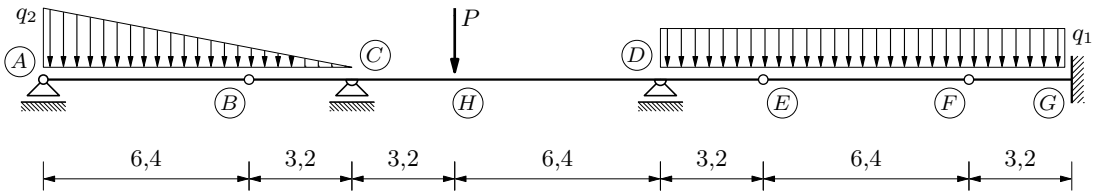
Gerberov nosač – analitički (raščlanjeni) postupak

Nacrtajte dijagrame unutarnjih sila! Izračunajte najveće intenzitete momenata savijanja u poljima $A-C$ i $D-G$!

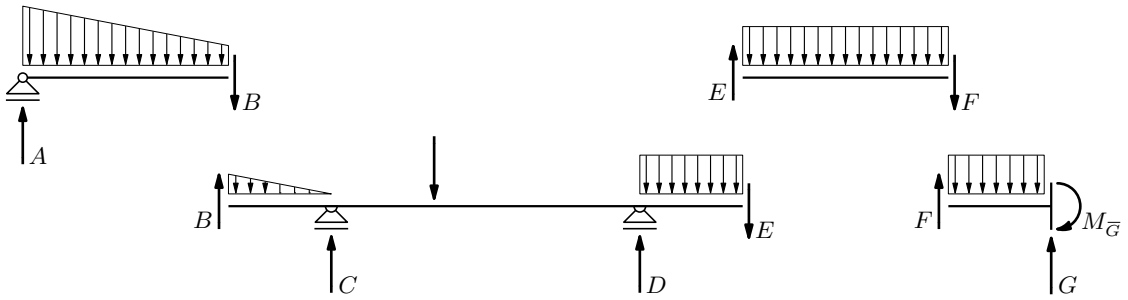
$q_1 = 32\,\mathrm{kN/m}$

$q_2 = 45\,\mathrm{kN/m}$

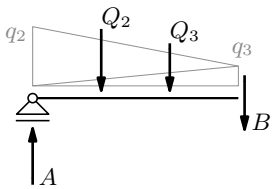
$P = 110\,\mathrm{kN}$



„mehanizam" prijenosa sila:



vrijednosti reakcija i spojnih sila:



$$q_3 = q_2 \frac{3{,}2}{6{,}4+3{,}2} = 15\,\mathrm{kN/m}$$

$$Q_2 = \frac{1}{2}\, q_2 \cdot 6{,}4 = 144\,\mathrm{kN}$$

$$Q_3 = \frac{1}{2}\, q_3 \cdot 6{,}4 = 48\,\mathrm{kN}$$

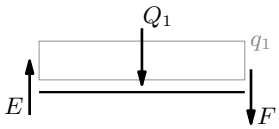
$$\sum\nolimits_{\overline{AB}} M_{/B} = 0: \qquad -6{,}4 \cdot A + \frac{2}{3} \cdot 6{,}4 \cdot Q_2 + \frac{1}{3} \cdot 6{,}4 \cdot Q_3 = 0$$

$$A = \frac{2}{3} \cdot Q_2 + \frac{1}{3} \cdot Q_3 = 112\,\mathrm{kN}$$

$$\sum\nolimits_{\overline{AB}} M_{/A} = 0: \qquad -\frac{1}{3} \cdot 6{,}4 \cdot Q_2 - \frac{2}{3} \cdot 6{,}4 \cdot Q_3 - 6{,}4 \cdot B = 0$$

$$B = -\frac{1}{3} \cdot Q_2 - \frac{2}{3} \cdot Q_3 = -80\,\mathrm{kN}$$

provjera: $\sum\nolimits_{\overline{AB}} F_z = -A + Q_2 + Q_3 + B = -112 + 144 + 48 + (-80) = 0$

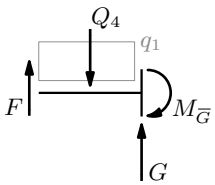


$$Q_1 \;=\; q_1 \cdot 6{,}4 \;=\; 204{,}8\,\text{kN}$$

$$\sum\nolimits_{\overline{EF}} M_{/F} \;=\; 0 \;:\qquad -6{,}4 \cdot E \;+\; \frac{1}{2} \cdot 6{,}4 \cdot Q_1 \;=\; 0 \qquad \Rightarrow \qquad E \;=\; \frac{1}{2}\, Q_1 = 102{,}4\,\text{kN}$$

$$\sum\nolimits_{\overline{EF}} M_{/E} \;=\; 0 \;:\qquad -\frac{1}{2} \cdot 6{,}4 \cdot Q_1 \;-\; 6{,}4 \cdot F \;=\; 0 \qquad \Rightarrow \qquad F \;=\; -\frac{1}{2}\, Q_1 = -102{,}4\,\text{kN}$$

provjera: $\quad \sum\nolimits_{\overline{EF}} F_z \;=\; -E \;+\; Q_1 \;+\; F \;=\; -102{,}4 \;+\; 204{,}8 \;+\; (-102{,}4) \;=\; 0$



$$Q_4 \;=\; q_1 \cdot 3{,}2 \;=\; 102{,}4\,\text{kN}$$

$$\sum\nolimits_{\overline{FG}} M_{/G} \;=\; 0 \;:\qquad -3{,}2 \cdot F \;+\; \frac{1}{2} \cdot 3{,}2 \cdot Q_4 \;-\; M_{\overline{G}} \;=\; 0$$

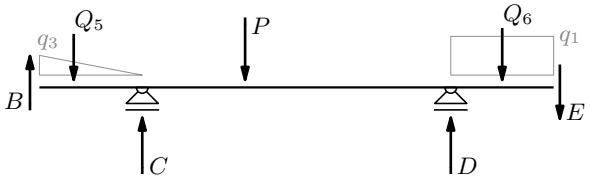
$$M_{\overline{G}} \;=\; -3{,}2 \cdot F \;+\; \frac{1}{2} \cdot 3{,}2 \cdot Q_4 \;=\; -3{,}2 \cdot (-102{,}4) \;+\; 1{,}6 \cdot 102{,}4 \;=\; 491{,}52\,\text{kNm}$$

$$\sum\nolimits_{\overline{FG}} F_z \;=\; 0 \;:\qquad -F \;+\; Q_4 \;-\; G \;=\; 0$$

$$G \;=\; -F \;+\; Q_4 \;=\; -(-102{,}4) \;+\; 102{,}4 \;=\; 204{,}8\,\text{kN}$$

provjera: $\quad \sum\nolimits_{\overline{FG}} M_{/F} \;=\; -\frac{1}{2} \cdot 3{,}2 \cdot Q_4 \;-\; M_{\overline{G}} \;+\; 3{,}2 \cdot G$

$$= \; -1{,}6 \cdot 102.4 \;-\; 491{,}52 \;+\; 3{,}2 \cdot 204{,}8 \;=\; 0$$



$$Q_5 \;=\; \frac{1}{2}\, q_3 \cdot 3{,}2 \;=\; 24\,\text{kN}$$

$$Q_6 \;=\; q_1 \cdot 3{,}2 \;=\; 102{,}4\,\text{kN}$$

$$\sum\nolimits_{\overline{BE}} M_{/D} \;=\; 0 \;:\qquad -(3{,}2 + 3{,}2 + 6{,}4) \cdot B \;+\; \left(\frac{2}{3} \cdot 3{,}2 \;+\; 3{,}2 \;+\; 6{,}4\right) \cdot Q_5$$

$$-\; (3{,}2 + 6{,}4) \cdot C \;+\; 6{,}4 \cdot P \;-\; \frac{1}{2} \cdot 3{,}2 \cdot Q_6 \;-\; 3{,}2 \cdot E \;=\; 0$$

$$C \;=\; \frac{1}{9{,}6} \cdot (-12{,}8 \cdot B \;+\; 11{,}733\,3 \cdot Q_5 \;+\; 6{,}4 \cdot P \;-\; 1{,}6 \cdot Q_6 \;-\; 3{,}2 \cdot E)$$

$$= \; \frac{1}{9{,}6} \cdot \big(-12{,}8 \cdot (-80) \;+\; 11{,}733\,3 \cdot 24 \;+\; 6{,}4 \cdot 110 \;-\; 1{,}6 \cdot 102{,}4 \;-\; 3{,}2 \cdot 102{,}4\big)$$

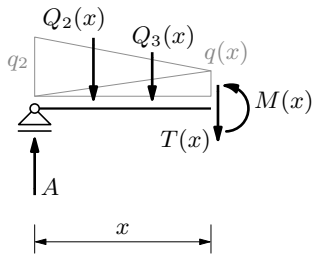
$$= \; 158{,}133\,\text{kN}$$

$$\sum\nolimits_{\overline{BE}} M_{/C} \;=\; 0 \;:\qquad -3{,}2\cdot B \;+\; \frac{2}{3}\cdot 3{,}2\cdot Q_5 \;-\; 3{,}2\cdot P \;+\; (3{,}2+6{,}4)\cdot D$$
$$-\left(3{,}2+6{,}4+\frac{1}{2}\cdot 3{,}2\right)\cdot Q_6 \;-\; (3{,}2+6{,}4+3{,}2)\cdot E \;=\; 0$$

$$D \;=\; \frac{1}{9{,}6}\cdot(3{,}2\cdot B \;-\; 2{,}133\,33\cdot Q_5 \;+\; 3{,}2\cdot P \;+\; 11.2\cdot Q_6 \;+\; 12{,}8\cdot E)$$
$$\;=\; \frac{1}{9{,}6}\cdot\big(3{,}2\cdot(-80) \;-\; 2{,}133\,33\cdot 24 \;+\; 3{,}2\cdot 110 \;+\; 11{,}2\cdot 102{,}4 \;+\; 12{,}8\cdot 102{,}4\big)$$
$$\;=\; 260{,}667\,\mathrm{kN}$$

provjera: $\sum\nolimits_{\overline{BE}} F_z \;=\; -B \;+\; Q_5 \;-\; C \;+\; P \;-\; D \;+\; Q_6 \;+\; E$
$$\;=\; -(-80) \;+\; 24 \;-\; 158{,}133 \;+\; 110 \;-\; 260{,}667 \;+\; 102{,}4 \;+\; 102{,}4 \;=\; 0$$

funkcije koje opisuju raspodjelu unutarnjih sila (i vrijednosti tih sila):



$$q(x) \;=\; q_2 \;-\; \frac{q_2}{9{,}6}\,x \;=\; 45 \;-\; 4{,}687\,5\cdot x^2$$

$$Q_2(x) \;=\; \frac{1}{2}\,q_2\,x \;=\; 22{,}5\cdot x$$

$$Q_3(x) \;=\; \frac{1}{2}\,q(x)\,x \;=\; \frac{1}{2}\left(q_2 \;-\; \frac{q_2}{9{,}6}\,x\right)x$$
$$\;=\; 22{,}5\cdot x \;-\; 2{,}343\,75\cdot x^2$$

$$\sum\nolimits_{\overline{Ax}} M_{/x} \;=\; 0 \;:\qquad -x\cdot A \;+\; \frac{2}{3}\cdot x\cdot Q_2(x) \;+\; \frac{1}{3}\cdot x\cdot Q_3(x) \;+\; M(x) \;=\; 0$$

$$M(x) \;=\; x\cdot A \;-\; \frac{2}{3}\cdot x\cdot Q_2(x) \;-\; \frac{1}{3}\cdot x\cdot Q_3(x)$$
$$\;=\; 112\cdot x \;-\; \frac{2}{3}\cdot x\cdot(22{,}5\cdot x) \;-\; \frac{1}{3}\cdot x\cdot(22{,}5\cdot x \;-\; 2{,}343\,75\cdot x^2)$$
$$\;=\; 112\cdot x \;-\; 22{,}5\cdot x^2 \;+\; 0{,}781\,25\cdot x^3$$

$$M(0) \;=\; 0 \qquad \& \qquad M(6{,}4) \;=\; 0 \qquad \& \qquad M(9{,}6) \;=\; -307{,}2\,\mathrm{kNm} \;=\; M_C$$

$$\sum\nolimits_{\overline{Ax}} F_z \;=\; 0 \;:\qquad -A \;+\; Q_2(x) \;+\; Q_3(x) \;+\; T(x) \;=\; 0$$

$$T(x) \;=\; A \;-\; Q_2(x) \;-\; Q_3(x) \;=\; 112 \;-\; 22{,}5\cdot x \;-\; (22{,}5\cdot x \;-\; 2{,}343\,75\cdot x^2)$$
$$\;=\; 112 \;-\; 45\cdot x \;+\; 2{,}343\,75\cdot x^2$$

$$T(0+\mathrm{d}x) \;=\; T(0^+) \;=\; 112\,\mathrm{kN} \;=\; T_{A^{\mathrm{d}}} \qquad \& \qquad T(9{,}6-\mathrm{d}x) \;=\; T(9{,}6^-) \;=\; -104\,\mathrm{kN} \;=\; T_{C^{\ell}}$$

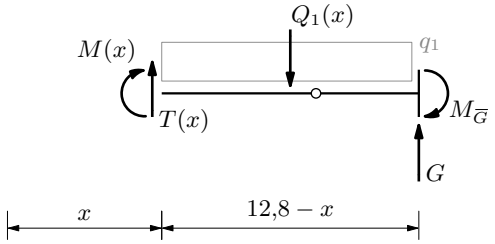
$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} \;=\; 112 \;-\; 2\cdot 22{,}5\cdot x \;+\; 3\cdot 0{,}781\,25\cdot x^2 \;=\; 112 \;-\; 45\cdot x \;+\; 2{,}343\,75\cdot x^2 \;=\; T(x)$$

$$\frac{\mathrm{d}T(x)}{\mathrm{d}x} \;=\; 45 \;+\; 2\cdot 2{,}343\,75\cdot x \;=\; 45 \;+\; 4{,}687\,5\cdot x \;=\; q(x)$$

provjera M_C & T_{C^ℓ}:

$$\sum\nolimits_{\overline{AC}} M_{/C} \;=\; -\frac{1}{3}\cdot 9{,}6\cdot Q_2(9{,}6) \;-\; \frac{2}{3}\cdot 9{,}6\cdot Q_3(9{,}6) \;+\; M_C \;-\; 9{,}6\cdot T_{C^\ell}$$

$$=\; -\frac{1}{3}\cdot 9{,}6\cdot 216 \;-\; \frac{2}{3}\cdot 9{,}6\cdot 0 \;+\; (-307{,}2) \;-\; 9{,}6\cdot(-104) \;=\; 0$$



$$Q_1(x) \;=\; q_1\cdot(12{,}8 - x) \;=\; 409{,}6 \;-\; 32\cdot x$$

$$\sum\nolimits_{\overline{xG}} M_{/x} \;=\; 0 \;:\qquad -M(x) \;-\; \frac{1}{2}\cdot(12{,}8 - x)\cdot Q_1(x) \;+\; (12{,}8 - x)\cdot G \;-\; M_{\overline{G}} \;=\; 0$$

$$M(x) \;=\; -\frac{1}{2}\cdot(12{,}8 - x)\cdot Q_1(x) \;+\; (12{,}8 - x)\cdot G \;-\; M_{\overline{G}}$$

$$=\; -\frac{1}{2}\cdot(12{,}8 - x)\cdot(409{,}6 - 32\cdot x) \;+\; (12{,}8 - x)\cdot 204{,}8 \;-\; 491{,}52$$

$$=\; -491{,}52 \;+\; 204{,}8\cdot x \;-\; 16\cdot x^2$$

$$M(0) \;=\; -491{,}52\,\mathrm{kNm} \;=\; M_D \qquad \& \qquad M(3{,}2) \;=\; 0$$

$$\& \qquad M(9{,}6) \;=\; 0 \qquad \& \qquad M(12{,}8) \;=\; -491{,}52\,\mathrm{kNm} \;=\; M_G$$

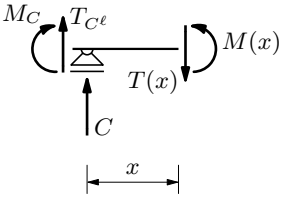
$$\sum\nolimits_{\overline{xG}} M_{/x} \;=\; 0 \;:\qquad -T(x) \;+\; Q_1(x) \;-\; G \;=\; 0$$

$$T(x) \;=\; Q_1(x) \;-\; G \;=\; (409{,}6 \;-\; 32\cdot x) \;-\; 204{,}8 \;=\; 204{,}8 \;-\; 32\cdot x$$

$$T(0^+) \;=\; 204{,}8\,\mathrm{kN} \;=\; T_{D^\mathrm{d}} \qquad \& \qquad T(12{,}8^-) \;=\; -204{,}8\,\mathrm{kN} \;=\; T_{G^\ell}$$

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} \;=\; 204{,}8 \;-\; 2\cdot 16\cdot x \;=\; 204{,}8 \;-\; 32\cdot x \;=\; T(x)$$

$$\frac{\mathrm{d}T(x)}{\mathrm{d}x} \;=\; 32 \;=\; q_1 \;=\; q(x)$$



$$\sum\nolimits_{\overline{Cx}} M_{/x} \;=\; 0 \;:\qquad -M_C \;-\; x\cdot T_{C^\ell} \;-\; x\cdot C \;+\; M(x) \;=\; 0$$

$$M(x) \;=\; M_C \;+\; x\cdot(T_{C^\ell} \;+\; C)$$

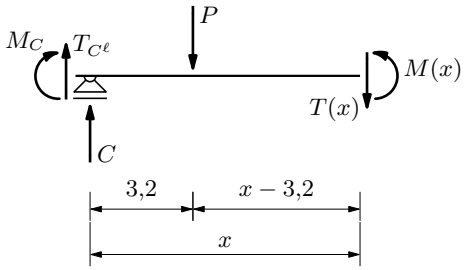
$$=\; -307{,}2 \;+\; x\cdot(-104 \;+\; 158{,}133) \;=\; -307{,}2 \;+\; 54{,}133\cdot x$$

$$M(0) \;=\; -307{,}2\,\mathrm{kNm} \;=\; M_C \qquad \& \qquad M(3{,}2) \;=\; -133{,}974\,\mathrm{kNm} \;=\; M_H$$

$$\sum\nolimits_{\overline{Cx}} F_z \;=\; 0 \;:\qquad -T_{C^\ell} \;-\; C \;+\; T(x) \;=\; 0$$

$$T(x) \;=\; T_{C^\ell} \;+\; C \;=\; 54{,}133$$

$$T(0^+) \;=\; 54{,}133\,\mathrm{kN} \;=\; T_{C^\mathrm{d}} \qquad \& \qquad T(3{,}2^-) \;=\; 54{,}133\,\mathrm{kN} \;=\; T_{H^\ell}$$



$$\sum\nolimits_{\overline{Cx}} M_{/x} \;=\; 0 \;:\qquad -M_C \;-\; x\cdot T_{C^\ell} \;-\; x\cdot C \;+\; (x-3{,}2)\cdot P \;+\; M(x) \;=\; 0$$

$$M(x) \;=\; M_C \;+\; x\cdot(T_{C^\ell} \;+\; C) \;+\; (3{,}2-x)\cdot P$$

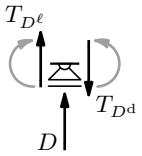
$$\;=\; -307{,}2 \;+\; 54{,}133\cdot x \;+\; 352 \;-\; 110\cdot x \;=\; 44{,}8 \;-\; 55{,}867\cdot x$$

$$M(3{,}2) \;=\; -133{,}974\,\mathrm{kNm} \;=\; M_H \qquad \& \qquad M(9{,}6) \;=\; -491{,}52(3)\,\mathrm{kNm} \;=\; M_D$$

$$\sum\nolimits_{\overline{Cx}} F_z \;=\; 0 \;:\qquad -T_{C^\ell} \;-\; C \;+\; P \;+\; T(x) \;=\; 0$$

$$T(x) \;=\; T_{C^\ell} \;+\; C \;-\; P \;=\; -55{,}867$$

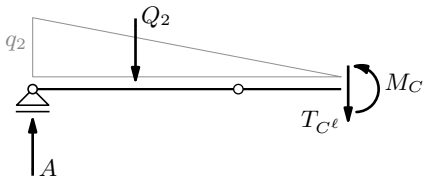
$$T(3{,}2^-) \;=\; -55{,}867\,\mathrm{kN} \;=\; T_{H^\mathrm{d}} \qquad \& \qquad T(9{,}6^-) \;=\; -55{,}867\,\mathrm{kN} \;=\; T_{D^\ell}$$



provjera: $\displaystyle\sum\nolimits_D F_z \;=\; -T_{D^\ell} \;-\; D \;+\; T_{D^\mathrm{d}}$

$$\;=\; -(-55{,}867) \;-\; 260{,}667 \;+\; 204{,}8 \;=\; 0$$

vrijednosti unutarnjih sila u karakterističnim točkama (izravno izračunavanje):

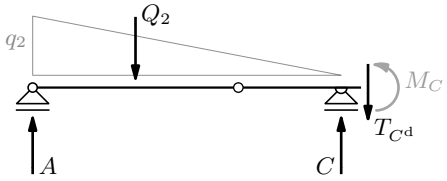


$$Q_2 \;=\; \frac{1}{2}\cdot q_2 \cdot 9{,}6 \;=\; 216\,\mathrm{kN}$$

$$\sum\nolimits_{\overline{AC}} M_{/C} \;=\; 0 \;:\qquad -9{,}6\cdot A \;+\; \frac{2}{3}\cdot 9{,}6\cdot Q_2 \;+\; M_C \;=\; 0$$

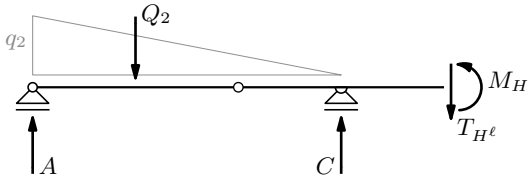
$$M_C \;=\; 9{,}6\cdot 112 \;-\; \frac{2}{3}\cdot 9{,}6\cdot 216 \;=\; -307{,}2\,\mathrm{kNm}$$

$$\sum\nolimits_{\overline{AC^\ell}} F_z \;=\; 0 \;:\qquad -A \;+\; Q_2 \;+\; T_{C^\ell} \;=\; 0 \qquad \Rightarrow \qquad T_{C^\ell} \;=\; 112 \;-\; 216 \;=\; -104\,\mathrm{kN}$$



$$\sum\nolimits_{\overline{AC^{\mathrm{d}}}} F_z \;=\; 0 \;:\qquad -A \;+\; Q_2 \;-\; C \;+\; T_{C^{\mathrm{d}}} \;=\; 0$$

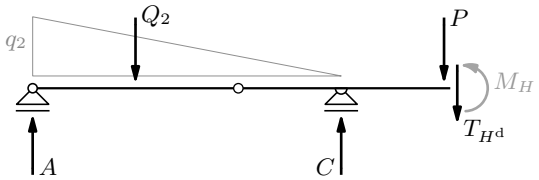
$$T_{C^{\mathrm{d}}} \;=\; 112 \;-\; 216 \;+\; 158{,}133 \;=\; 54{,}133\,\mathrm{kN}$$



$$T_{H^{\ell}} \;=\; T_{C^{\mathrm{d}}} \;=\; 54{,}133\,\mathrm{kN}$$

$$\sum\nolimits_{\overline{AH}} M_{/H} \;=\; 0 \;:\qquad -12{,}8\cdot A \;+\; \left(\frac{2}{3}\cdot 9{,}6 \;+\; 3{,}2\right)\cdot Q_2 \;-\; 3{,}2\cdot C \;+\; M_H \;=\; 0$$

$$M_H \;=\; 12{,}8\cdot 112 \;-\; 9{,}6\cdot 216 \;+\; 3{,}2\cdot 158{,}133 \;=\; -133{,}974\,\mathrm{kNm}$$

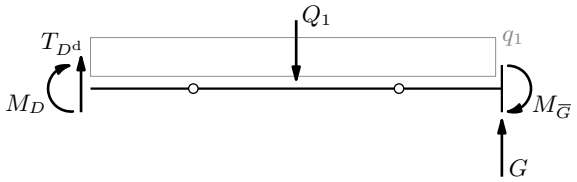


$$\sum\nolimits_{\overline{AH^{\mathrm{d}}}} F_z \;=\; 0 \;:$$

$$-A \;+\; Q_2 \;-\; C \;+\; P \;+\; T_{H^{\mathrm{d}}} \;=\; 0$$

$$T_{H^{\mathrm{d}}} \;=\; 112 \;-\; 216 \;+\; 158{,}133 \;-\; 110$$

$$=\; -55{,}867\,\mathrm{kN}$$

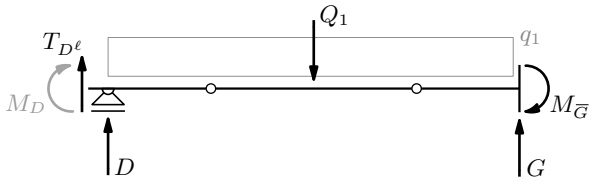


$$Q_1 \;=\; q_1\cdot 12{,}8 \;=\; 409{,}6\,\mathrm{kN}$$

$$\sum\nolimits_{\overline{DG}} M_{/D} \;=\; 0 \;:\qquad -M_D \;-\; 6{,}4\cdot Q_1 \;+\; 12{,}8\cdot G \;-\; M_{\overline{G}} \;=\; 0$$

$$M_D \;=\; -6{,}4\cdot 409{,}6 \;+\; 12{,}8\cdot 204{,}8 \;-\; 491{,}52 \;=\; -491{,}52\,\mathrm{kNm}$$

$$\sum\nolimits_{\overline{D^{\mathrm{d}}G}} F_z \;=\; 0 \;:\qquad -T_{D^{\mathrm{d}}} \;+\; Q_1 \;-\; G \;=\; 0 \qquad\Rightarrow\qquad T_{D^{\mathrm{d}}} \;=\; 409{,}6 \;-\; 204{,}8 \;=\; 204{,}8\,\mathrm{kN}$$

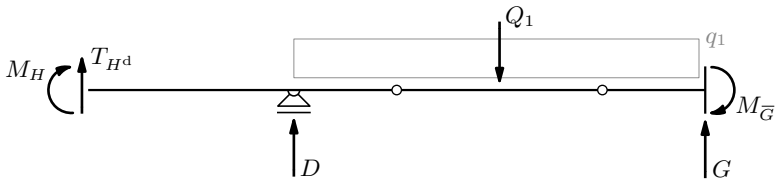


$$\sum\nolimits_{\overline{D^{\mathrm{d}}G}} F_z \;=\; 0 \;:$$

$$-T_{D^{\ell}} \;-\; D \;+\; Q_1 \;-\; G \;=\; 0$$

$$T_{D^{\ell}} \;=\; -260{,}667 \;+\; 409{,}6 \;-\; 204{,}8$$

$$=\; -55{,}867\mathrm{kN}$$



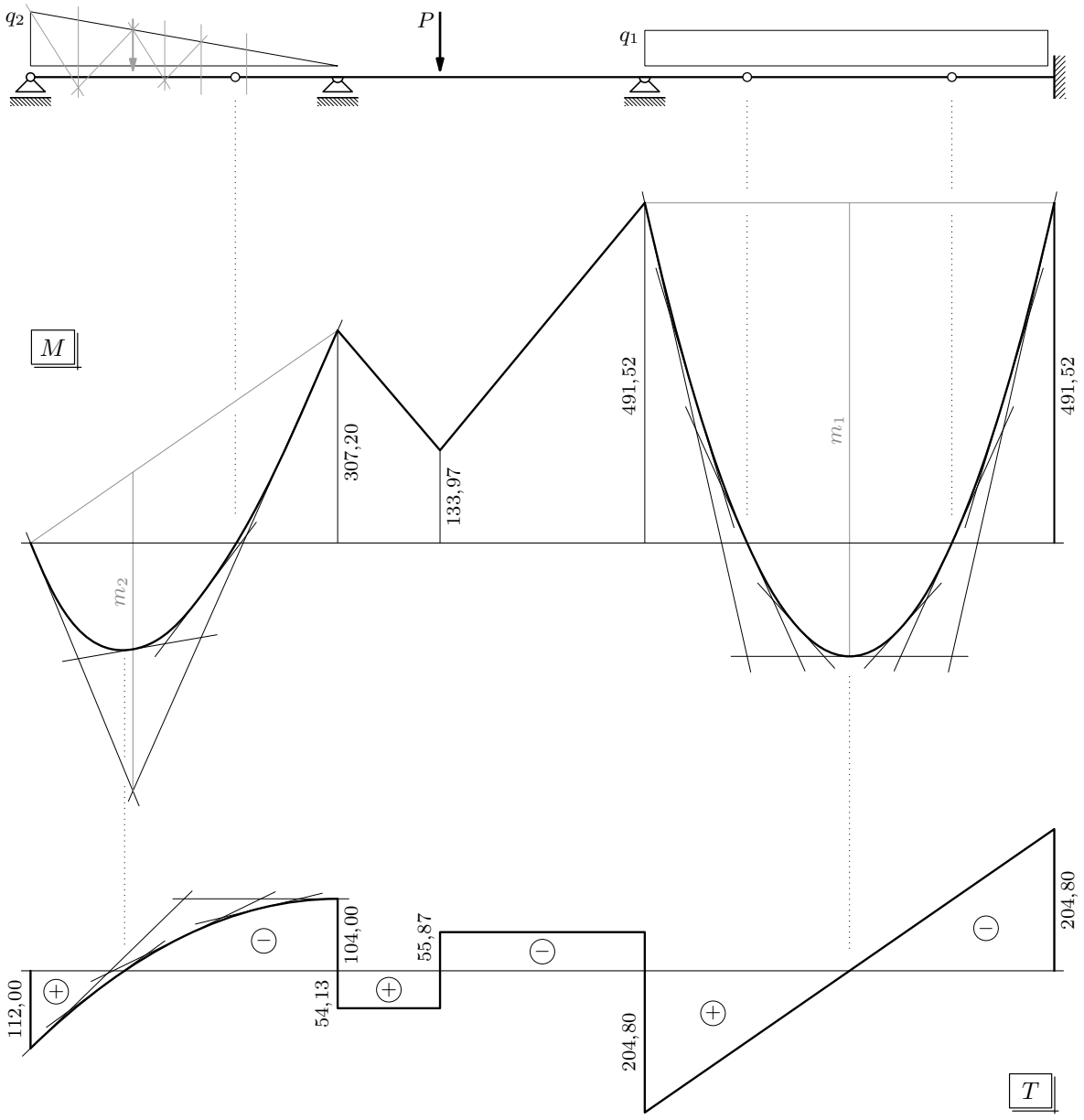
$$T_{H^{\mathrm{d}}} \;=\; T_{D^{\ell}} \;=\; -55{,}867\,\mathrm{kN}$$

$$\sum\nolimits_{\overline{HG}} M_{/H} \;=\; 0 \;:\qquad -M_H \;+\; 6{,}4\cdot D \;-\; 12{,}8\cdot Q_1 \;+\; 19{,}2\cdot G \;-\; M_{\overline{G}} \;=\; 0$$

$$M_H \;=\; 6{,}4\cdot 260{,}667 \;-\; 12{,}8\cdot 409{,}6 \;+\; 19{,}2\cdot 204{,}8 \;-\; 491{,}52 \;=\; -133{,}971\,\mathrm{kNm}$$

$$\simeq\; -133{,}974 \;\;\text{(izračunano provjere radi)}$$

dijagrami:



$$m_2 = \frac{q_2\,\ell_{AC}^2}{9} = \frac{45\cdot 9{,}6^2}{9} = 460{,}8\,\text{kNm}$$ (od spojnice M_A i M_C do sjecišta tangenata u tim točkama)

$$m_1 = \frac{q_1\,\ell_{DG}^2}{8} = \frac{32\cdot 12{,}8^2}{8} = 655{,}36\,\text{kNm}$$ (od spojnice M_D i M_G do tjemena parabole)

$$\left(\overline{m}_1 = \frac{q_1\,\ell_{DG}^2}{4} = 2\,m_1 \text{ od spojnice } M_D \text{ i } M_G \text{ do sjecišta tangenata u tim točkama}\right)$$

najveći intenziteti momenata savijanja:

u polju $A-C$:

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = 0 \qquad \Rightarrow \qquad 112 - 45 \cdot x + 2{,}343\,75 \cdot x^2 = 0$$

$$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-45 \mp \sqrt{45^2 - 4 \cdot 2{,}343\,75 \cdot 112}}{2 \cdot 2{,}343\,75}$$

$$x_1 = 2{,}938\,67\,\mathrm{m} \qquad \& \qquad x_2 = 16{,}261\,3\,\mathrm{m}$$

$$M_{\max} = 112 \cdot x_1 - 22{,}5 \cdot x_1^2 + 0{,}781\,25 \cdot x_1^3 = 154{,}652\,\mathrm{kNm}$$

u polju $D-G$:

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x} = 0 \qquad \Rightarrow \qquad 204{,}8 - 32 \cdot x = 0 \qquad \Rightarrow \qquad x_0 = 6{,}4\,\mathrm{m}$$

$$M_{\max} = -491{,}52 + 204{,}8 \cdot x_0 - 16 \cdot x_0^2 = 163{,}84\,\mathrm{kNm}$$

$$\left(-491{,}52 + \frac{32 \cdot 12{,}^2}{8} = 163{,}84\right)$$