\frac{0}{l} = 0$$

$$\eta_{T_{x_t}}(l) = -\frac{l}{l} = -1$$

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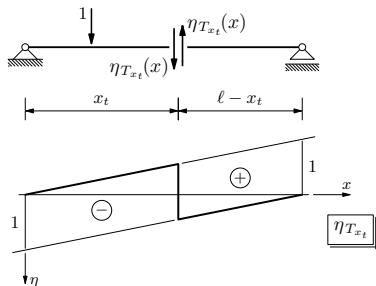
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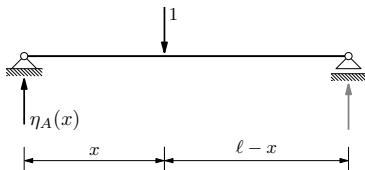
kinematički postupak:

teorem o virtualnom radu za sistem krutih tijela:

1. ako su tijela sistema krutih tijela, na koja djeluju sile i momenti, u ravnoteži, onda je zbroj radova tih sila i momenata na bilo kojim virtualnim pomacima i zaokretima tijelâ jednak nuli;
2. i obratno, ako je zbroj radova sila i momenata na bilo kojim virtualnim pomacima i zaokretima tijelâ sistema krutih tijela na koja djeluju jednak nuli, onda su tijela u ravnoteži.

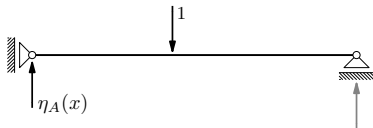
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UL za reakcije:



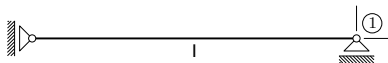
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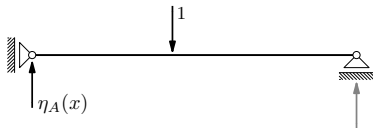
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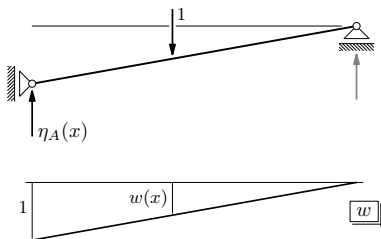
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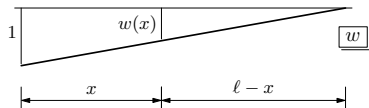
$$-\eta_A(x) \cdot 1 + 1 \cdot w(x) = 0$$

$$\eta_A(x) \cdot 1 = 1 \cdot w(x)$$

$$\eta_A(x) = w(x)$$

kinematički postupak — jednostavno oslonjena greda:

UL za reakcije:

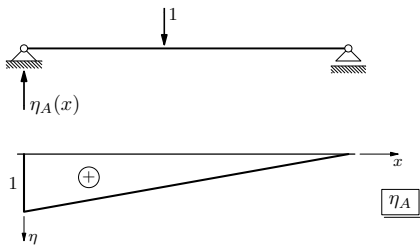


$$\frac{1}{l} = \frac{w(x)}{l - x} \quad \Rightarrow \quad w(x) = \frac{l - x}{l}$$

$$\eta_A(x) = \frac{l - x}{l}$$

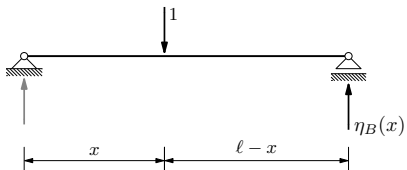
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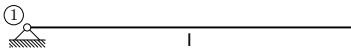
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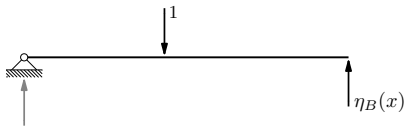
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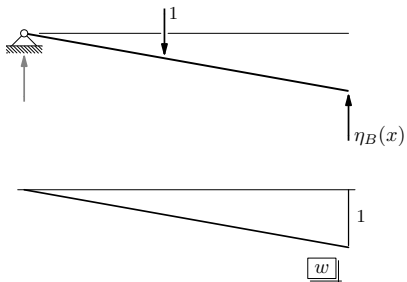
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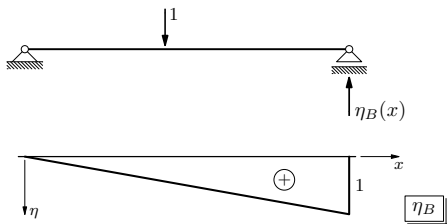
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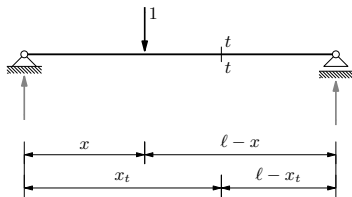
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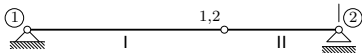
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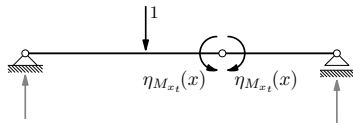
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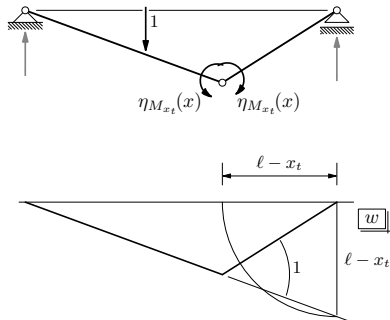
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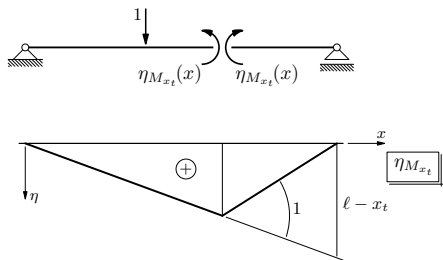
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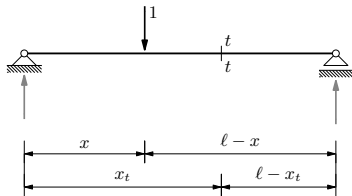
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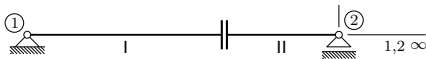
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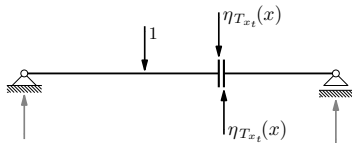
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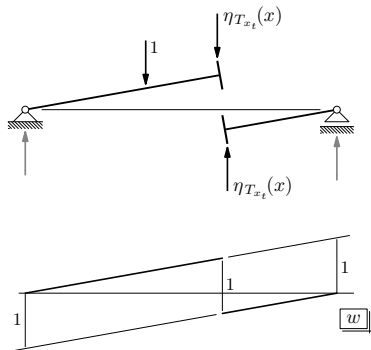
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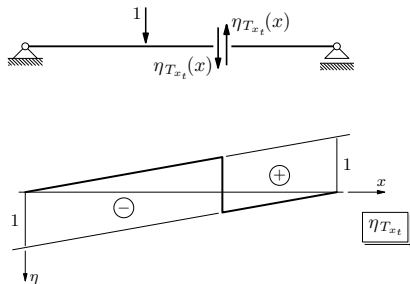
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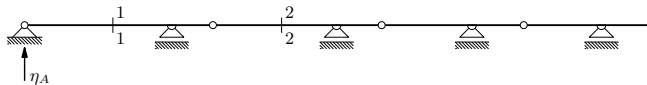


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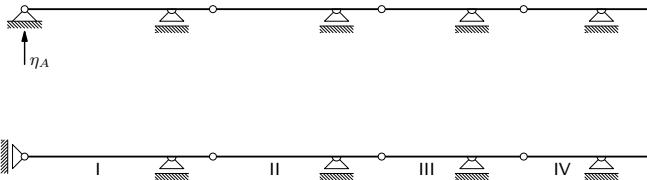
UL za poprečnu silu u presjeku x_t :



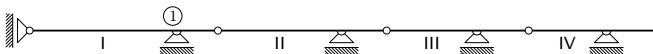
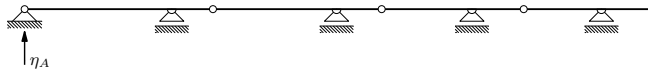
kinematički postupak — Gerberov nosač:



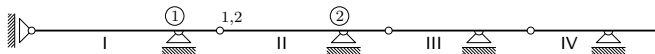
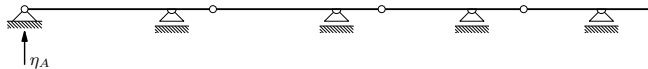
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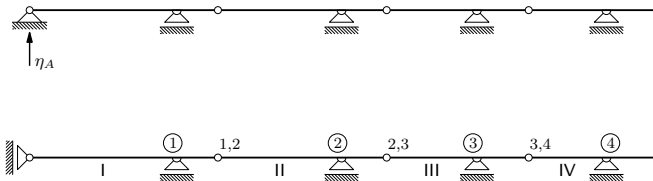
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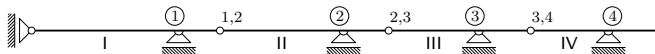
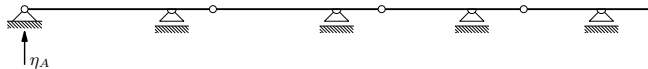
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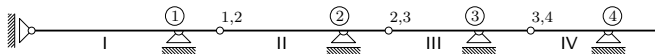
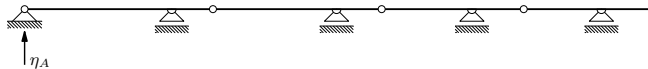
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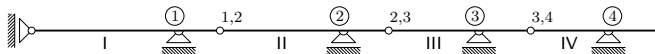
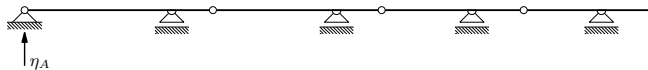
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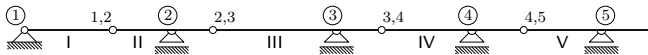
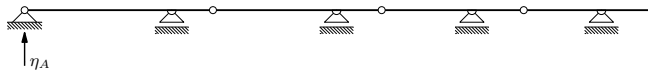
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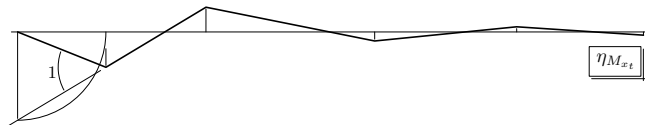
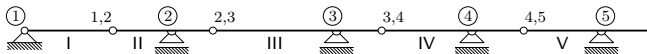
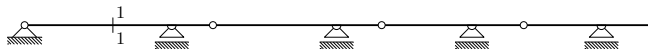
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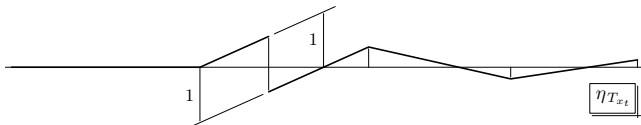
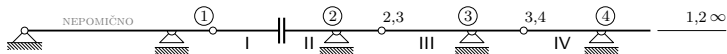
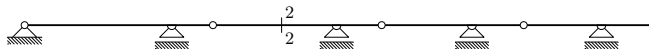
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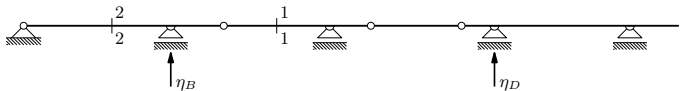
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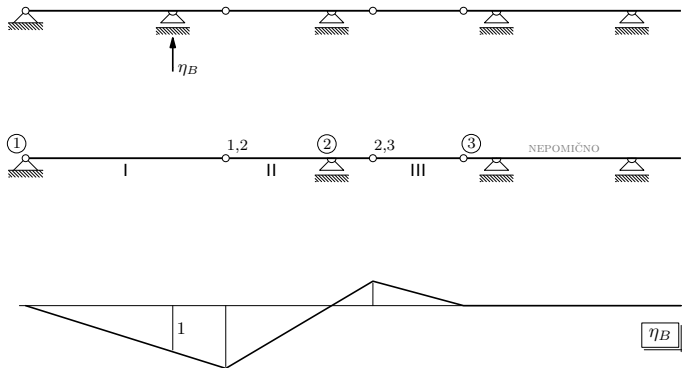
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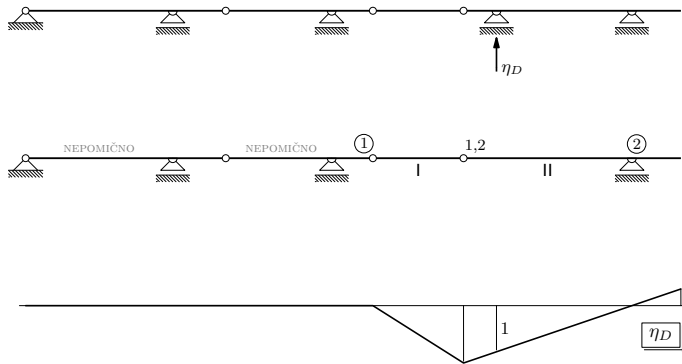
kinematički postupak — Gerberov nosač (primjer 2.):



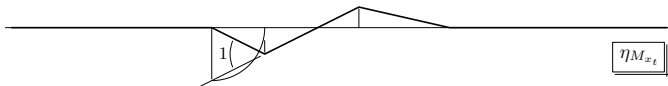
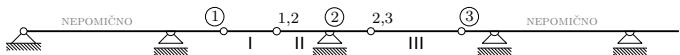
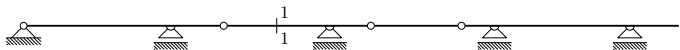
kinematički postupak — Gerberov nosač (primjer 2.):



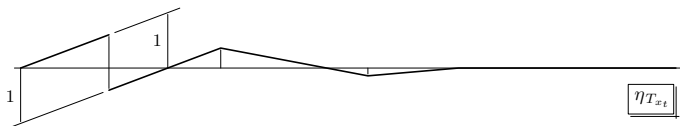
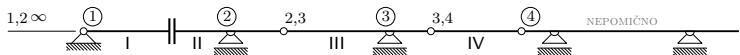
kinematički postupak — Gerberov nosač (primjer 2.):



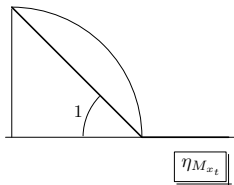
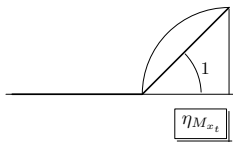
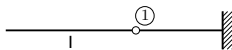
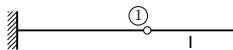
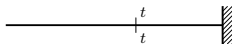
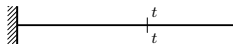
kinematički postupak — Gerberov nosač (primjer 2.):



kinematički postupak — Gerberov nosač (primjer 2.):



kinematički postupak — konzola:



kinematički postupak — konzola:

