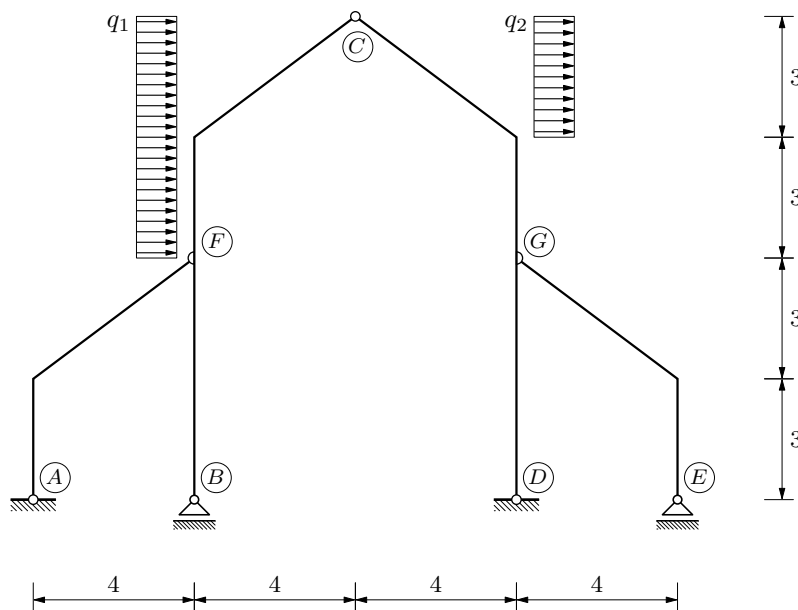


GS 1. — 20. veljače 2024.

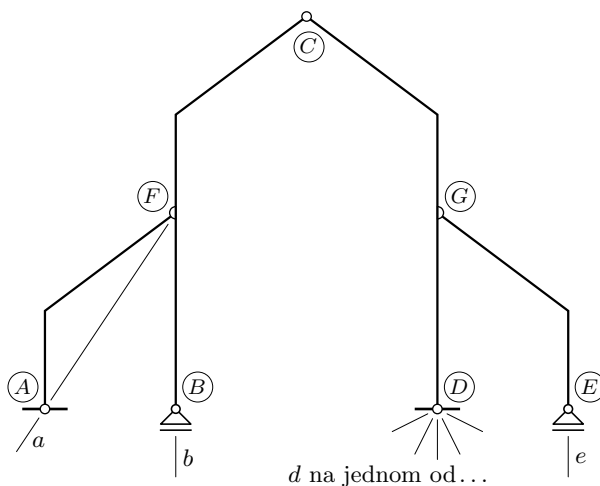
Zadatak 1.

- Za opterećenje q_1 nacrtajte dijagrame unutarnjih sila!
- Za opterećenje q_1 i q_2 grafičkim postupkom odredite vrijednosti sila u presjecima neposredno iznad točke G i neposredno ispod točke F !

$$q_1 = q_2 = 50 \text{ kN/m'}$$



„prepoznavanje” sistema:



dio $A-F$ neopterećen \Rightarrow reakcija u A na spojnici točaka A i F (na pravcu a) (♠)

reakcija u B na pravcu b (♣)

(♠) \mathcal{E} (♣) \Rightarrow F je zamišljeni nepomični zglobni ležaj trozglobnoga sistema $F-C-D$

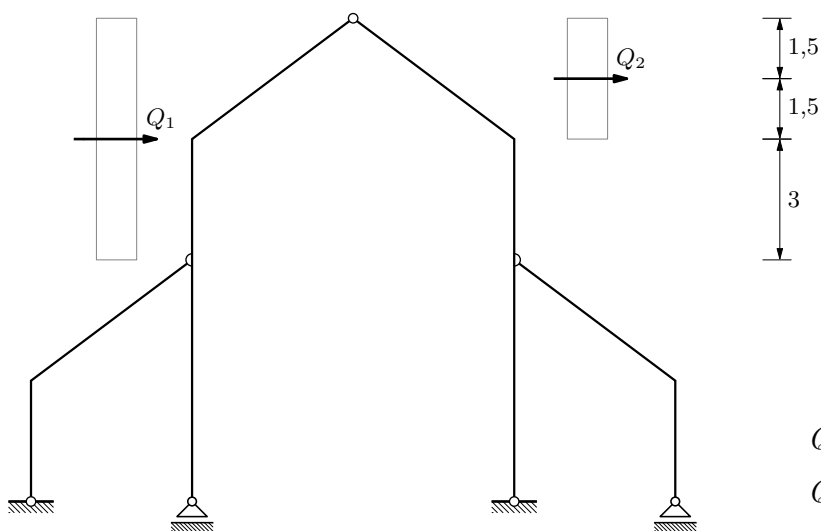
dio $G-E$ se „oslanja” na $F-C-D$; neopterećen \Rightarrow možemo ga zaboraviti

grafički postupak (b.):

I videl sem daljine, meglene i kalne,
i videl sem glibine, se dalne i dalne.
I videl sem strele, žveplene, blisiču.
Občutel sem merliča perst na ogerliču:
na jabuki Adama hladni tuji palec,
bil sem v čarnom plašču, hman kak zvezdoznalec.

Miroslav Krežža: *Planetarijom*

rezultante distribuiranih sila:

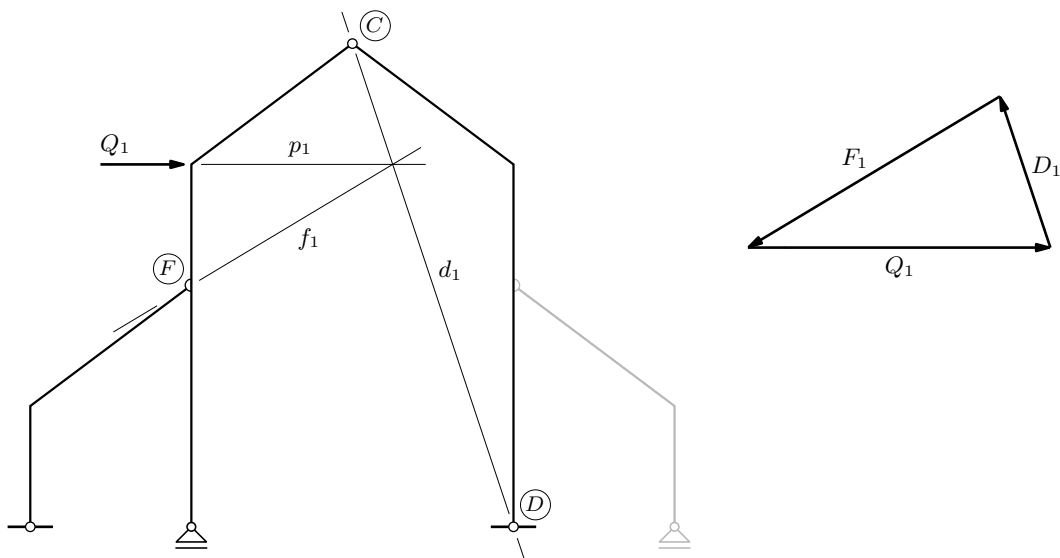


$$Q_1 = 50 \cdot 6 = 300 \text{ kN}$$

$$Q_2 = 50 \cdot 3 = 150 \text{ kN}$$

opterećen lijevi dio:

desni dio neopterećen \Rightarrow pravac djelovanja reakcije u D prolazi kroz C



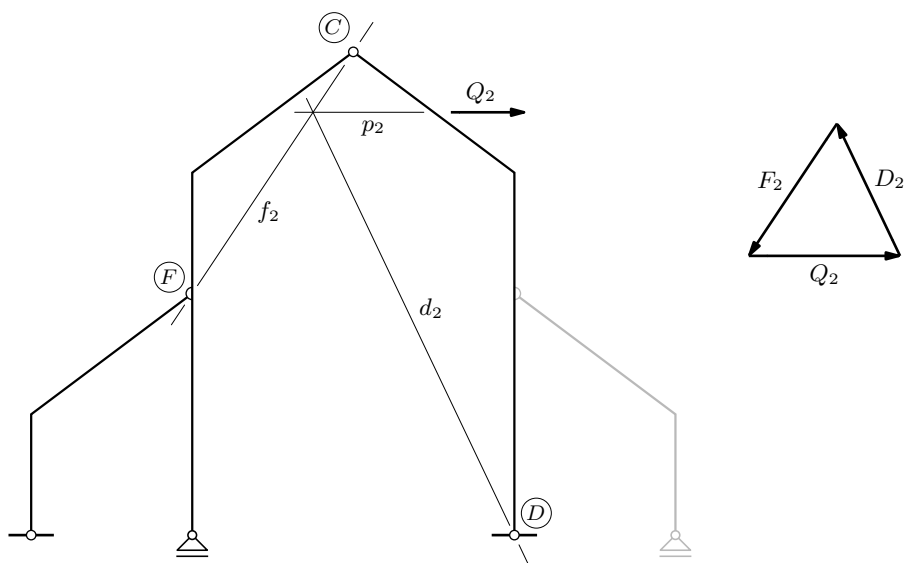
mjerilo sila: 1 cm $::$ 75 kN

$$\text{očitano: } \tilde{F}_1 = 39 \text{ mm} \Rightarrow F_1 = 292,5 \text{ kN}$$

$$\tilde{D}_1 = 21 \text{ mm} \Rightarrow D_1 = 157,5 \text{ kN}$$

opterećen desni dio:

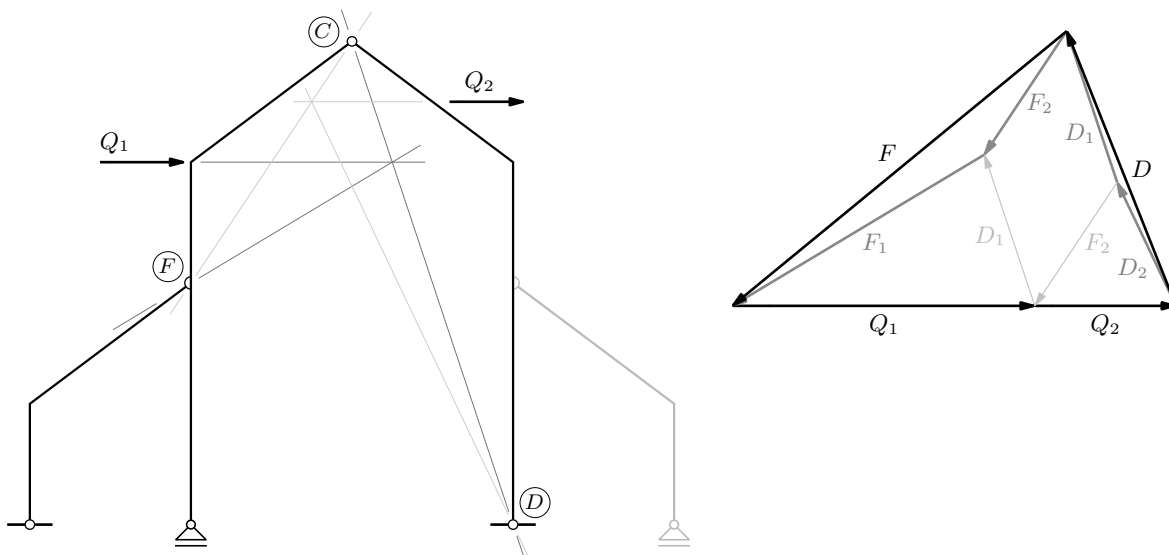
lijevi dio neopterećen \Rightarrow pravac djelovanja „reakcije” u F prolazi kroz C



očitano: $\tilde{F}_2 = 21 \text{ mm} \Rightarrow F_2 = 157,5 \text{ kN}$

$\tilde{D}_2 = 19 \text{ mm i } \sim \frac{1}{2} \text{ mm} \Rightarrow D_2 = 146,3 \text{ kN}$

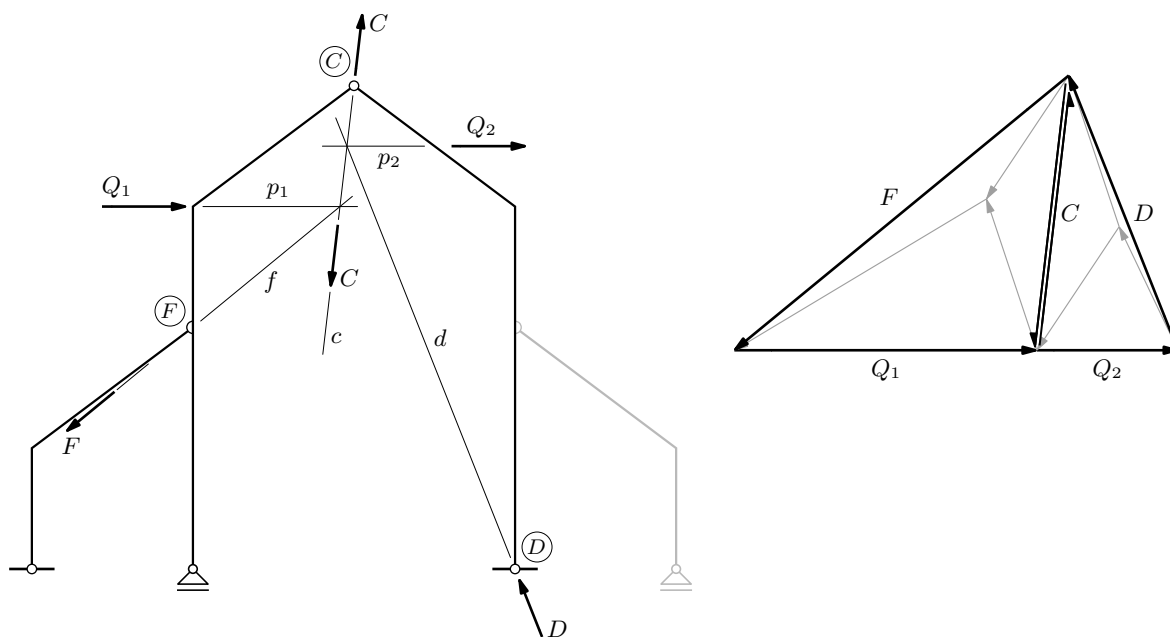
zbrojevi sila:



očitano: $\tilde{F} = 57 \text{ mm i } \sim \frac{1}{4} \text{ mm} \Rightarrow F = 429,4 \text{ kN}$

$\tilde{D} = 39 \text{ mm} \Rightarrow D = 292,5 \text{ kN}$

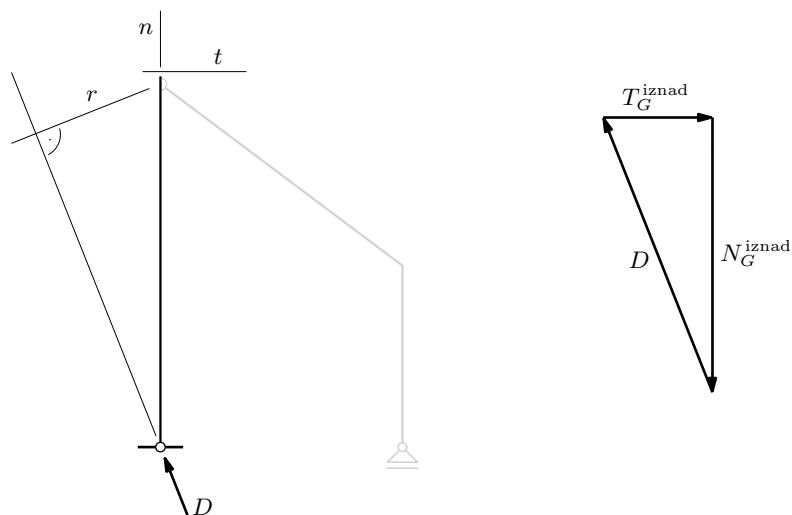
pravci djelovanja sila i kontrola:



kontrola:

rezultanta sila \vec{Q}_1 i \vec{F} mora djelovati na pravcu c koji prolazi zglobov C
 rezultanta sila \vec{Q}_2 i \vec{D} mora djelovati na istom pravcu c

sile u presjeku neposredno iznad točke G :



mjerilo duljina: 1 cm :: 1,25 m

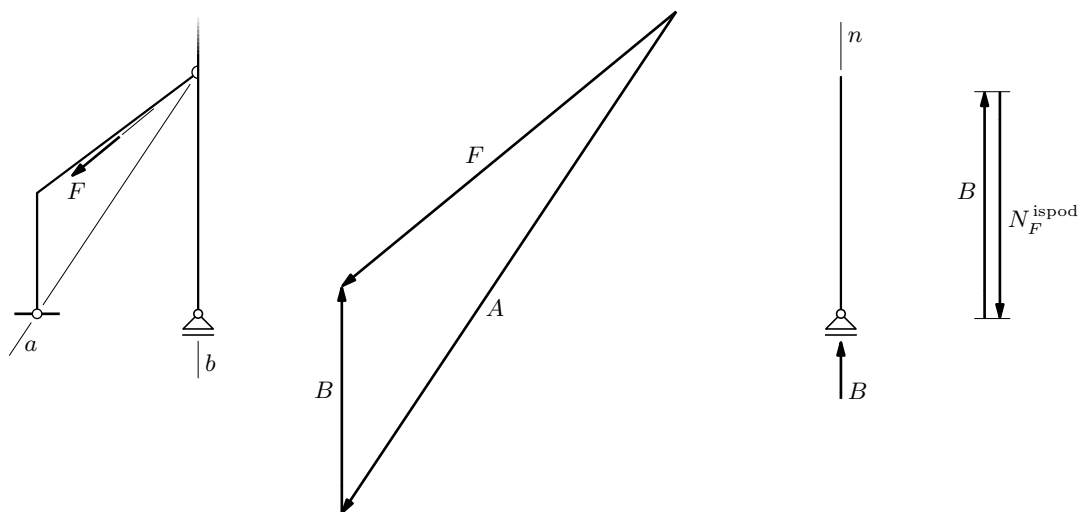
očitano: $\tilde{r} = 17 \text{ mm}$ i $\sim \frac{3}{4} \text{ mm} \Rightarrow r = 2,22 \text{ m}$

$M_G = D \cdot r = 649,4 \text{ kNm}$ ↷

očitano: $\tilde{T}_G^{\text{iznad}} = 14 \text{ mm}$ i $\sim \frac{1}{2} \text{ mm} \Rightarrow T_G^{\text{iznad}} = 108,8 \text{ kN}$

očitano: $\tilde{N}_G^{\text{iznad}} = 36 \text{ mm}$ i $\sim \frac{1}{3} \text{ mm} \Rightarrow N_G^{\text{iznad}} = 272,5 \text{ kN}$

sile u presjeku neposredno ispod točke F :



rastavljanje sile \vec{F} u sile na pravcima a i b (u reakcije \vec{A} i \vec{B} u ležajevima A i B):

$$\text{očitano: } \tilde{A} = 79 \text{ mm i } \sim \frac{3}{4} \text{ mm} \Rightarrow F = 598,1 \text{ kN}$$

$$\tilde{D} = 30 \text{ mm} \Rightarrow D = 225 \text{ kN}$$

sile u presjeku:

$$N_F^{\text{ispod}} = B = 225 \text{ kN}$$

$$T_F^{\text{ispod}} = 0$$

$$M_F^{\text{ispod}} = 0$$

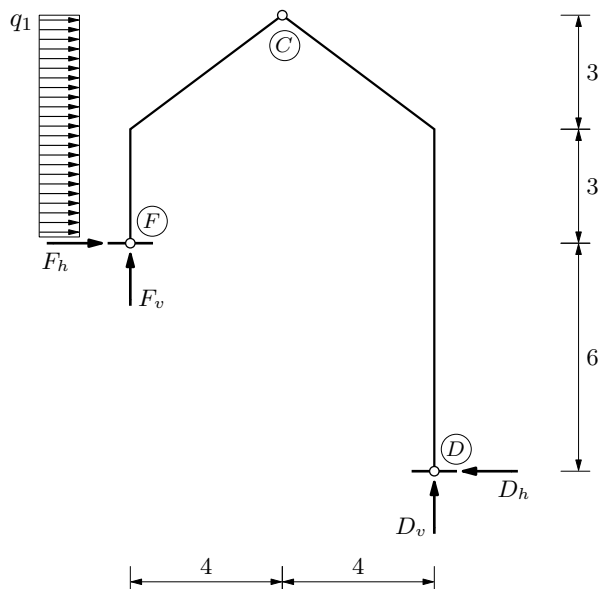
analitički postupak i dijagrami (a.):

Čuli su se glasi kak germlavine basi:

»Bežete, bežete, vi pogane žuči,
tuče se čarne spremaju potuči
se vas v pivnice, v liže i po kuči,
kaj blodite bez doma po vetru i po tuči.«

Miroslav Krleža: *Planetarijom*

vrijednosti „reakcije” \vec{F} i reakcije \vec{D} trozglobnoga sistema:



$$\left. \begin{aligned} \sum_{\widehat{FD}} M_{/D} = 0 : -9 \cdot (q_1 \cdot 6) - 6 \cdot F_h - 8 \cdot F_v &= 0 \\ \sum_{\widehat{FC}} M_{/C} = 0 : 3 \cdot (q_1 \cdot 6) + 6 \cdot F_h - 4 \cdot F_v &= 0 \end{aligned} \right\}$$

$$\left. \begin{aligned} 6 \cdot F_h + 8 \cdot F_v &= -2700 \\ 6 \cdot F_h - 4 \cdot F_v &= -900 \end{aligned} \right\}$$

$$F_h = -250 \text{ kN} \quad \& \quad F_v = -150 \text{ kN}$$

$$(F_x = -250 \text{ kN} \quad \& \quad F_z = 150 \text{ kN} \rightarrow \vec{F} = F_x \vec{i} + F_z \vec{k} = -250 \vec{i} + 150 \vec{k})$$

$$\left. \begin{aligned} \sum_{\widehat{FD}} M_{/F} = 0 : -3 \cdot (q_1 \cdot 6) - 6 \cdot F_h + 8 \cdot D_v &= 0 \\ \sum_{\widehat{CD}} M_{/C} = 0 : -12 \cdot F_h + 4 \cdot D_v &= 0 \end{aligned} \right\}$$

$$\left. \begin{aligned} 6 \cdot D_h - 8 \cdot D_v &= -900 \\ 12 \cdot D_h - 4 \cdot D_v &= 0 \end{aligned} \right\}$$

$$D_h = 50 \text{ kN} \quad \& \quad D_v = 150 \text{ kN}$$

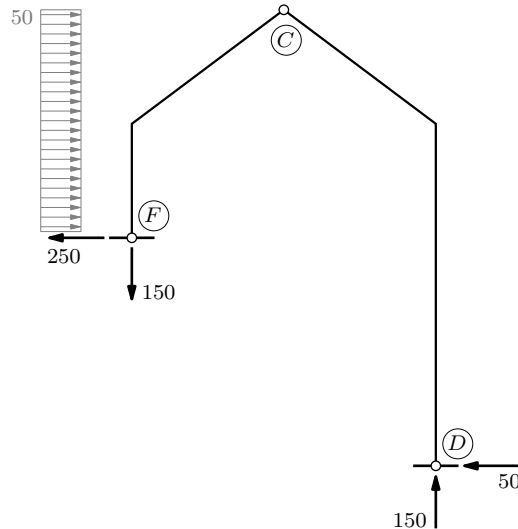
$$(D_x = -50 \text{ kN} \quad \& \quad D_z = -150 \text{ kN} \rightarrow \vec{D} = D_x \vec{i} + D_z \vec{k} = -50 \vec{i} - 150 \vec{k})$$

kontrola:

$$\sum_{\widehat{FD}} P_h : \quad q_1 \cdot 6 + F_h - D_h = 300 + (-250) - 50 = 0$$

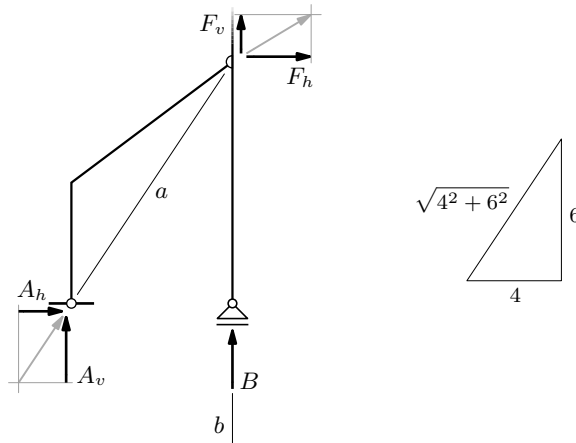
$$\sum_{\widehat{FD}} P_v : \quad -F_v - D_v = -(-150) - 150 = 0$$

skica „stvarnih” djelovanja (potpunosti i zora radi):



(na crtežima koji slijede zadržat ćemo orijentacije sila prikazane na slici na prethodnoj stranici, a u izraze uvrštavati vrijednosti s predznacima)

rastavljanje „reakcije” \vec{F} komponente na pravcima a i b (u reakcije \vec{A} i \vec{B}):



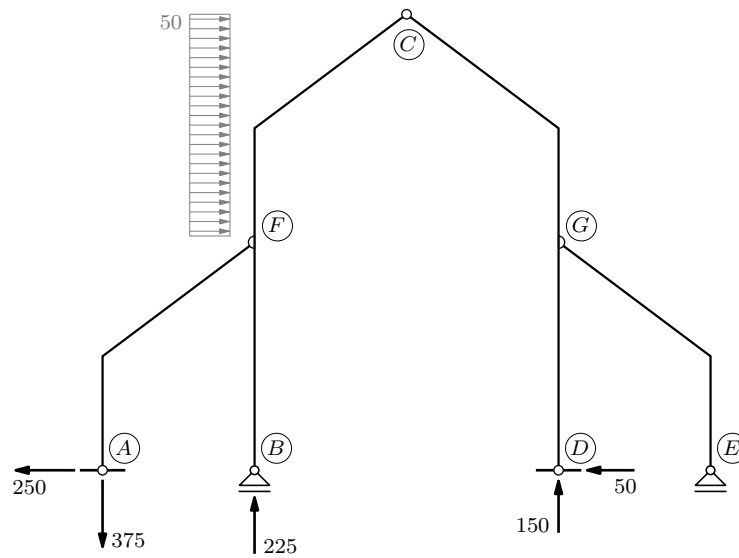
$$\frac{A}{A_h} = \frac{\sqrt{4^2 + 6^2}}{4} \quad \Rightarrow \quad A = \frac{\sqrt{4^2 + 6^2}}{4} A_h = 1,80278 \cdot A_h$$

$$\frac{A_v}{A_h} = \frac{6}{4} = \frac{3}{2} \quad \Rightarrow \quad A_v = \frac{3}{2} A_h$$

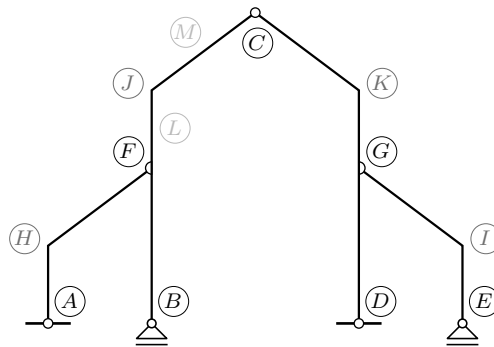
$$A_h = F_h = -250 \text{ kN} \quad \Rightarrow \quad A = -450,695 \text{ kN} \quad \& \quad A_v = -375 \text{ kN}$$

$$F_v = A_v + B \quad \Rightarrow \quad B = F_v - A_v = -150 - (-375) = 225 \text{ kN}$$

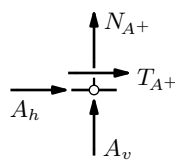
skica „stvarnih” djelovanja:



vrijednosti unutarnjih sila u karakterističnim točkama:



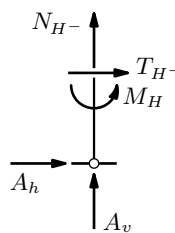
kao gornji indeks + označava „neposredno iznad”
 ++ označava „neposredno desno od”
 - označava „neposredno ispod”
 -- označava „neposredno lijevo od”



$$A_h + T_{A+} = 0 \Rightarrow T_{A+} = -A_h = -(-250) = 250 \text{ kN}$$

$$-A_v - N_{A+} = 0 \Rightarrow N_{A+} = -A_v = -(-375) = 375 \text{ kN}$$

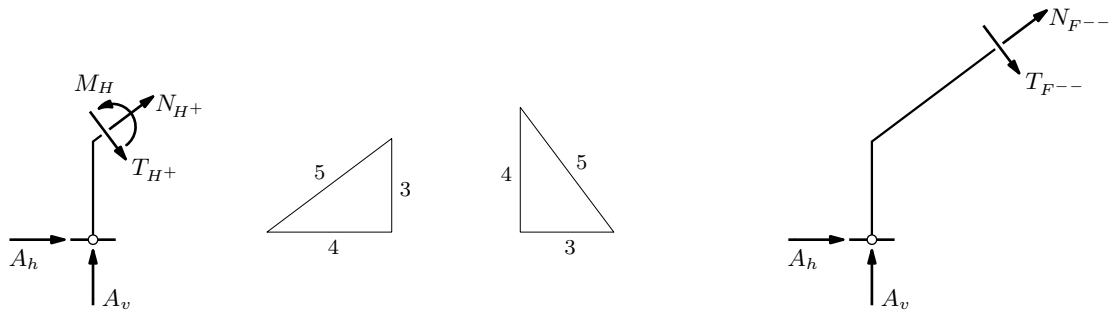
$$M_{A+} = 0$$



$$T_{H-} = T_{A+} = 250 \text{ kN}$$

$$N_{H-} = N_{A+} = 375 \text{ kN}$$

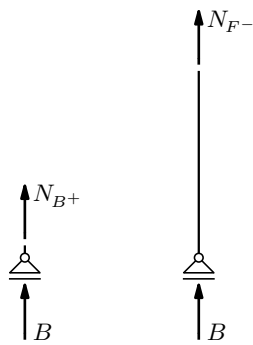
$$3 \cdot A_h + M_H = 0 \Rightarrow M_H = -3 \cdot A_h = -3 \cdot (-250) = 750 \text{ kNm}$$



$$\left. \begin{aligned} A_h + \frac{4}{5} N_{H+} + \frac{3}{5} T_{H+} &= 0 \\ -A_v - \frac{3}{5} N_{H+} + \frac{4}{5} T_{H+} &= 0 \end{aligned} \right\} \begin{aligned} N_{H+} &= 425 \text{ kN} & \mathcal{E} & T_{H+} = -150 \text{ kN} \end{aligned}$$

$$M_H = 750 \text{ kNm}$$

$$N_{F--} = N_{H+} = 425 \text{ kN} \quad \mathcal{E} \quad T_{F--} = T_{H+} = -150 \text{ kN} \quad \mathcal{E} \quad M_{F--} = 0$$

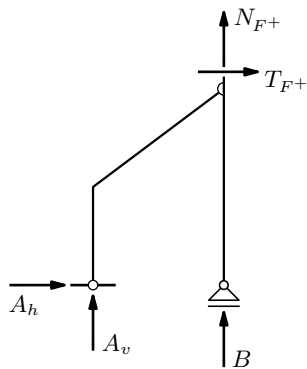


$$-B - N_{B+} = 0 \Rightarrow N_{B+} = -B = -225 \text{ kN}$$

$$T_{B+} = 0 \quad \mathcal{E} \quad M_B = 0$$

$$-B - N_{F-} = 0 \Rightarrow N_{F-} = -B = -225 \text{ kN}$$

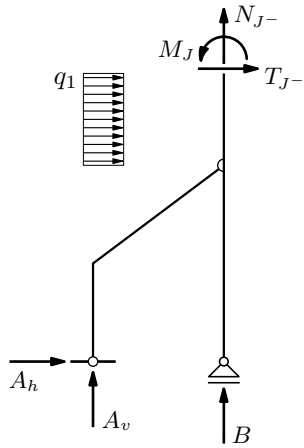
$$T_{F-} = 0 \quad \mathcal{E} \quad M_F = 0$$



$$\begin{aligned} A_h + T_{F+} &= 0 \Rightarrow T_{F+} = -A_h = -(-250) \\ &= 250 \text{ kN} \end{aligned}$$

$$\begin{aligned} -A_v - B - N_{F+} &= 0 \Rightarrow N_{F+} = -A_v - B \\ &= -(-375) - 225 \\ &= 150 \text{ kN} \end{aligned}$$

$$M_F = 0$$



$$A_h + 3 \cdot q_1 + T_{J-} = 0 \quad \Rightarrow \quad T_{J-} = -A_h - 3 \cdot q_1$$

$$= 250 - 150$$

$$= 100 \text{ kN}$$

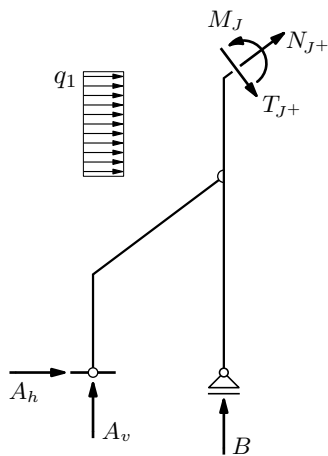
$$N_{J-} = N_{F+} = 150 \text{ kN}$$

$$9 \cdot A_h - 4 \cdot A_v + 1,5 \cdot (q_1 \cdot 3) + M_J = 0$$

$$\Rightarrow M_J = -9 \cdot A_h + 4 \cdot A_v - 1,5 \cdot (q_1 \cdot 3)$$

$$= -9 \cdot (-250) + 4 \cdot (-375) - 1,5 \cdot 150$$

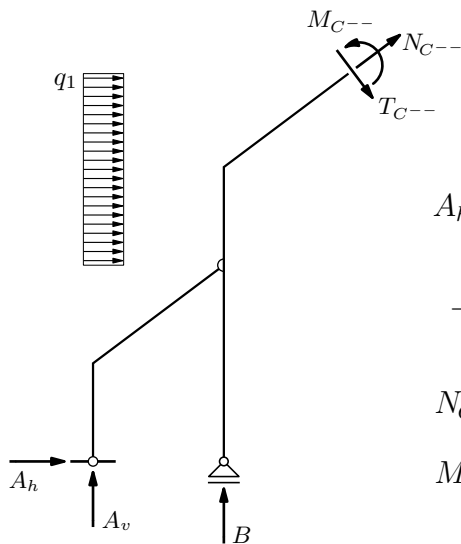
$$= 525 \text{ kNm}$$



$$\left. \begin{aligned} A_h + q_1 \cdot 3 + \frac{4}{5} N_{J+} + \frac{3}{5} T_{J+} &= 0 \\ -A_v - B - \frac{3}{5} N_{J+} + \frac{4}{5} T_{J+} &= 0 \end{aligned} \right\}$$

$$N_{J+} = 170 \text{ kN} \quad \& \quad T_{J+} = -60 \text{ kN}$$

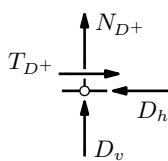
$$M_J = 525 \text{ kNm}$$



$$\left. \begin{aligned} A_h + q_1 \cdot 6 + \frac{4}{5} N_{C--} + \frac{3}{5} T_{C--} &= 0 \\ -A_v - B - \frac{3}{5} N_{C--} + \frac{4}{5} T_{C--} &= 0 \end{aligned} \right\}$$

$$N_{C--} = 50 \text{ kN} \quad \& \quad T_{C--} = -150 \text{ kN}$$

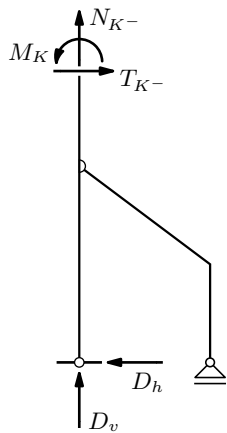
$$M_{C--} = 0$$



$$T_{D+} - D_h = 0 \quad \Rightarrow \quad T_{D+} = D_h = 50 \text{ kN}$$

$$-D_v - N_{D+} = 0 \quad \Rightarrow \quad N_{D+} = -D_v = -150 \text{ kN}$$

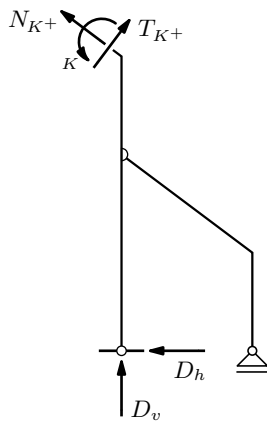
$$M_{D+} = 0$$



$$T_{K-} = T_{D+} = 50 \text{ kN}$$

$$N_{K-} = N_{D+} = -150 \text{ kN}$$

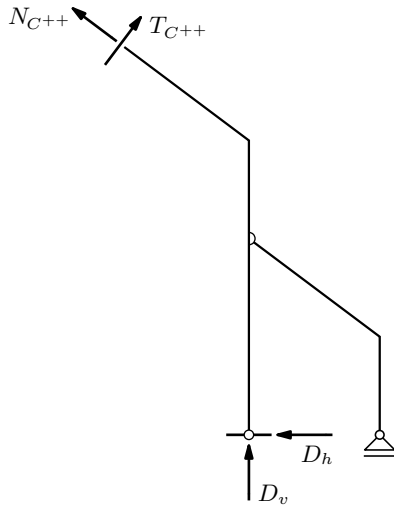
$$-9 \cdot D_h + M_K = 0 \Rightarrow M_K = 9 \cdot D_h = 450 \text{ kNm}$$



$$\left. \begin{aligned} -D_h - \frac{4}{5} N_{K+} + \frac{3}{5} T_{K+} &= 0 \\ -D_v - \frac{3}{5} N_{K+} - \frac{4}{5} T_{K+} &= 0 \end{aligned} \right\}$$

$$N_{K+} = -130 \text{ kN} \quad \& \quad T_{K+} = -90 \text{ kN}$$

$$M_K = 450 \text{ kNm}$$

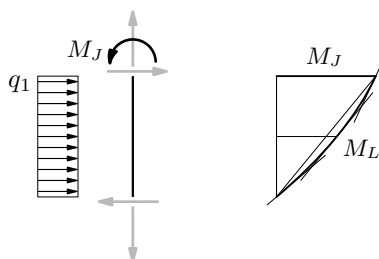


$$T_{C++} = T_{K+} = -90 \text{ kN}$$

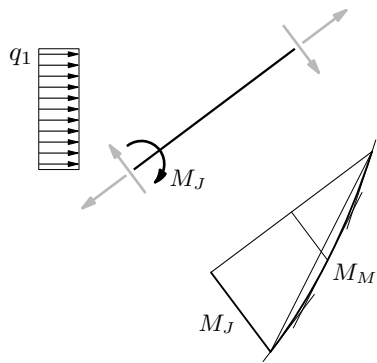
$$N_{C++} = N_{K+} = -130 \text{ kN}$$

$$M_{C++} = 0$$

„pomoćne” vrijednosti za crtanje momentnoga dijagrama:

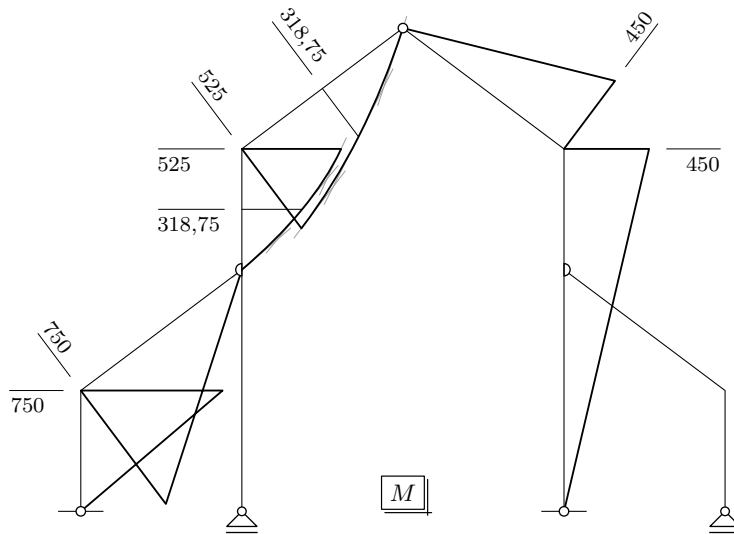


$$M_M = \frac{M_J}{2} + \frac{q_1 \cdot 3^2}{8} = 318,75 \text{ kNm}$$

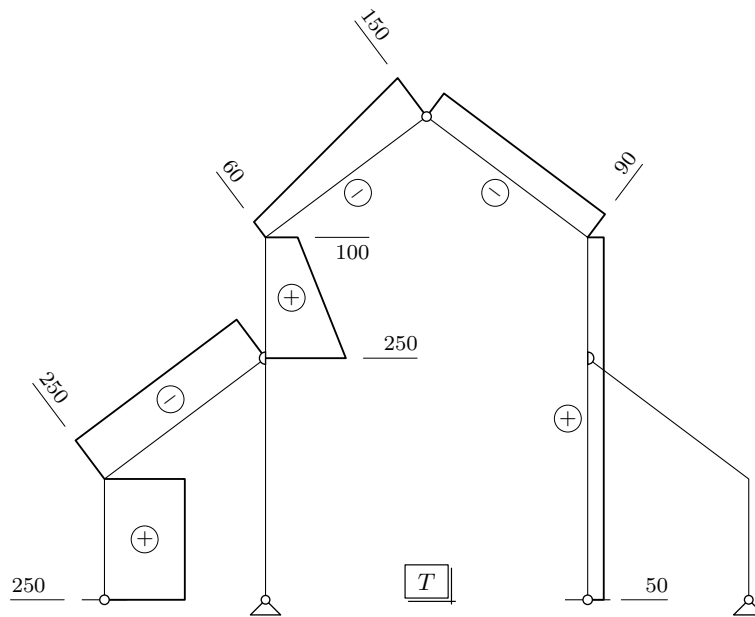


$$M_M = \frac{M_J}{2} + \frac{q_1 \cdot 3^2}{8} = 318,75 \text{ kNm}$$

dijagram momenata savijanja:



dijagram poprečnih sila:



dijagram uzdužnih sila:

