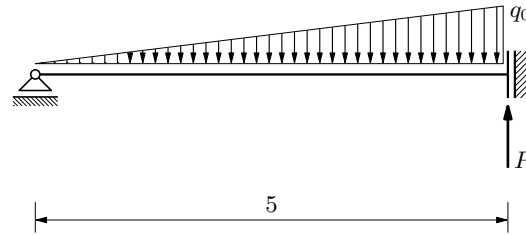


Jednostavno oslonjena greda

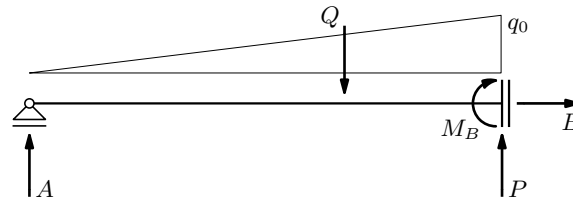
Nacrtajte dijagrame momenata savijanja i poprečnih sila! Izračunajte najveći intenzitet momenta savijanja!

$$q_0 = 50 \text{ kN/m}$$

$$P = 100 \text{ kN}$$



vrijednosti reakcija:



$$\sum F_x = 0 : \quad B = 0$$

$$\sum F_z = 0 : \quad -A + \underbrace{\frac{1}{2} q_0 \ell}_{Q} - P = 0$$

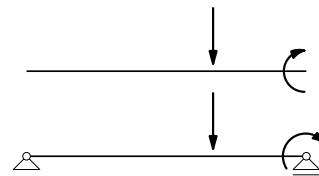
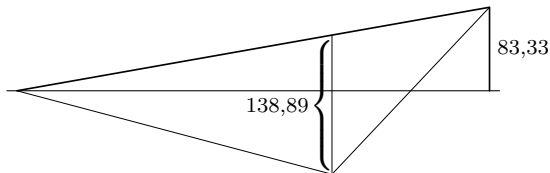
$$Q = \frac{1}{2} \cdot 50 \cdot 5 = 125 \text{ kN}$$

$$A = Q - P = 125 - 100 = 25 \text{ kN}$$

$$\sum M_{/A} = 0 : \quad -\frac{2}{3} \ell Q - M_B + \ell P = 0$$

$$M_B = -\frac{2}{3} \ell Q + \ell P = -\frac{2}{3} \cdot 5 \cdot 125 + 5 \cdot 100 = 83,333 \text{ kNm}$$

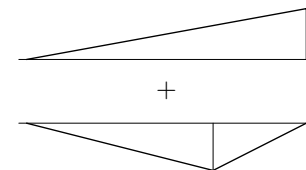
dijagrami:

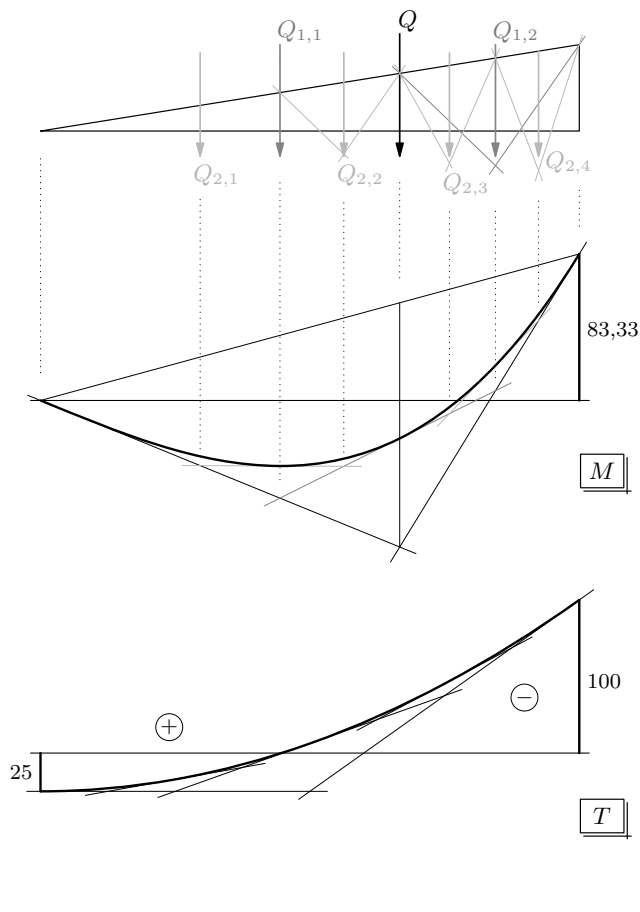


$$\frac{1}{3} \ell Q - \ell \cdot \bar{A} = 0 \quad \Rightarrow \quad \bar{A} = \frac{1}{3} Q$$

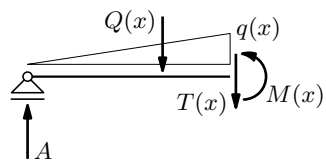
$$\bar{M}_Q = \frac{2}{3} \ell \bar{A} = \frac{2}{9} Q \ell = \frac{2}{9} \left(\frac{1}{2} q_0 \ell \right) \ell = \frac{q_0 \ell^2}{9}$$

$$= \frac{50 \cdot 5^2}{9} = 138,889 \text{ kNm}$$





moment najvećega intenziteta:



$$q(x) = \frac{q_0}{\ell} x = \frac{50}{5} x = 10 \cdot x$$

$$Q(x) = \frac{1}{2} q(x) x = \frac{1}{2} \cdot (10 \cdot x) \cdot x = 5 \cdot x^2$$

$$\sum M_{/x} = 0 : \quad -x A + \frac{1}{3} x Q(x) + M(x) = 0$$

$$M(x) = x \cdot 25 - \frac{x}{3} \cdot (5 \cdot x^2) = 25 \cdot x - \frac{5}{3} \cdot x^3$$

$$\text{provjera: } M(\ell) = 25 \cdot 5 - \frac{5}{3} \cdot 5^3 = -83,3333 \text{ kNm} \quad \checkmark$$

$$\sum F_z = 0 : \quad -A + Q(x) + T(x) = 0$$

$$T(x) = 25 - 5 \cdot x^2$$

$$\text{provjera: } T(0) = 25 \text{ kN} \quad \& \quad T(\ell) = 25 - 5 \cdot 5^2 = -100 \text{ kN}$$

$$\frac{dM(x)}{dx} = 25 - 3 \cdot \frac{5}{3} \cdot x^2 = 25 - 5 \cdot x^2 = T(x)$$

$$\frac{dM(x)}{dx} = 0 \quad (\text{ili } T(x) = 0)$$

$$25 - 5 \cdot x^2 = 0 \quad \Rightarrow \quad x^2 = 5 \quad \Rightarrow \quad x_{1,2} = \mp \sqrt{5} = \mp 2,23607 \text{ [m]}$$

$$M_{\max} = M(2,236) = 37,2678 \text{ kNm}$$

varijacija na temu:

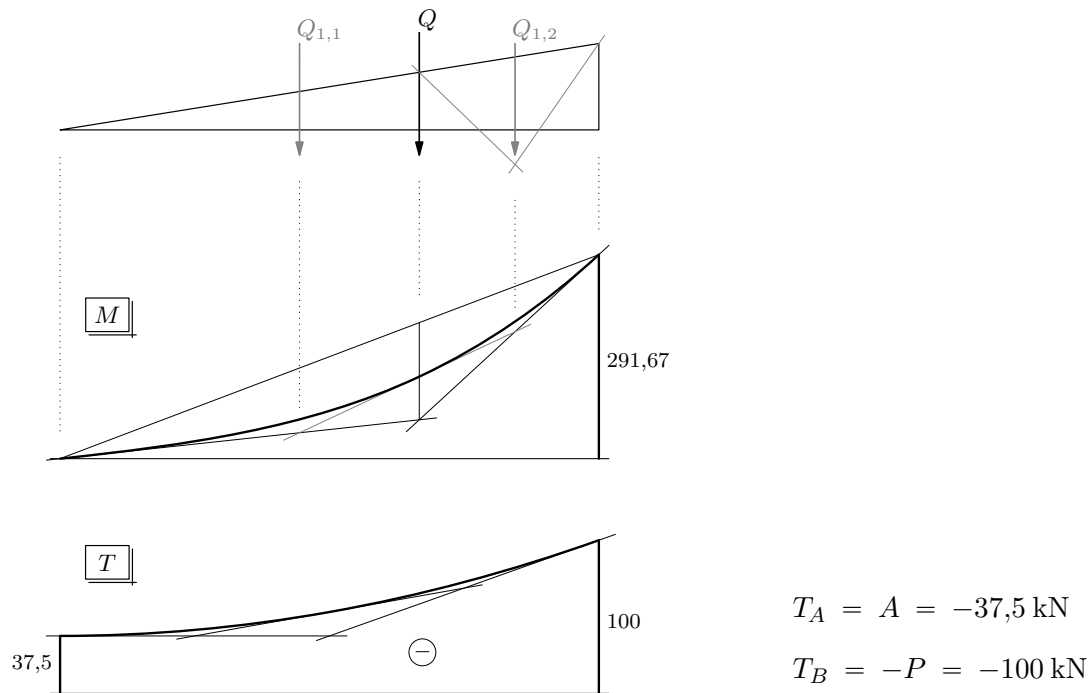
$$q_0 = 25 \text{ kN/m} \quad Q = \frac{1}{2} q_0 \ell = \frac{1}{2} \cdot 25 \cdot 5 = 62,5 \text{ kN}$$

$$P = 100 \text{ kN}$$

$$A = Q - P = 62,5 - 100 = -37,5 \text{ kN} \quad \downarrow$$

$$M_B = -\frac{2}{3} \ell Q + \ell P = -\frac{2}{3} \cdot 5 \cdot 62,5 + 5 \cdot 100 = 291,667 \text{ kNm}$$

$$\bar{M}_Q = \frac{q_0 \ell^2}{9} = \frac{25 \cdot 5^2}{9} = 69,4444 \text{ kNm}$$



$$q(x) = \frac{25}{5} x = 5 \cdot x \quad Q(x) = \frac{1}{2} \cdot (5 \cdot x) \cdot x = \frac{5}{2} \cdot x^2$$

$$M(x) = x A - \frac{x}{3} Q(x) = x \cdot (-37,5) - \frac{x}{3} \cdot \left(\frac{5}{2} \cdot x^2 \right) = -37,5 \cdot x - \frac{5}{6} \cdot x^3$$

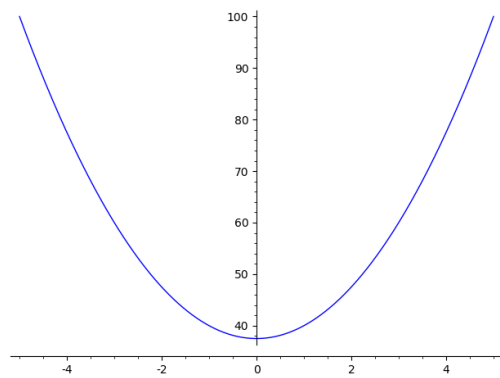
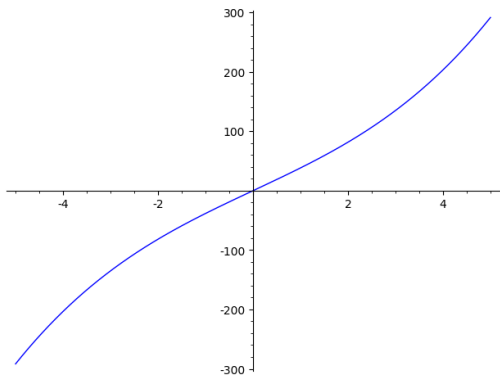
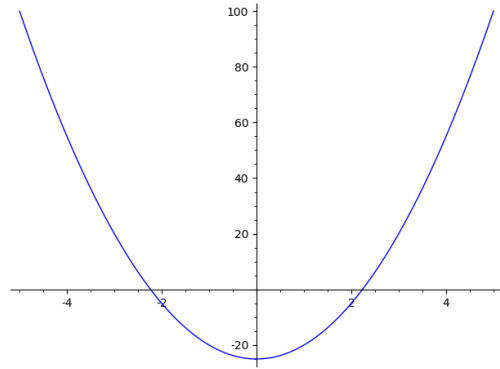
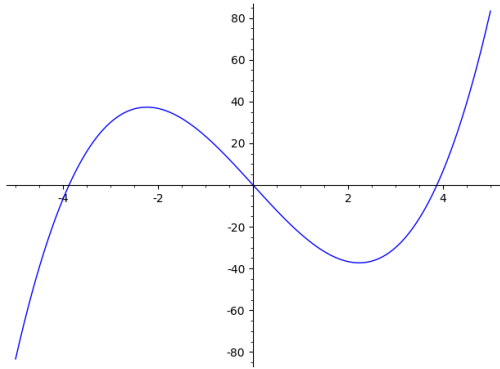
$$\text{provjera: } M(\ell) = -37,5 \cdot 5 - \frac{5}{6} \cdot 5^3 = -291,667 \text{ kNm} \quad \checkmark$$

$$T(x) = A - Q(x) = -37,5 - \frac{5}{2} \cdot x^2$$

$$\text{provjera: } T(\ell) = -37,5 - \frac{5}{2} \cdot 5^2 = -100 \text{ kNm}$$

$$\frac{dM(x)}{dx} = -37,5 - 3 \cdot \frac{5}{6} \cdot x^2 = -37,5 - \frac{5}{2} \cdot x^2 = T(x)$$

$$\frac{dM(x)}{dx} = 0 \quad \Rightarrow \quad x^2 = \frac{2}{5} (-37,5) \quad \Rightarrow \quad x = \mp \sqrt{-15}$$

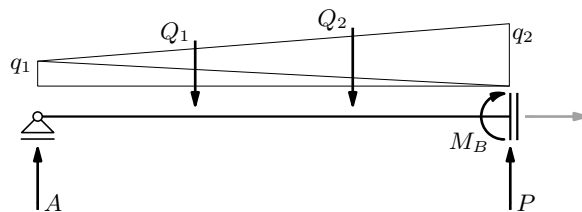
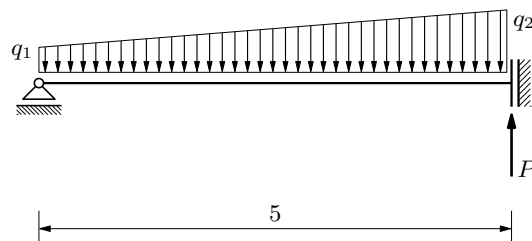


još jedna varijacija:

$$q_1 = 10 \text{ kNm}$$

$$q_2 = 25 \text{ kN/m}$$

$$P = 100 \text{ kN}$$



$$Q_1 = \frac{1}{2} q_1 \ell = \frac{1}{2} \cdot 10 \cdot 5 = 25 \text{ kN}$$

$$Q_2 = \frac{1}{2} q_2 \ell = \frac{1}{2} \cdot 25 \cdot 5 = 62,5 \text{ kN}$$

$$\sum F_z = 0 : \quad -A + Q_1 + Q_2 - P = 0$$

$$A = Q_1 + Q_2 - P = -12,5 \text{ kN}$$

$$\sum M_{/A} = 0 : \quad -\frac{1}{3} \ell Q_1 - \frac{2}{3} \ell Q_2 - M_B + \ell P = 0$$

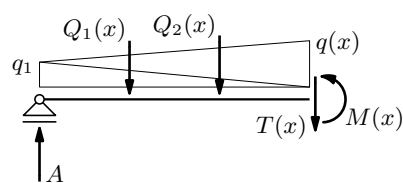
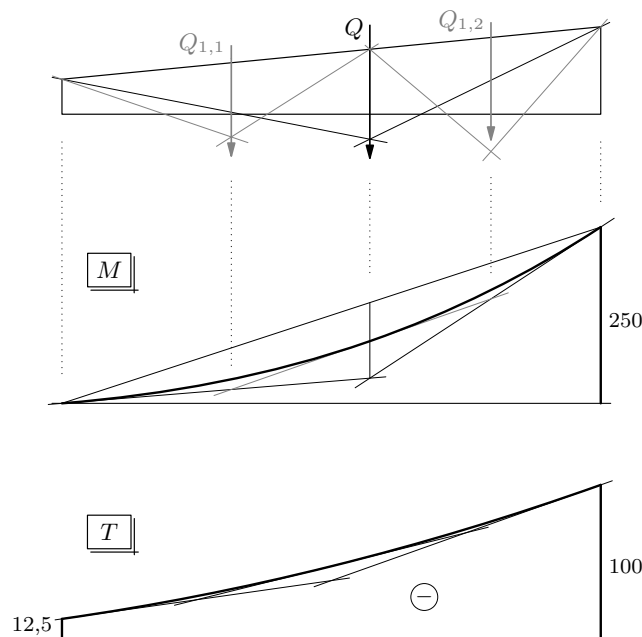
$$M_B = -\frac{1}{3} \cdot 5 \cdot 25 - \frac{2}{3} \cdot 5 \cdot 62,5 + 5 \cdot 100 = 250 \text{ kNm}$$

$$Q = Q_1 + Q_2 = 25 + 62,5 = 87,5 \text{ kN}$$

$$x_Q Q = \frac{1}{3} \ell Q_1 + \frac{2}{3} \ell Q_2 \quad \Rightarrow \quad x_Q = \frac{4}{7} \ell = \frac{20}{7} = 2,85714 \text{ m}$$

$$-\ell \bar{A} + (\ell - x_Q) Q = 0 \quad \Rightarrow \quad \bar{A} = \left(1 - \frac{4}{7}\right) Q = \frac{3}{7} Q = 37,5 \text{ kN}$$

$$\bar{M}_Q = x_Q \bar{A} = \frac{12}{49} \ell Q = 107,143 \text{ kNm}$$



$$q(x) = q_1 + \frac{q_2 - q_1}{\ell} x = 10 + \frac{15}{5} x = 10 + 3 \cdot x$$

$$Q_1(x) = \frac{1}{2} \cdot 10 \cdot x = 5 \cdot x$$

$$Q_2(x) = \frac{1}{2} \cdot (10 + 3 \cdot x) \cdot x = 5 \cdot x + \frac{3}{2} \cdot x^2$$

$$-x A + \frac{2}{3} x Q_1(x) + \frac{1}{3} x Q_2(x) + M(x) = 0$$

$$M(x) = x \cdot (-12,5) - \frac{2}{3} \cdot x \cdot (5 \cdot x) - \frac{1}{3} \cdot x \cdot \left(5 \cdot x + \frac{3}{2} \cdot x^2\right) = -12,5 \cdot x - 5 \cdot x^2 - \frac{1}{2} \cdot x^3$$

$$\text{provjera: } M(\ell) = -12,5 \cdot 5 - 5 \cdot 5^2 - \frac{1}{2} \cdot 5^3 = -250 \text{ kNm} \quad \checkmark$$

$$-A + Q_1(x) + Q_2(x) + T(x) = 0$$

$$T(x) = -12,5 - 5 \cdot x - 5 \cdot x - \frac{3}{2} \cdot x^2 = -12,5 - 10 \cdot x - \frac{3}{2} \cdot x^2$$

$$\text{provjera: } T(\ell) = -12,5 - 10 \cdot 5 - \frac{3}{2} \cdot 5^2 = -100 \text{ kN}$$

$$\frac{dM(x)}{dx} = -12,5 - 2 \cdot 5 \cdot x - 3 \cdot \frac{1}{2} \cdot x^2 = -12,5 - 10 \cdot x - \frac{3}{2} \cdot x^2 = T(x)$$

$$\frac{dM(x)}{dx} = 0 : \quad -\frac{3}{2} \cdot x^2 - 10 \cdot x - 12,5 = 0 \quad \rightarrow \quad \frac{3}{2} \cdot x^2 + 10 \cdot x + 12,5 = 0$$

$$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \mp \sqrt{10^2 - 4 \cdot \frac{3}{2} \cdot 12,5}}{2 \cdot \frac{3}{2}} \Rightarrow x_1 = -5 \quad \& \quad x_2 = -\frac{5}{3}$$

