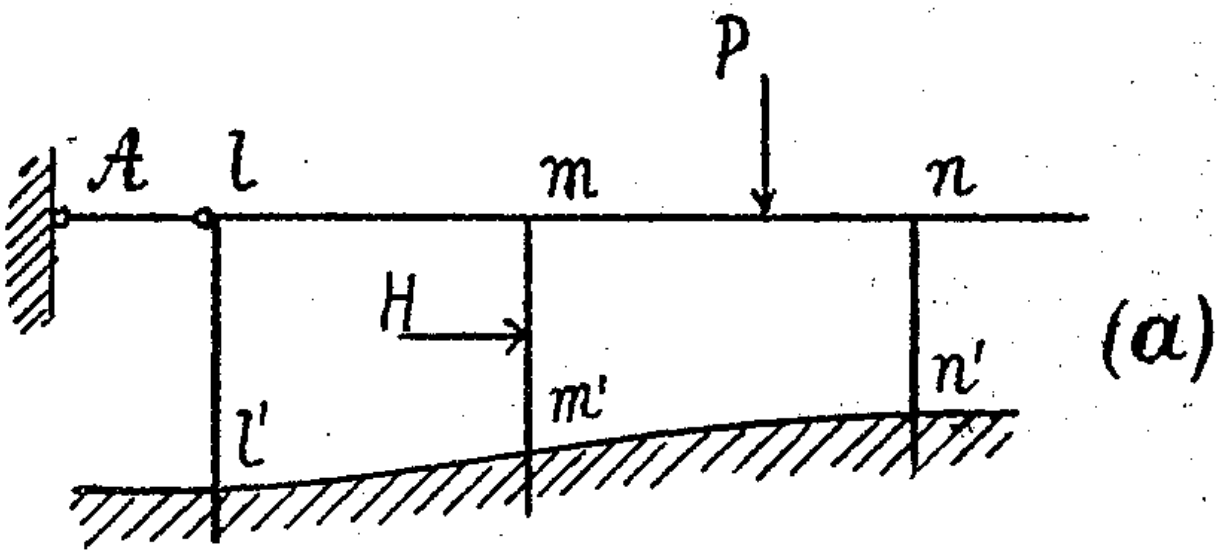
Metode pomakâ

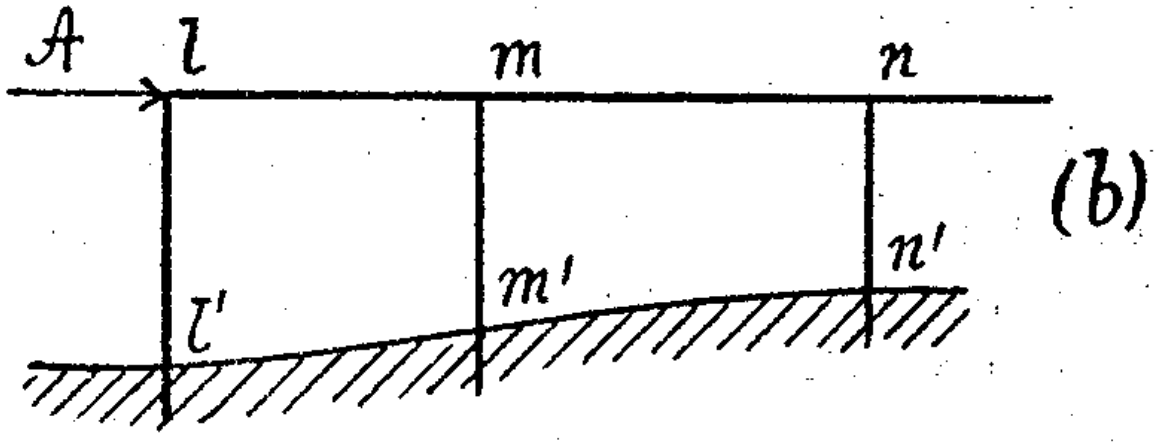
Relaksacijski postupci

III. Postupak H. Crossa, ponovno

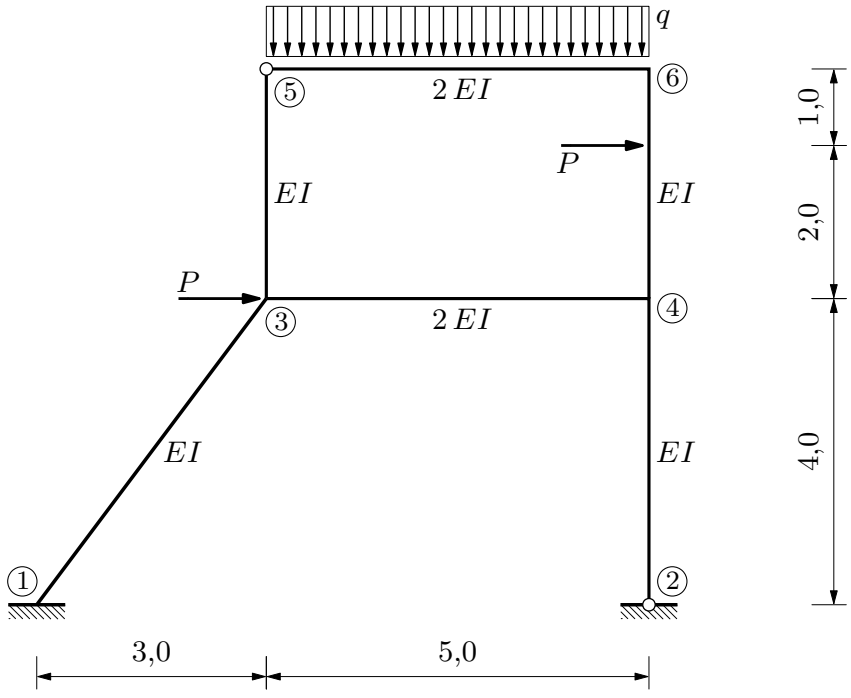
(sada za pomične sisteme)



„Rastavit ćemo našu zadaću u dva dijela. U prvom dijelu odredit ćemo momente na krajevima pojedinih štapova, koji nastaju uslijed djelovanja vanjskih sila, te ćemo uz to pretpostaviti, da je jedan čvor nosača spojen sa čvrstim tlom još pomoću horizontalnog štapa (štap $\mathcal{A}$), te će radi toga pomaknuće $\mathfrak{u}$ biti jednako nuli."

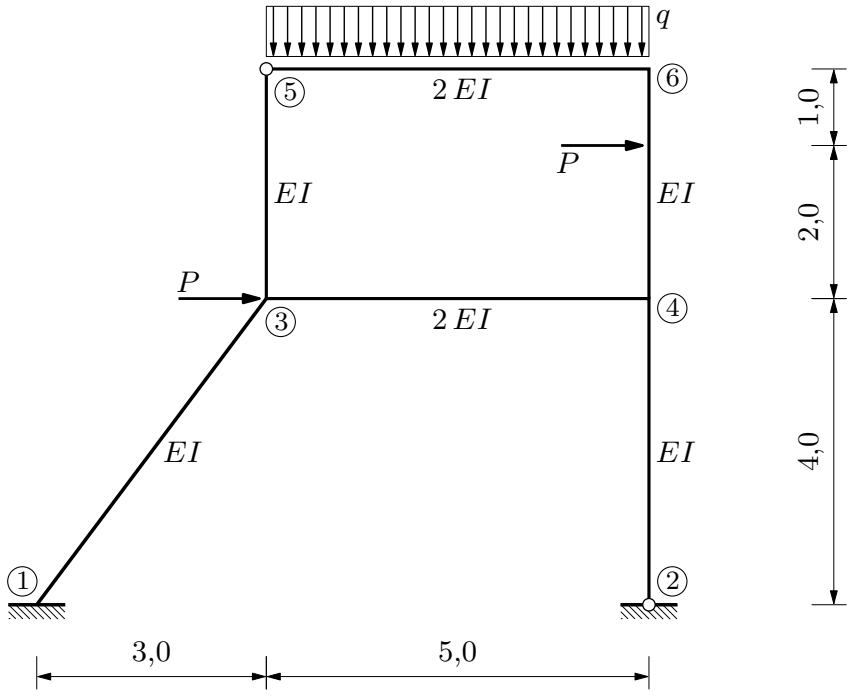


„U drugom dijelu odredit ćemo momente, koji nastaju pod djelovanjem same horizontalne sile, koja je jednaka i protusmjerna sili u štapu $\mathcal{A}$."



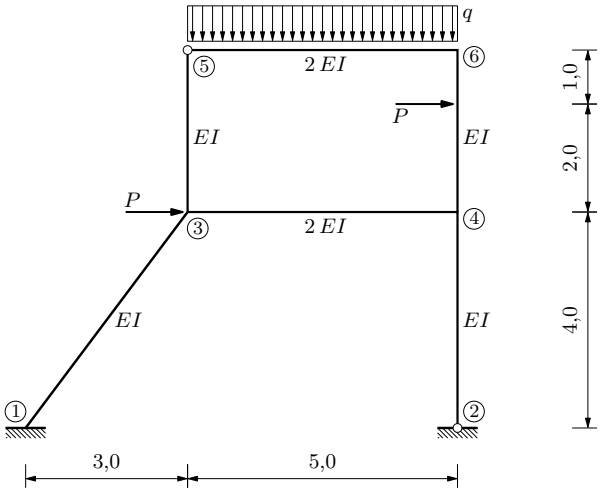
$q = 20\ \mathrm{kN/m'}$

$P = 75\ \mathrm{kN}$



$$k_{\{1,3\}} = \frac{EI}{5} \qquad k_{\{3,5\}} = k_{\{4,6\}} = \frac{EI}{3}$$

$$k_{\{2,4\}} = \frac{EI}{4} \qquad k_{\{3,4\}} = k_{\{5,6\}} = \frac{2\,EI}{3}$$



$$k_3 = 4\,k_{\{1,3\}} + 4\,k_{\{3,4\}} + 3\,k_{\{3,5\}}$$

$$= 4 \cdot \frac{EI}{5} + 4 \cdot \frac{2\,EI}{5} + 3 \cdot \frac{EI}{3}$$

$$= 3{,}40 \cdot EI$$

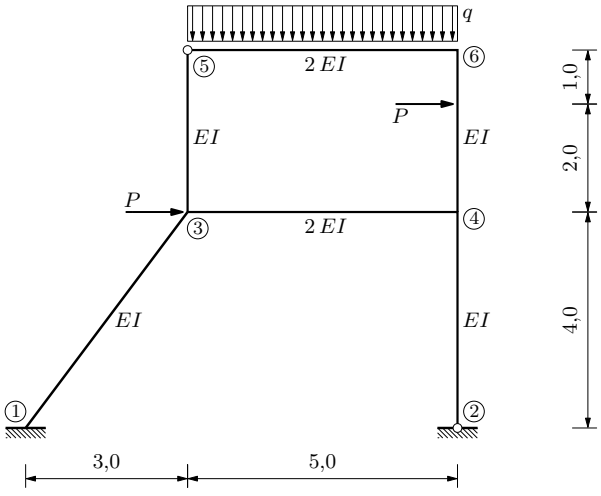
$$\mu_{3,1} = \frac{4\,k_{\{1,3\}}}{k_3} = \frac{0{,}80\,EI}{3{,}40\,EI} = 0{,}24$$

$$\mu_{3,4} = \frac{4\,k_{\{3,4\}}}{k_3} = \frac{1{,}60\,EI}{3{,}40\,EI} = 0{,}47$$

$$\mu_{3,5} = \frac{3\,k_{\{3,5\}}}{k_3} = \frac{1{,}00\,EI}{3{,}40\,EI} = 0{,}29$$

$$\overline{\qquad\qquad}$$

$$1{,}00$$



$$k_4 = 4\,k_{\{3,4\}} + 3\,k_{\{2,4\}} + 4\,k_{\{4,6\}}$$

$$= 4 \cdot \frac{2\,EI}{5} + 3 \cdot \frac{EI}{4} + 4 \cdot \frac{EI}{3}$$

$$= 3{,}68 \cdot EI$$

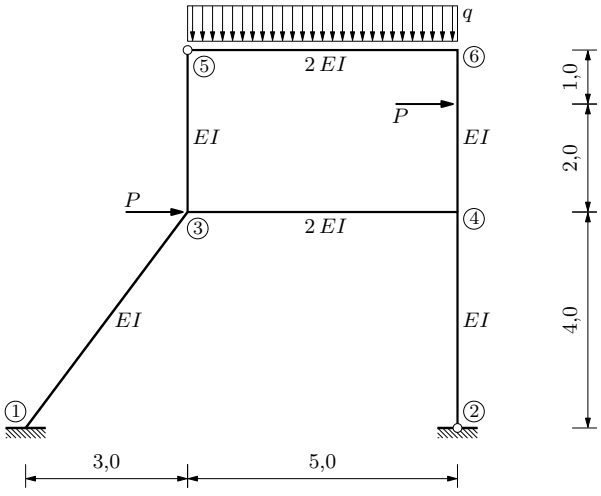
$$\mu_{4,3} = \frac{4\,k_{\{3,4\}}}{k_4} = \frac{1{,}60\,EI}{3{,}68\,EI} = 0{,}43$$

$$\mu_{4,2} = \frac{3\,k_{\{2,4\}}}{k_4} = \frac{0{,}75\,EI}{3{,}68\,EI} = 0{,}20$$

$$\mu_{4,6} = \frac{4\,k_{\{4,6\}}}{k_4} = \frac{1{,}33\,EI}{3{,}68\,EI} = 0{,}36$$

$$\overline{\qquad\qquad}$$

$$0{,}99$$



$$k_4 = 4\,k_{\{3,4\}} + 3\,k_{\{2,4\}} + 4\,k_{\{4,6\}}$$

$$= 4 \cdot \frac{2\,EI}{5} + 3 \cdot \frac{EI}{4} + 4 \cdot \frac{EI}{3}$$

$$= 3{,}68 \cdot EI$$

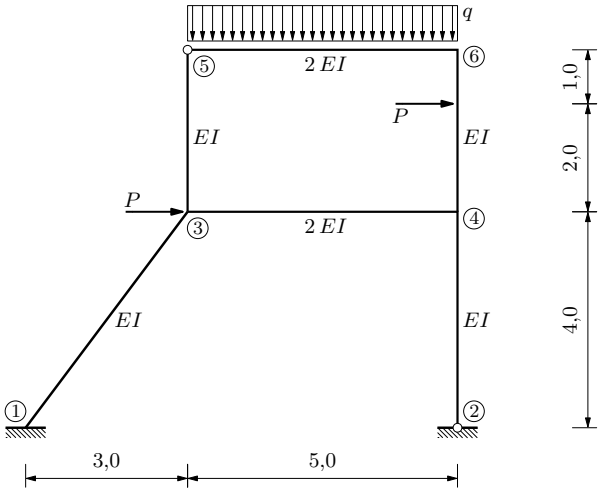
$$\mu_{4,3} = \frac{4\,k_{\{3,4\}}}{k_4} = \frac{1{,}60\,EI}{3{,}68\,EI} = 0{,}44$$

$$\mu_{4,2} = \frac{3\,k_{\{2,4\}}}{k_4} = \frac{0{,}75\,EI}{3{,}68\,EI} = 0{,}20$$

$$\mu_{4,6} = \frac{4\,k_{\{4,6\}}}{k_4} = \frac{1{,}33\,EI}{3{,}68\,EI} = 0{,}36$$

$$\overline{\qquad\qquad}$$

$$1{,}00$$



$$k_6 = 3\,k_{\{5,6\}} + 4\,k_{\{4,6\}}$$

$$= 3 \cdot \frac{2\,EI}{5} + 4 \cdot \frac{EI}{3}$$

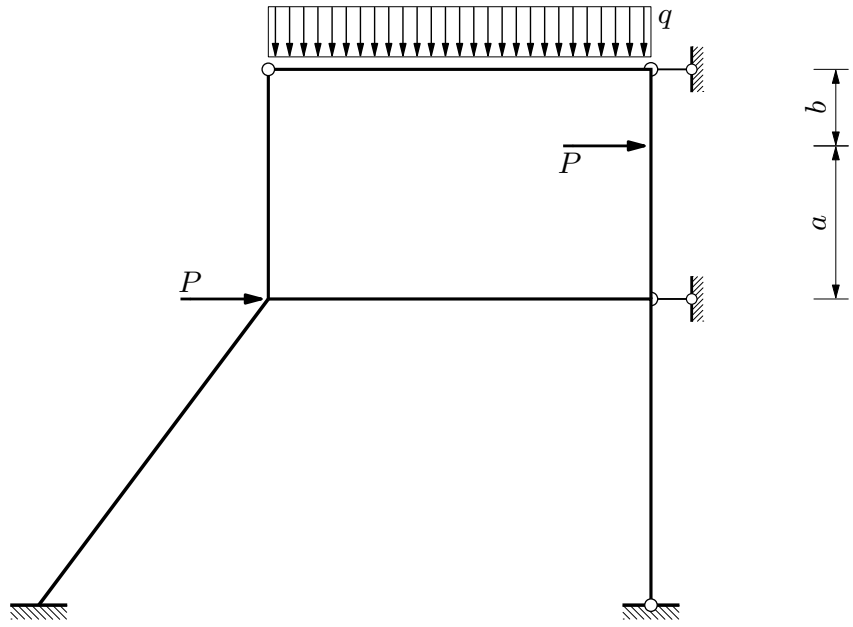
$$= 2{,}53 \cdot EI$$

$$\mu_{6,5} = \frac{3\,k_{\{5,6\}}}{k_6} = \frac{1{,}20\,EI}{2{,}53\,EI} = 0{,}47$$

$$\mu_{6,4} = \frac{4\,k_{\{4,6\}}}{k_6} = \frac{1{,}33\,EI}{2{,}53\,EI} = 0{,}53$$

$$\overline{\qquad\qquad}$$

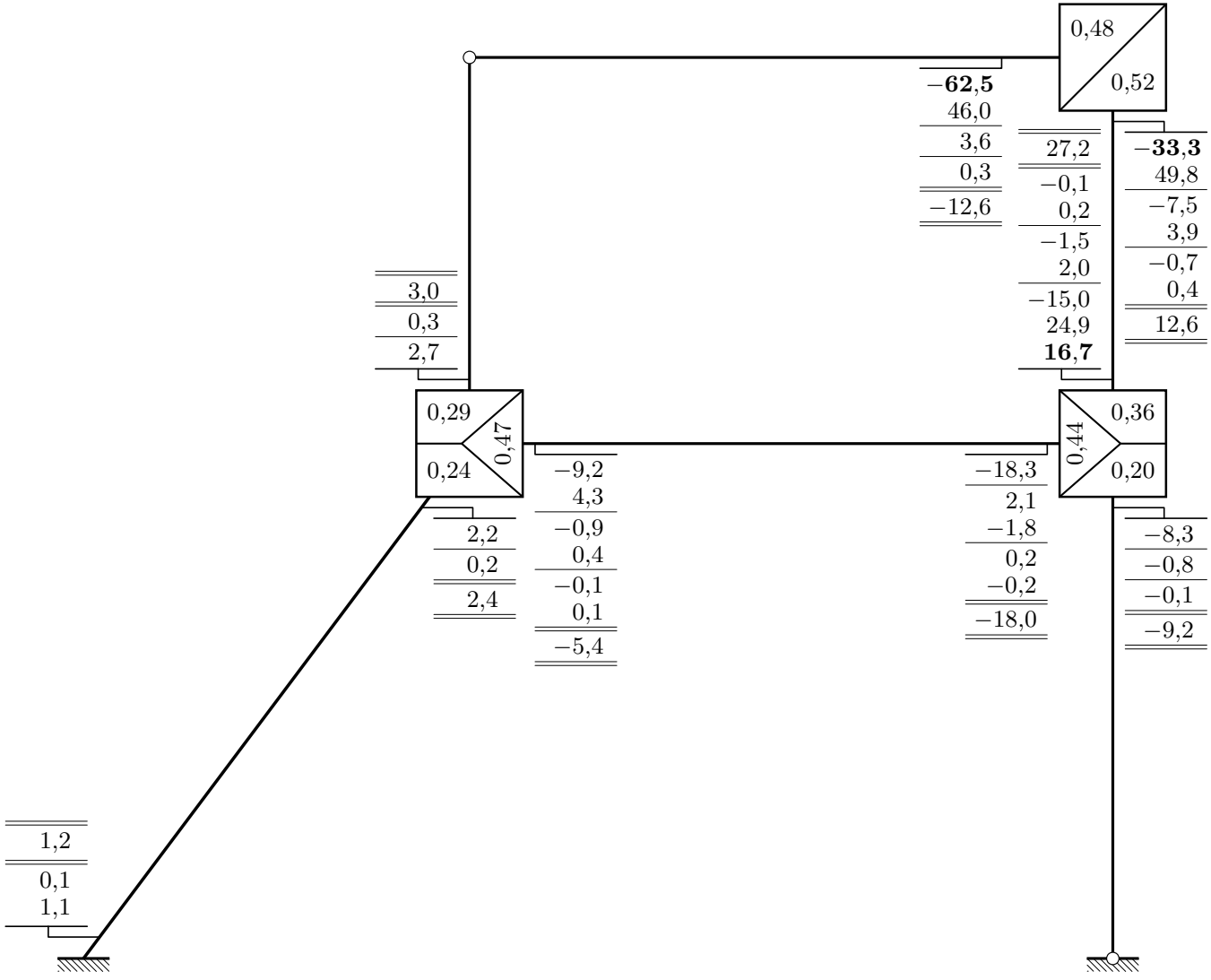
$$1{,}00$$

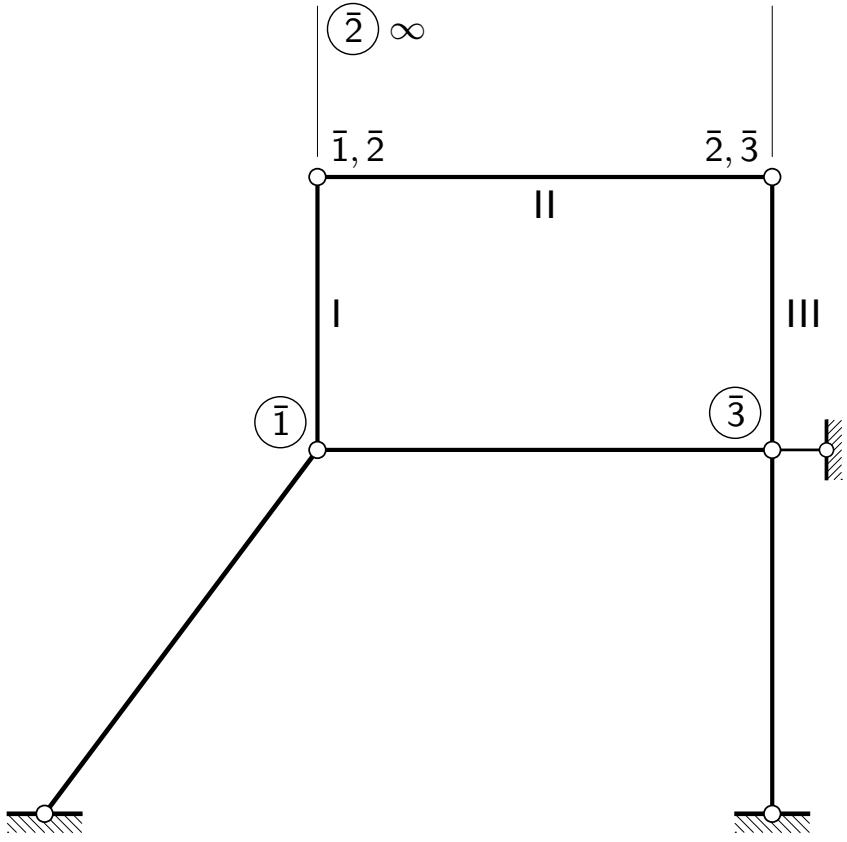


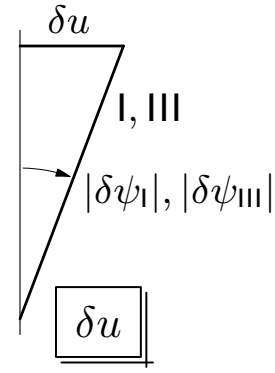
$$\overline{M}_{\{6,5\}} = -\frac{3\,q\,\ell_{\{5,6\}}^2}{24} = -62{,}5\ \text{kNm}$$

$$\overline{M}_{\{4,6\}} = \frac{P \cdot a \cdot b^2}{\ell_{\{4,6\}}^2} = \frac{P \cdot 2 \cdot 1^2}{3^2} = 16{,}7\ \text{kNm}$$

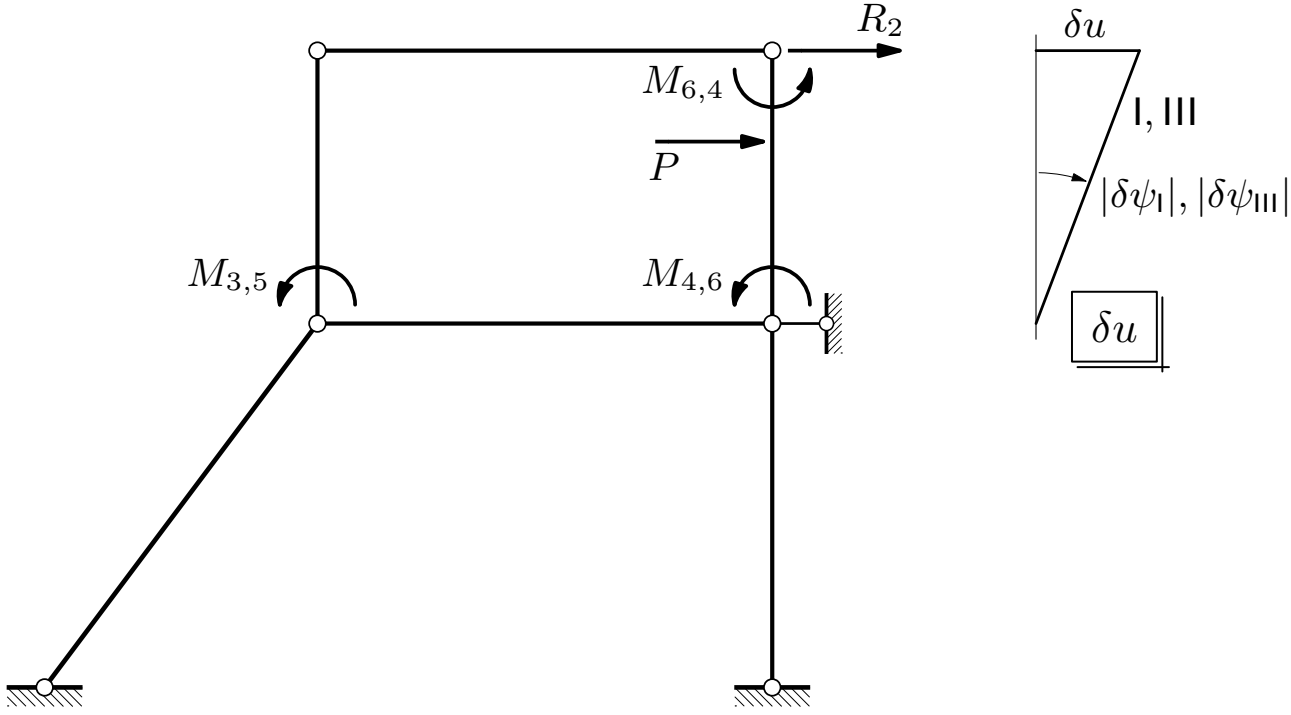
$$\overline{M}_{\{6,4\}} = \frac{P \cdot a^2 \cdot b}{\ell_{\{4,6\}}^2} = -\frac{P \cdot 2^2 \cdot 1}{3^2} = -33{,}3\ \text{kNm}$$


0,48
0,52
−62,5
46,0
3,6
0,3
−12,6
27,2
−0,1
0,2
−1,5
2,0
−15,0
24,9
16,7
−33,3
49,8
−7,5
3,9
−0,7
0,4
12,6
3,0
0,3
2,7
0,29
0,47
0,24
−9,2
4,3
−0,9
0,4
−0,1
0,1
−5,4
2,2
0,2
2,4
0,44
0,36
0,20
−18,3
2,1
−1,8
0,2
−0,2
−18,0
−8,3
−0,8
−0,1
−9,2
1,2
0,1
1,1





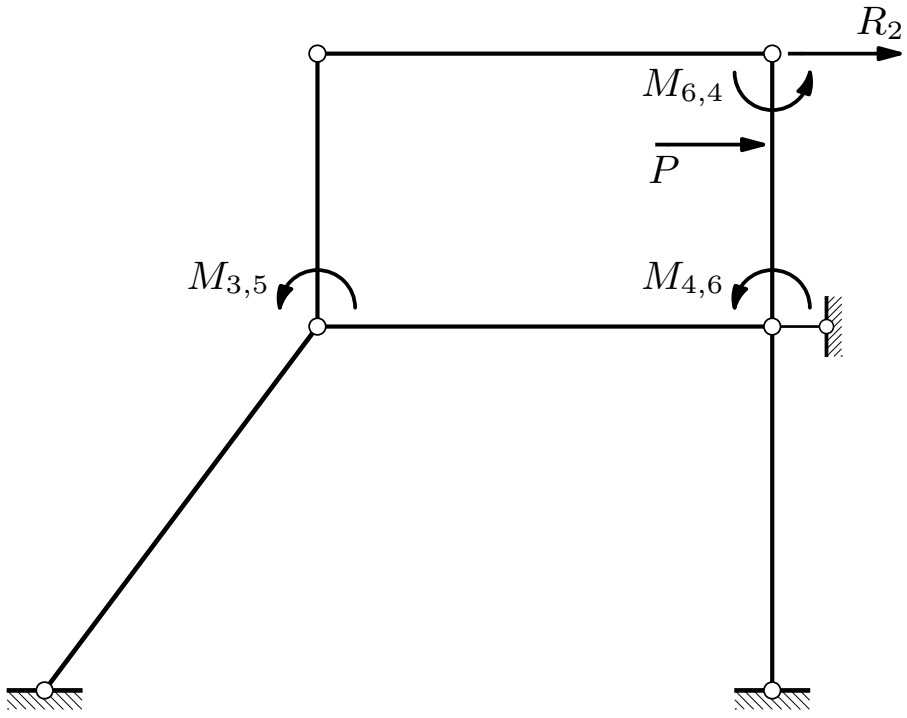
$$\delta\psi_{\{3,5\}} = \delta\psi_{\{4,6\}} = \delta\psi_{\mathrm{I}} = -\frac{\delta u}{\ell_{\{3,5\}}} = -\frac{\delta u}{3}$$

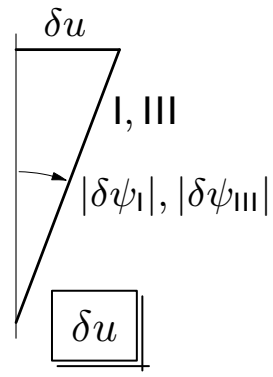


$$M_{3,5}\,\delta\psi_{\{3,5\}} + \left(M_{4,6} + M_{6,4}\right)\delta\psi_{6,4} + P\cdot\frac{2}{3}\,\delta u + R_2\,\delta u = 0$$

$$-M_{3,5}\,\frac{\delta u}{3} - \left(M_{4,6} + M_{6,4}\right)\frac{\delta u}{3} + P\cdot\frac{2}{3}\,\delta u + R_2\,\delta u = 0 \qquad \forall\,\delta u$$

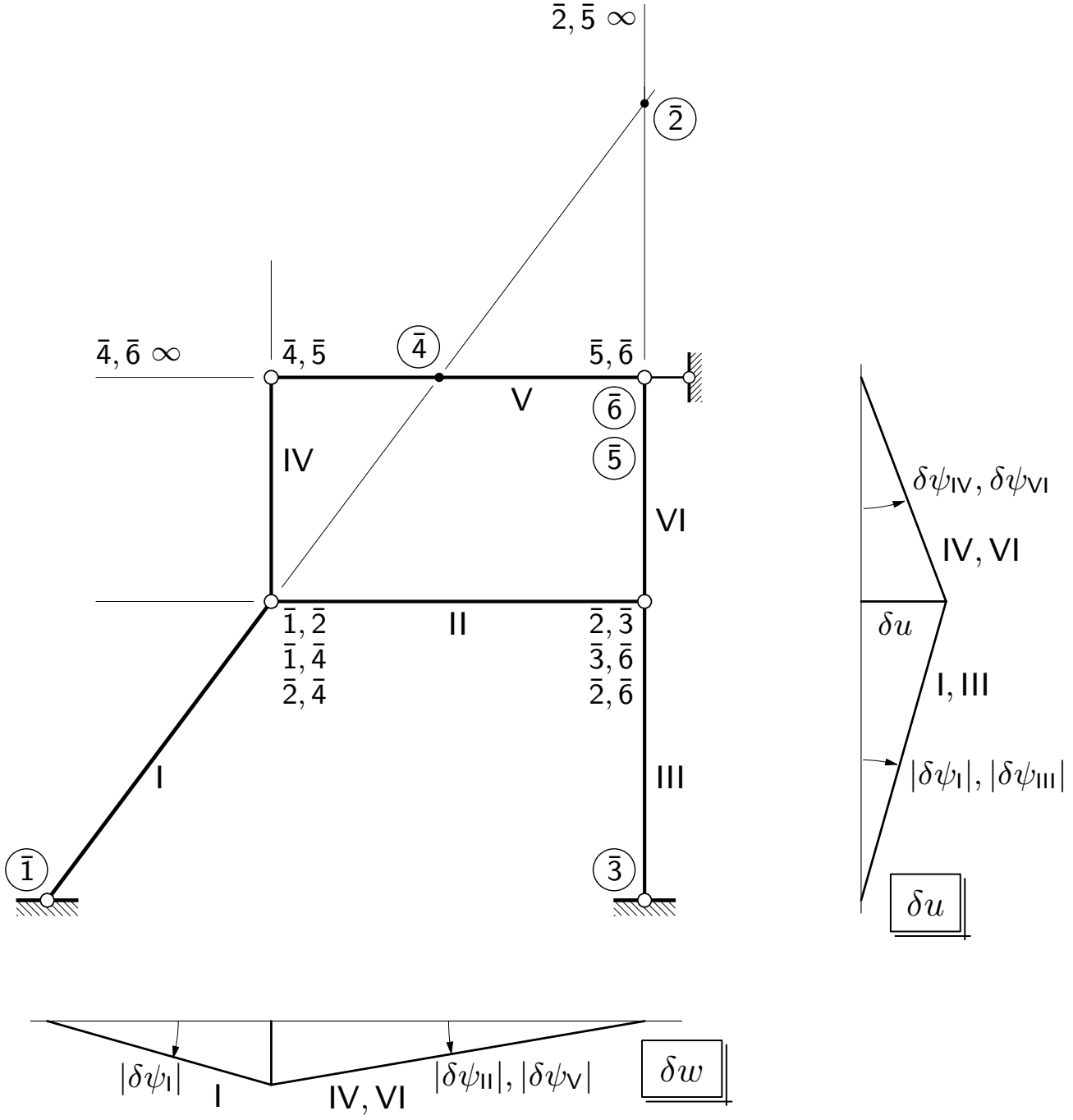
$$-\frac{M_{3,5}}{3} - \frac{M_{4,6}}{3} - \frac{M_{6,4}}{3} + \frac{2P}{3} + R_2 = 0$$

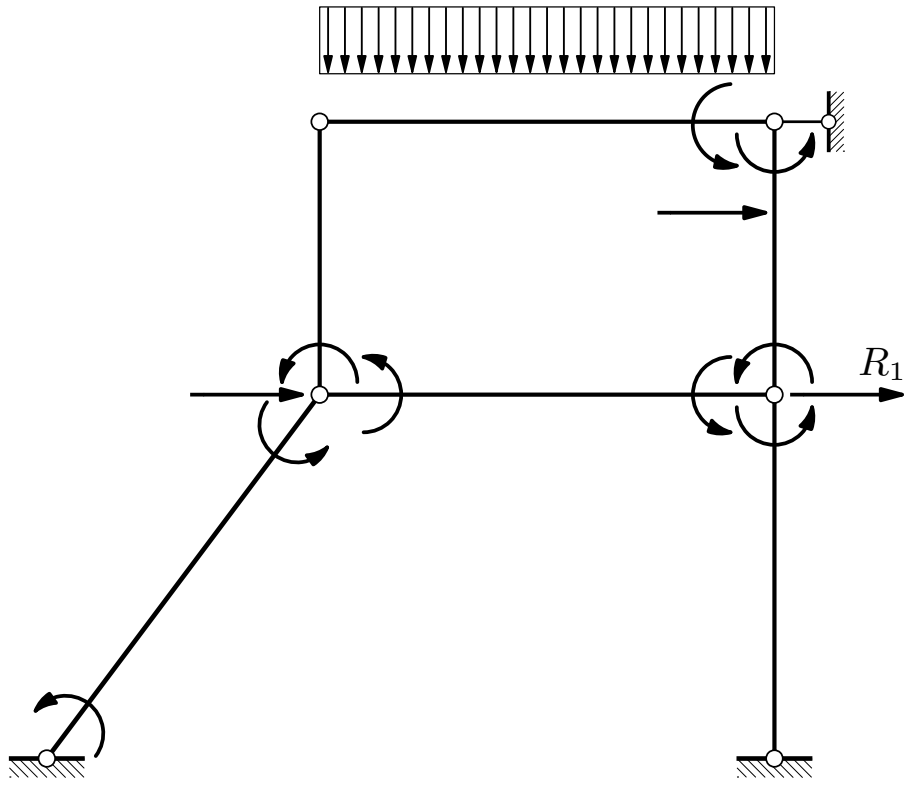


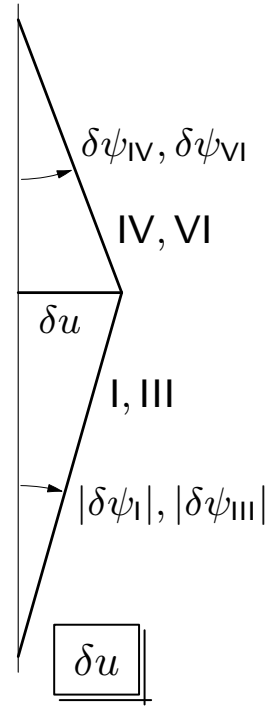


$$\mathrm{R}_2 = \frac{1}{3}\left(\mathrm{M}_{3,5} + \mathrm{M}_{4,6} + \mathrm{M}_{6,4} - 2\,\mathrm{P}\right)$$

$$= \frac{1}{3}\left(3{,}0 + 27{,}2 + 12{,}6 - 2 \cdot 75\right) = -35{,}7\ \mathrm{kN}$$

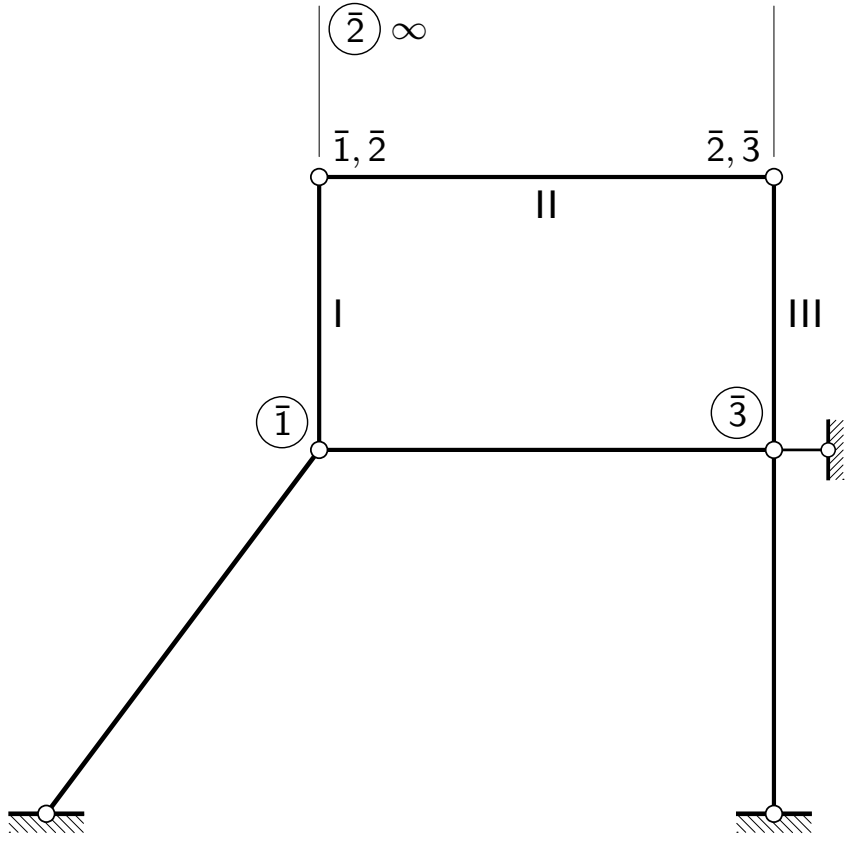

$\bar{2}, \bar{5}\ \infty$
$\bar{2}$
$\bar{4}, \bar{6}\ \infty$
$\bar{4}, \bar{5}$
$\bar{4}$
$\bar{5}, \bar{6}$
V
$\bar{6}$
$\bar{5}$
IV
VI
$\bar{1}, \bar{2}$
$\bar{1}, \bar{4}$
$\bar{2}, \bar{4}$
II
$\bar{2}, \bar{3}$
$\bar{3}, \bar{6}$
$\bar{2}, \bar{6}$
I
III
$\bar{1}$
$\bar{3}$
$\delta\psi_{\mathrm{IV}}, \delta\psi_{\mathrm{VI}}$
IV, VI
δu
I, III
$|\delta\psi_{\mathrm{I}}|, |\delta\psi_{\mathrm{III}}|$
δu
$|\delta\psi_{\mathrm{I}}|$
I
IV, VI
$|\delta\psi_{\mathrm{II}}|, |\delta\psi_{\mathrm{V}}|$
δw

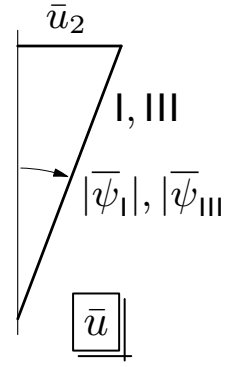

R_1


$\delta\psi_{IV}, \delta\psi_{VI}$
IV, VI
δu
I, III
$|\delta\psi_I|, |\delta\psi_{III}|$
δu


$|\delta\psi_I|$
I
IV, VI
$|\delta\psi_{II}|, |\delta\psi_V|$
δw

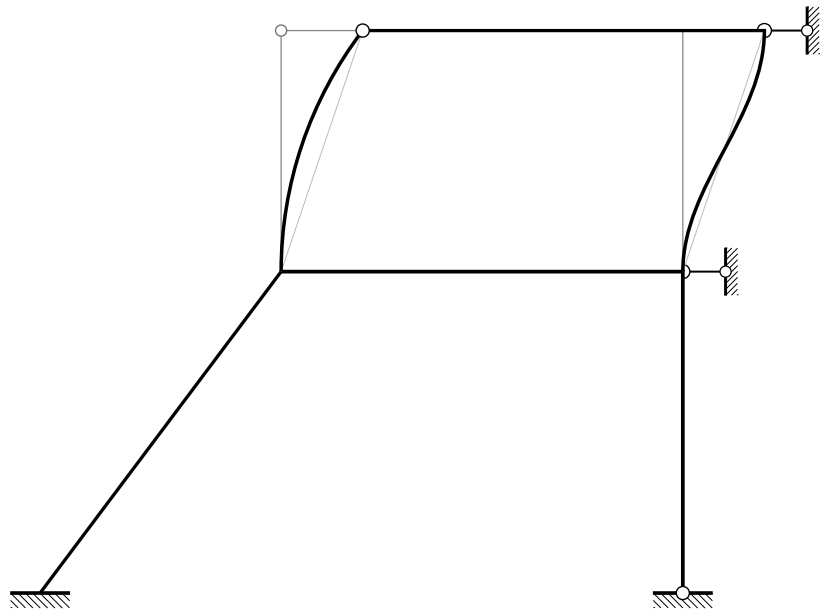
$R_1 = -147{,}8$ kN





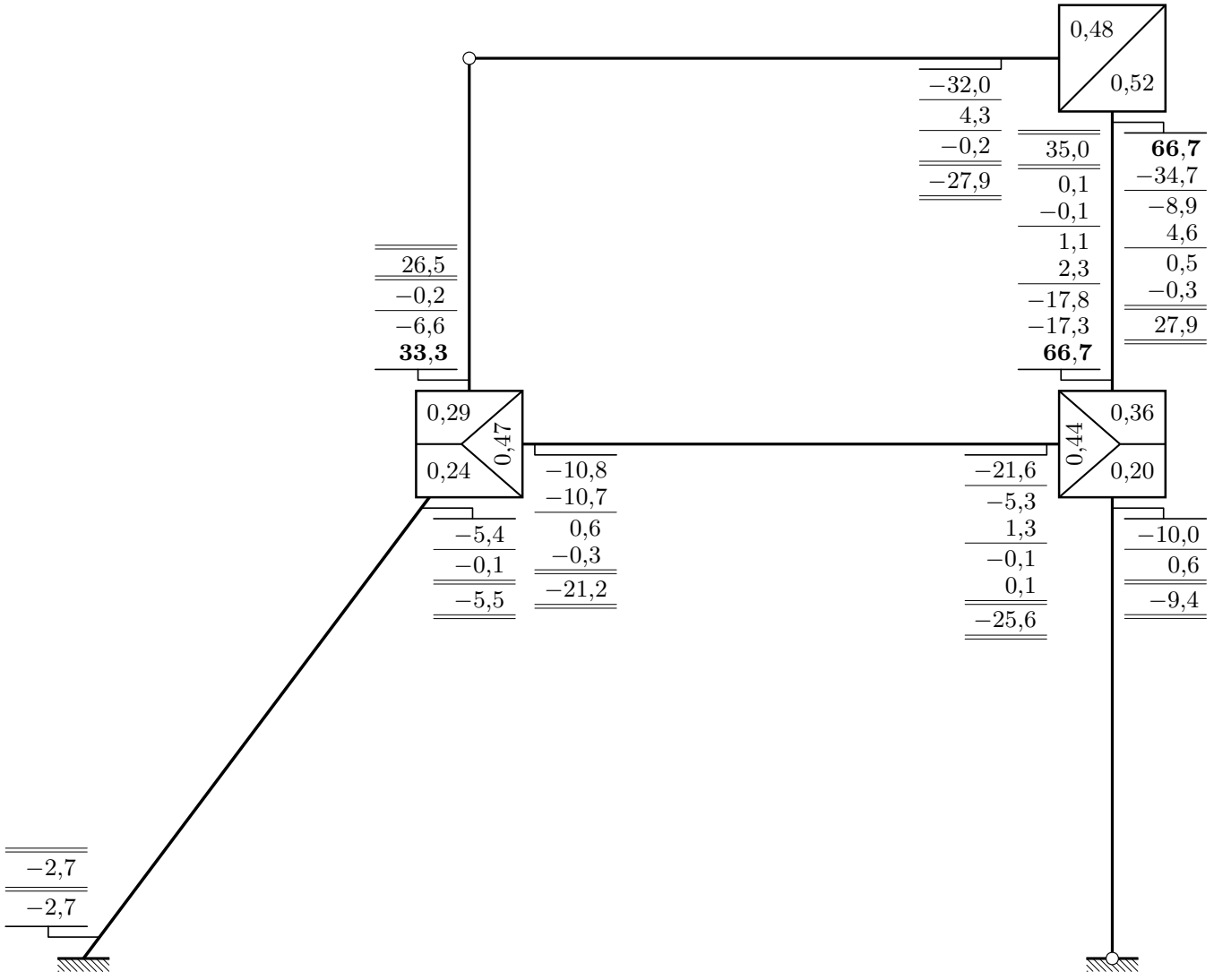
$$\bar{u}_2 = \frac{100}{EI}$$

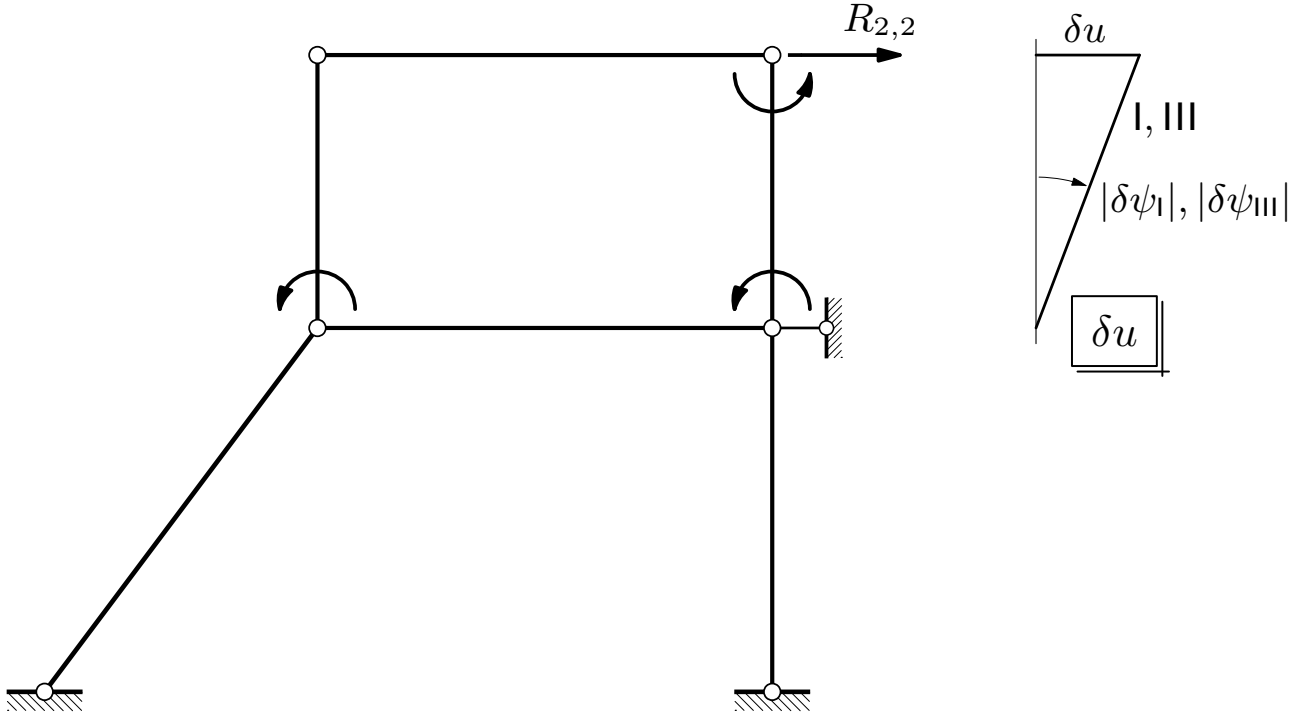
$$\overline{\psi}_{\{3,5\}} = \overline{\psi}_{\{4,6\}} = -\frac{\bar{u}_2}{\ell_{\{3,5\}}} = -\frac{100}{3\,EI} = -\frac{33{,}3}{EI}$$



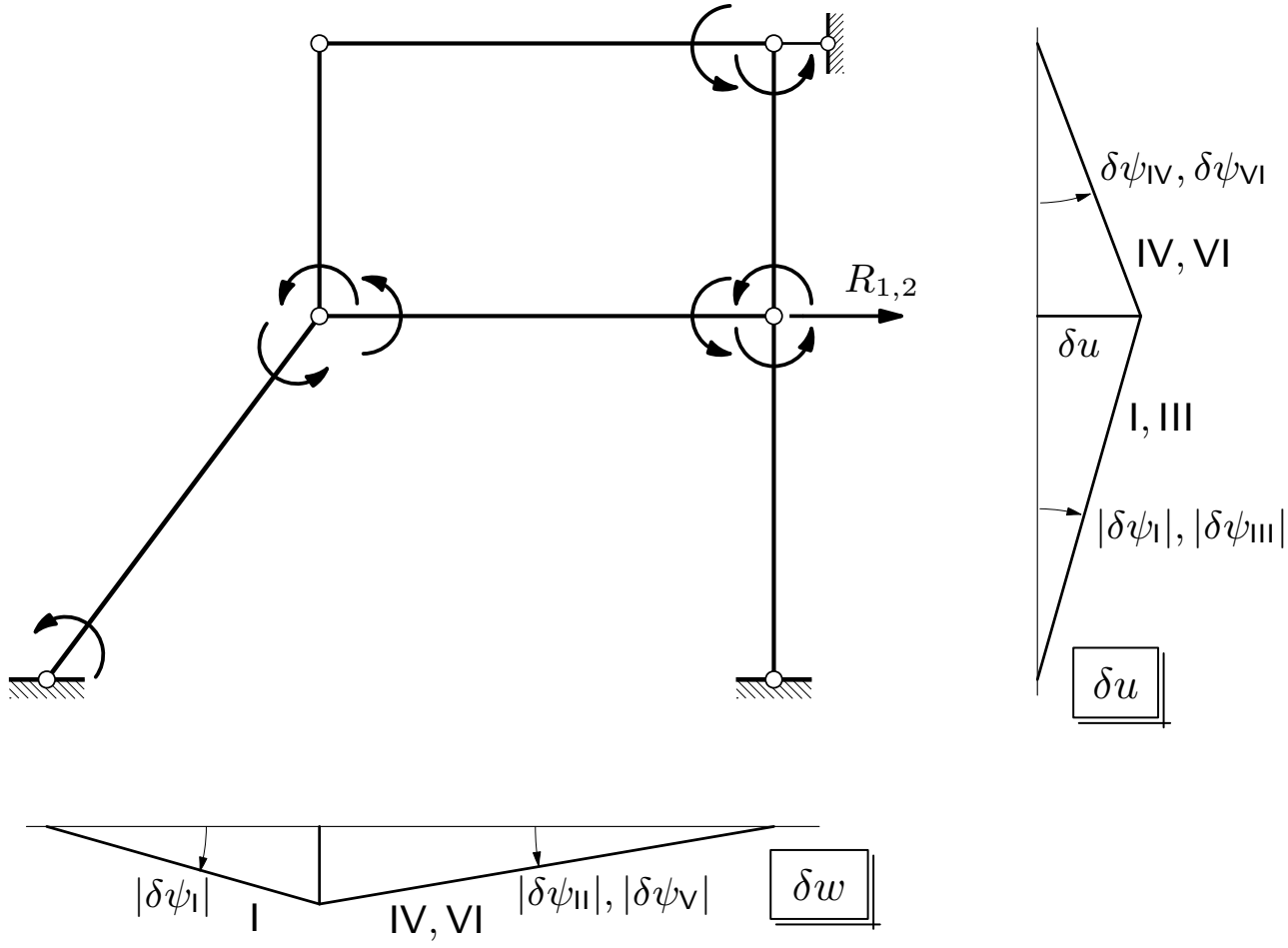
$$\overline{M}_{3,5} = -3\,k_{\{3,5\}}\,\overline{\psi}_{\{3,5\}} = -3\,\frac{EI}{3}\left(-\frac{33{,}3}{EI}\right) = 33{,}33$$

$$\overline{M}_{4,6} = \overline{M}_{6,4} = -6\,k_{\{4,6\}}\,\overline{\psi}_{\{4,6\}} = 66{,}7$$

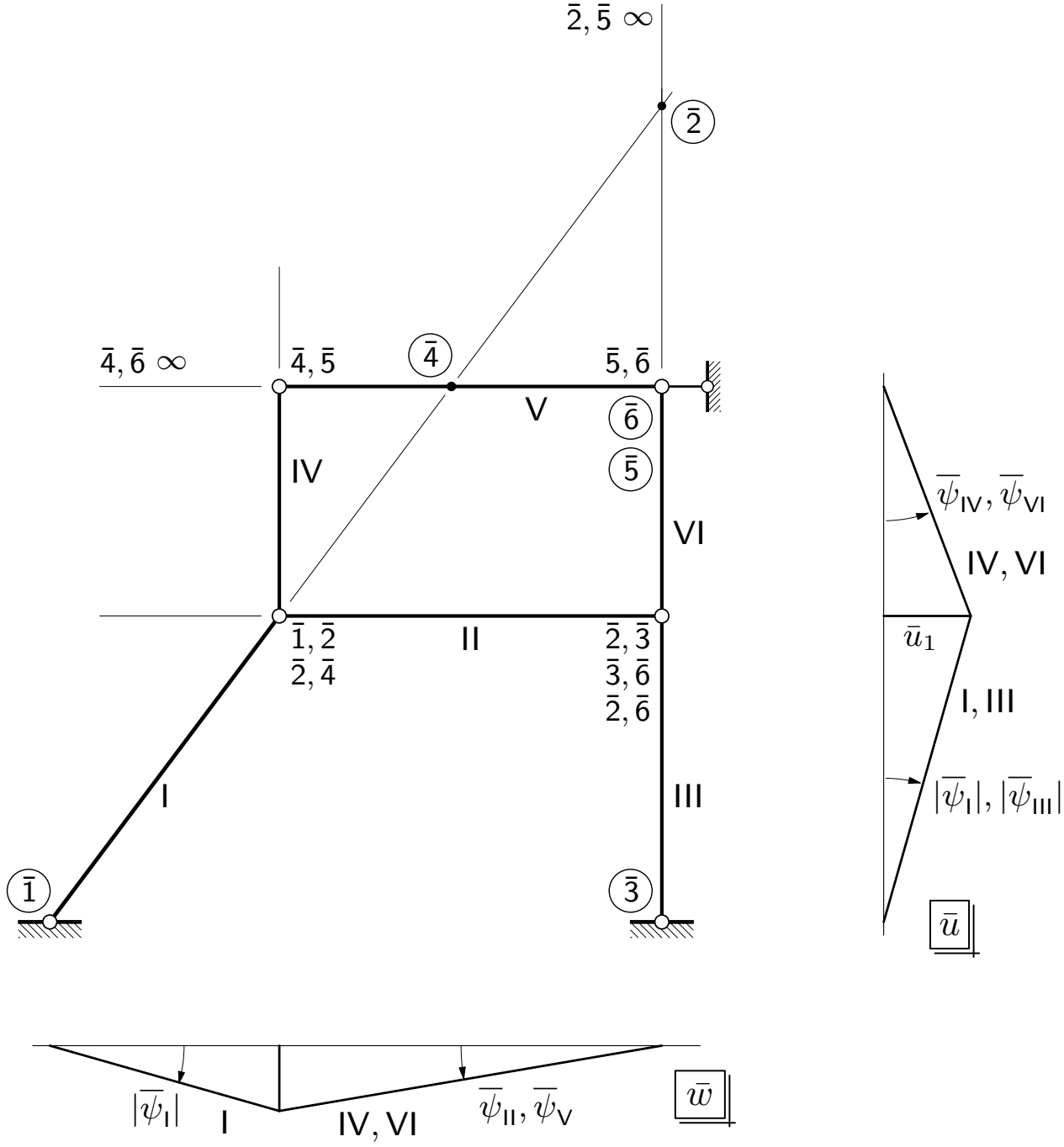

0,48
0,52
−32,0
4,3
−0,2
−27,9
35,0
0,1
−0,1
1,1
2,3
−17,8
−17,3
66,7
66,7
−34,7
−8,9
4,6
0,5
−0,3
27,9
26,5
−0,2
−6,6
33,3
0,29
0,47
0,24
−10,8
−10,7
0,6
−0,3
−21,2
−5,4
−0,1
−5,5
0,44
0,36
0,20
−21,6
−5,3
1,3
−0,1
0,1
−25,6
−10,0
0,6
−9,4
−2,7
−2,7



$\mathrm{R}_{2,2} = 29{,}8\ \mathrm{kN}$


$R_{1,2}$
$\delta\psi_{IV}, \delta\psi_{VI}$
IV, VI
δu
I, III
$|\delta\psi_I|, |\delta\psi_{III}|$
δu
$|\delta\psi_I|$
I
IV, VI
$|\delta\psi_{II}|, |\delta\psi_V|$
δw

$R_{1,2} = -23{,}1$ kN


$\bar{2}, \bar{5} \infty$
$\bar{2}$
$\bar{4}, \bar{6} \infty$
$\bar{4}, \bar{5}$
$\bar{4}$
$\bar{5}, \bar{6}$
V
$\bar{6}$
$\bar{5}$
IV
VI
$\bar{1}, \bar{2}$
$\bar{2}, \bar{4}$
II
$\bar{2}, \bar{3}$
$\bar{3}, \bar{6}$
$\bar{2}, \bar{6}$
I
III
$\bar{1}$
$\bar{3}$
$\overline{\psi}_{\mathrm{IV}}, \overline{\psi}_{\mathrm{VI}}$
IV, VI
$\bar{u}_1$
I, III
$|\overline{\psi}_{\mathrm{I}}|, |\overline{\psi}_{\mathrm{III}}|$
$\bar{u}$
$|\overline{\psi}_{\mathrm{I}}|$
I
IV, VI
$\overline{\psi}_{\mathrm{II}}, \overline{\psi}_{\mathrm{V}}$
$\bar{w}$

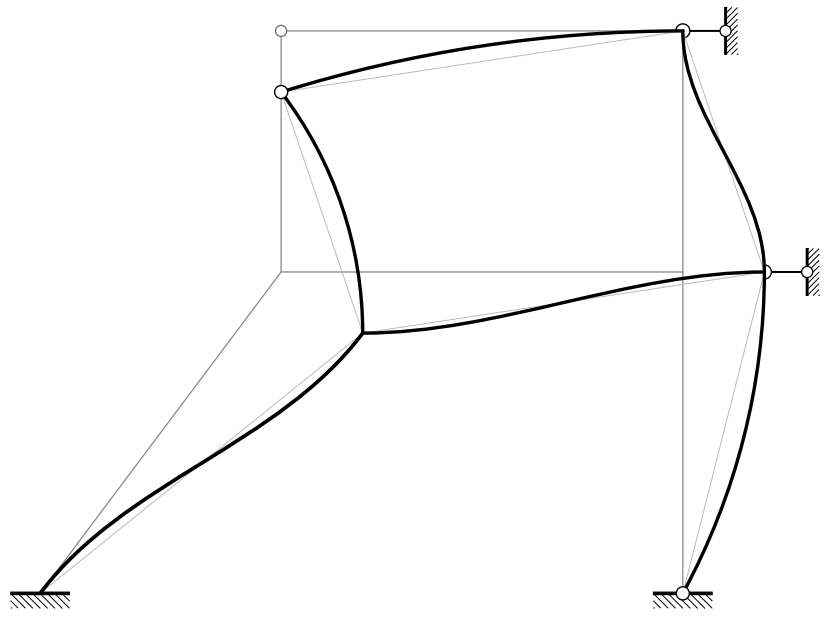
$$\bar{u}_1 = \frac{100}{EI}$$

$$\overline{\psi}_{\{1,3\}} = \overline{\psi}_{\{2,4\}} = \overline{\psi}_I = -\frac{\bar{u}_1}{4} = -\frac{100}{4\,EI} = -\frac{25{,}0}{EI}$$

$$\overline{\psi}_{\{3,5\}} = \overline{\psi}_{\{4,6\}} = \overline{\psi}_{IV} = -\frac{\bar{u}_1}{3} = -\frac{100}{3\,EI} = -\frac{33{,}3}{EI}$$

$$\overline{\psi}_I \cdot 3 = \overline{\psi}_{II} \cdot 5 \qquad \Rightarrow \qquad \overline{\psi}_{II} = -\frac{3}{5}\,\overline{\psi}_I$$

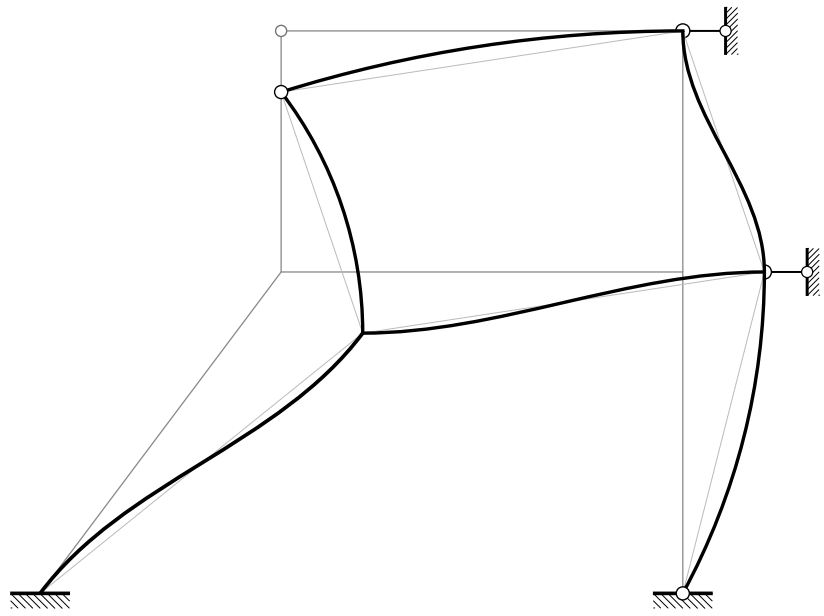
$$\overline{\psi}_{\{3,4\}} = \overline{\psi}_{\{5,6\}} = \overline{\psi}_{II} = = \frac{15{,}0}{EI}$$



$$\overline{M}_{1,3} = \overline{M}_{3,1} = -6\, k_{\{1,3\}}\, \overline{\psi}_{\{1,3\}} = 30{,}0$$

$$\overline{M}_{4,2} = -3\, k_{\{2,4\}}\, \overline{\psi}_{\{2,4\}} = 18{,}8$$

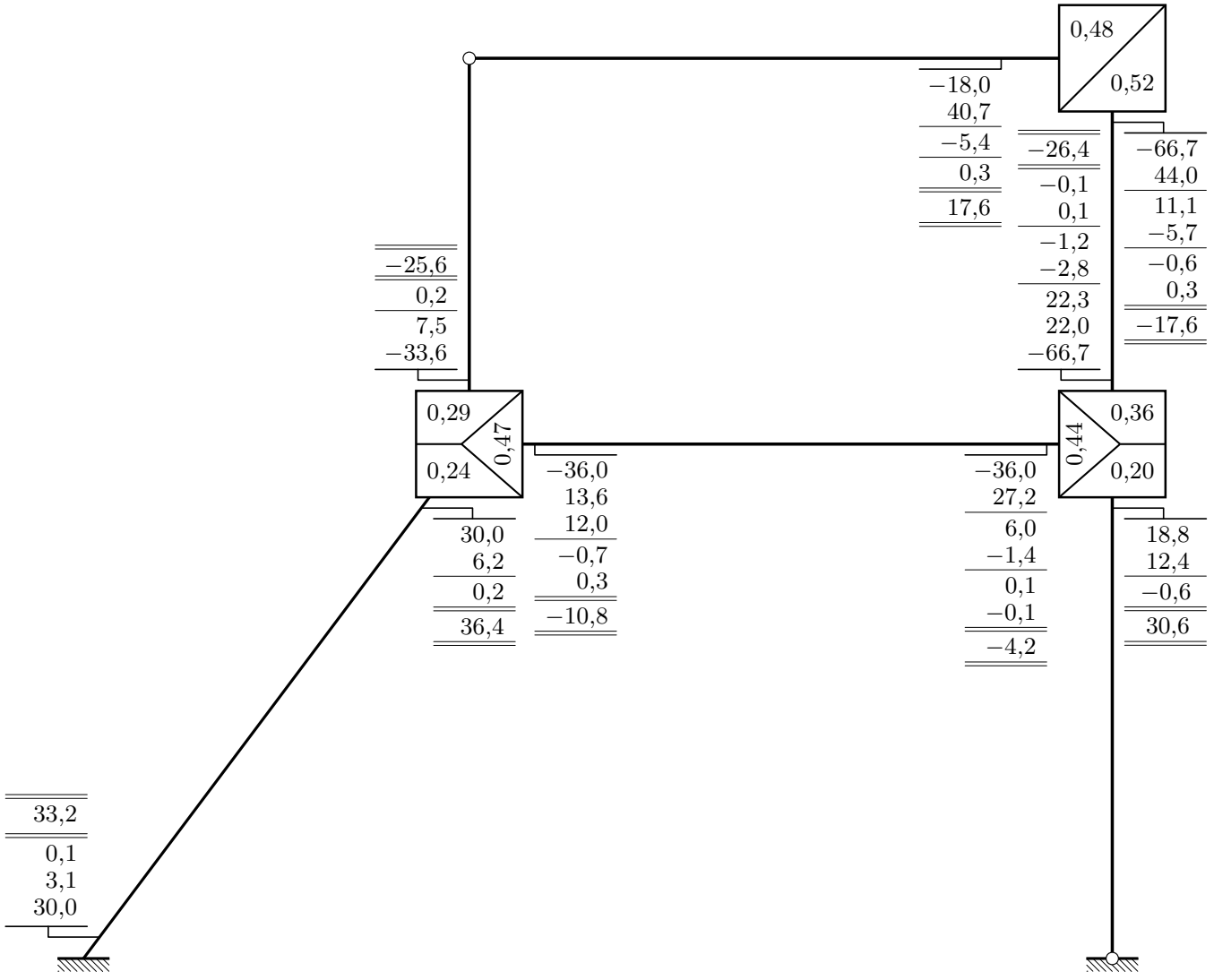
$$\overline{M}_{3,5} = -3\, k_{\{3,5\}}\, \overline{\psi}_{\{3,5\}} = -33{,}3$$

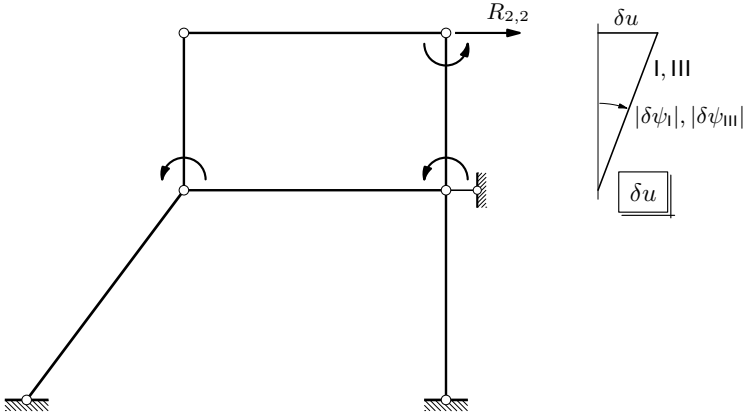


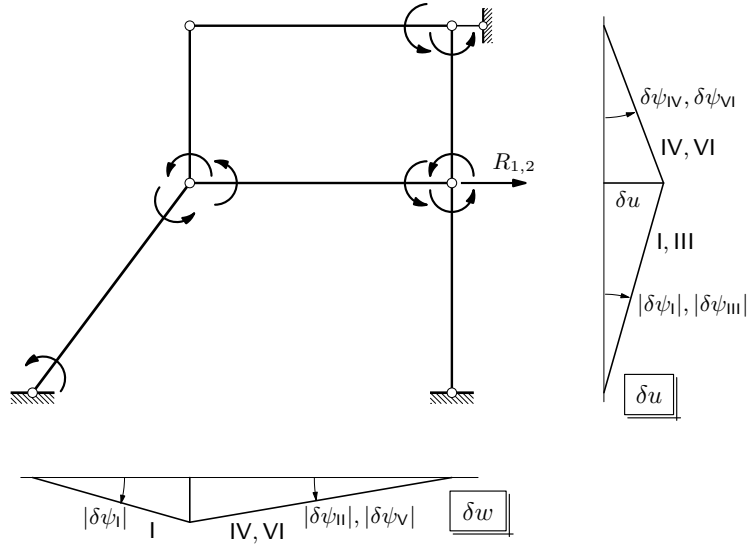
$$\overline{M}_{4,6} = \overline{M}_{6,4} = -6\,k_{\{4,6\}}\,\overline{\psi}_{\{4,6\}} = -66{,}7$$

$$\overline{M}_{3,4} = \overline{M}_{4,3} = -6\,k_{\{3,4\}}\,\overline{\psi}_{\{3,4\}} = -36{,}0$$

$$\overline{M}_{6,5} = -3\,k_{\{5,6\}}\,\overline{\psi}_{\{5,6\}} = -18{,}0$$


0,48
0,52
−18,0
40,7
−5,4
0,3
17,6
−26,4
−0,1
0,1
−1,2
−2,8
22,3
22,0
−66,7
−66,7
44,0
11,1
−5,7
−0,6
0,3
−17,6
−25,6
0,2
7,5
−33,6
0,29
0,47
0,24
−36,0
13,6
12,0
−0,7
0,3
−10,8
30,0
6,2
0,2
36,4
0,44
0,36
0,20
−36,0
27,2
6,0
−1,4
0,1
−0,1
−4,2
18,8
12,4
−0,6
30,6
33,2
0,1
3,1
30,0





$$R_{2,1} = -23{,}2$$

$$R_{1,1} = 47{,}9$$

$$R_{1,1}\,\rho_1 + R_{1,2}\,\rho_2 + R_1 = 0$$

$$R_{2,1}\,\rho_1 + R_{2,2}\,\rho_2 + R_2 = 0$$

$$47{,}9\,\rho_1 - 23{,}1\,\rho_2 - 147{,}8 = 0$$

$$-23{,}2\,\rho_1 + 29{,}8\,\rho_2 - 37{,}5 = 0$$

$$\rho_1 = 5{,}91$$

$$\rho_2 = 5{,}86$$

$$u_1 = \rho_1\,\bar{u}_1 = 5{,}91 \cdot \frac{100}{EI}$$

$$u_2 = \rho_2\,\bar{u}_2 = 5{,}86 \cdot \frac{100}{EI}$$

$$M = M_{P,q} + \rho_1 M_{\bar{u}_1} + \rho_2 M_{\bar{u}_2}$$