

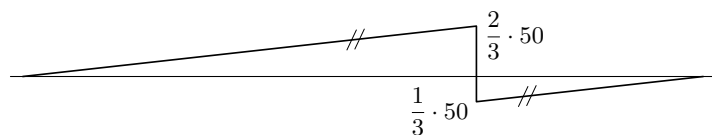
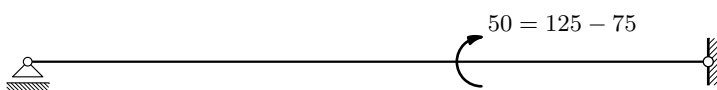
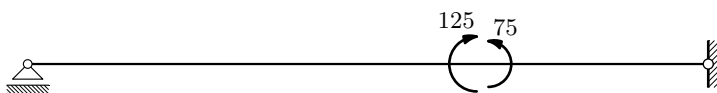
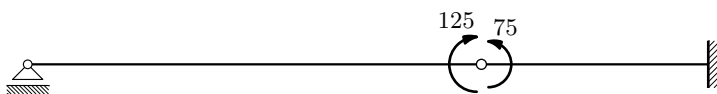
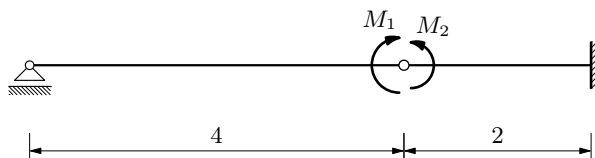
GS 1. — 1. kolokvij (B) (2023./2024.)

Zadatak 1.

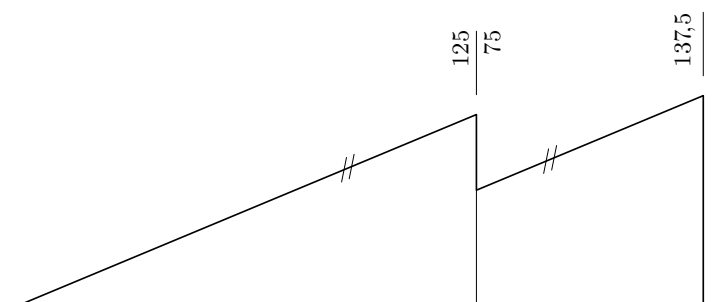
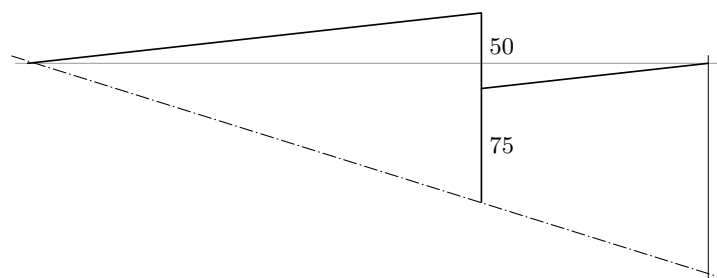
Superpozicijskim postupkom nacrtajte dijagram momenata savijanja!

$$M_1 = 125 \text{ kNm}$$

$$M_2 = 75 \text{ kNm}$$



M_0

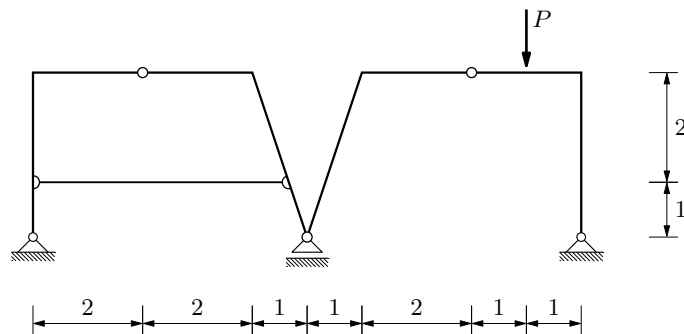


M

Zadatak 2.

Nacrtajte dijagrame unutarnjih sila!

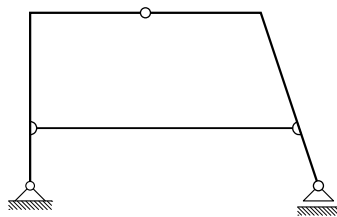
$$P = 100 \text{ kN}$$



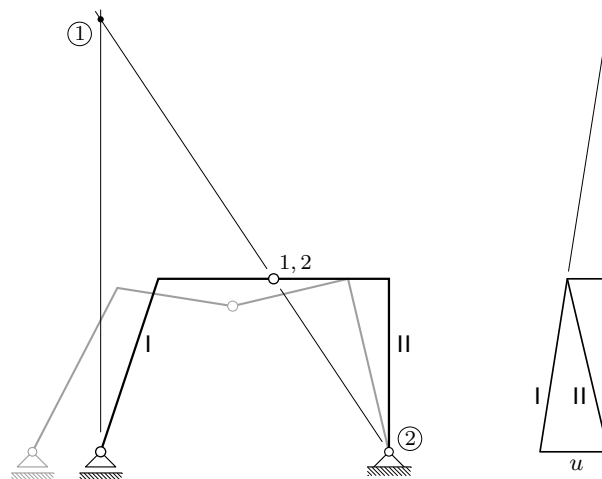
(Za potpuniju i detaljniju priču pročitajte primjer rješavanja analognoga¹ sistema — trozglobnoga nosača koji se „oslanja” na trozglobni nosač sa zategom — u datoteci <http://master.grad.hr/nastava/gs/gs1/isp/1-220906-2.pdf>, dostupnoj na stranici s bilješkama s predavanja (<http://master.grad.hr/nastava/gs/gs1/>)².)

prepoznavanje statičkoga sistema: složeni ulančeni sistem

„temeljni” podsistem: lijevi podsistem — trozglobni nosač sa zategom:



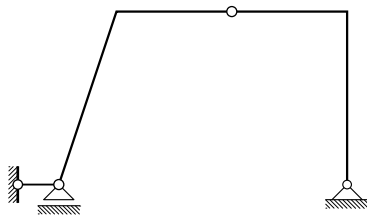
desni podsistem bez lijevoga podsistema — mehanizam:



¹ „Analogan — koji je nečemu nalik, koji je po nekoj osobini nečemu sličan, koji nečemu odgovara; istovrstan” [*Hrvatski jezični portal*, <https://hjp.znanje.hr>].

² Poveznicu na tu stranicu možete naći i na stranici predmeta na *Merlinu*.

desni podsistem slijeva „temeljen” na lijevom podsistemu — trozglobni nosač:



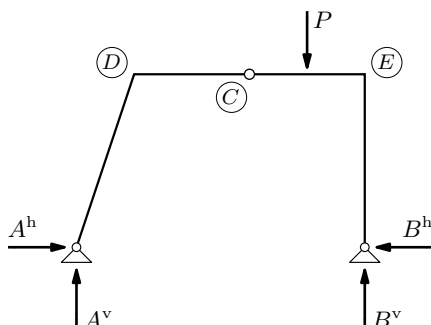
redosljed rješavanja:

reakcije trozglobnoga nosača → opterećenje trozglobnoga nosača sa zategom

⇒ (1) trozglobni nosač pa (2) trozglobni nosač sa zategom

(1) trozglobni nosač:

vrijednosti reakcija:



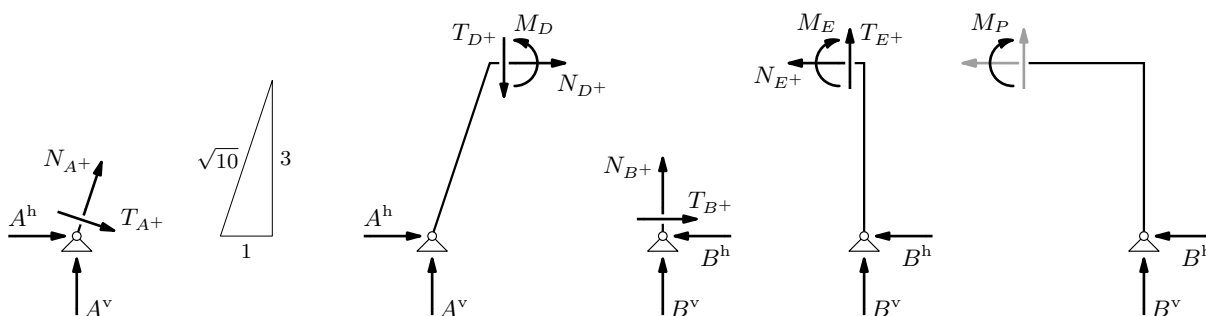
$$\sum_{\widehat{AB}} M_{/B} = 0 : \quad -5 \cdot A^v + 1 \cdot P = 0 \quad \Rightarrow \quad A^v = \frac{P}{5} = 20 \text{ kN}$$

$$\sum_{\widehat{AB}} M_{/A} = 0 : \quad -4 \cdot P + 5 \cdot B^v = 0 \quad \Rightarrow \quad B^v = \frac{4 \cdot P}{5} = 80 \text{ kN}$$

$$\sum_{\widehat{AC}} M_{/C} = 0 : \quad -3 \cdot A^v + 3 \cdot A^h = 0 \quad \Rightarrow \quad A^h = A^v = 20 \text{ kN}$$

$$\sum_{\widehat{AB}} F_x = 0 : \quad A^h - B^h = 0 \quad \Rightarrow \quad B^h = A^h = 20 \text{ kN}$$

vrijednosti unutarnjih sila:

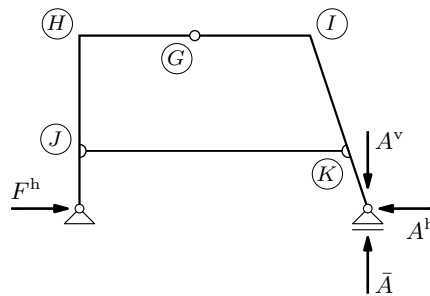


$$\sum_{\widehat{AD}} M_{/D} = 0 : \quad 3 \cdot A^h - 1 \cdot A^v + M_D = 0 \quad \Rightarrow \quad M_D = -40 \text{ kNm}$$

$$\begin{aligned}
\sum_{\widehat{BE}} M_{/E} = 0 : \quad & -3 \cdot B^h - M_E = 0 \quad \Rightarrow \quad M_E = -60 \text{ kNm} \\
\sum_{\widehat{BP}} M_{/P} = 0 : \quad & -3 \cdot B^h + 1 \cdot B^h - M_P = 0 \quad \Rightarrow \quad M_P = 20 \text{ kNm} \\
\left. \begin{aligned}
\sum_{A^+} F_x = 0 : \quad & \frac{1}{\sqrt{10}} N_{A^+} + \frac{3}{\sqrt{10}} T_{A^+} + A^h = 0 \\
\sum_{A^+} F_z = 0 : \quad & -\frac{3}{\sqrt{10}} N_{A^+} + \frac{1}{\sqrt{10}} T_{A^+} - A^v = 0
\end{aligned} \right\} \begin{aligned}
N_{A^+} &= -8\sqrt{10} \text{ kN} \\
T_{A^+} &= -4\sqrt{10} \text{ kN}
\end{aligned} \\
\sum_{B^+} F_x = 0 : \quad & -B^h + T_{B^+} = 0 \quad \Rightarrow \quad T_{B^+} = B^h = 20 \text{ kN} \\
\sum_{B^+} F_z = 0 : \quad & -B^v - N_{B^+} = 0 \quad \Rightarrow \quad N_{B^+} = -B^v = -80 \text{ kN} \\
\sum_{D^+} F_x = 0 : \quad & A^h + N_{D^+} = 0 \quad \Rightarrow \quad N_{D^+} = -A^h = -20 \text{ kN} \\
\sum_{D^+} F_z = 0 : \quad & -A^v + T_{D^+} = 0 \quad \Rightarrow \quad T_{D^+} = A^v = 20 \text{ kN} \\
\sum_{E^+} F_x = 0 : \quad & -B^h - N_{E^+} = 0 \quad \Rightarrow \quad N_{E^+} = -B^h = -20 \text{ kN} \\
\sum_{E^+} F_z = 0 : \quad & -B^v - T_{E^+} = 0 \quad \Rightarrow \quad T_{E^+} = -B^v = -80 \text{ kN}
\end{aligned}$$

(2) trozglobni nosač sa zategom:

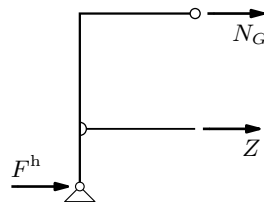
vrijednosti reakcija:



$$F^h = A^h = 20 \text{ kN}$$

$$\bar{A} = A^v = 20 \text{ kN} \quad \& \quad F^v = 0$$

vrijednost sile u zategi:



$$\sum_{\widehat{FG}} M_{/G} = 0 : \quad 3 \cdot F^h + 2 \cdot Z = 0 \quad \Rightarrow \quad Z = -\frac{3}{2} \cdot F^h = -30 \text{ kN}$$

$$\sum_{\widehat{FG}} F_x = 0 : \quad F^h + Z + N_G = 0 \quad \Rightarrow \quad N_G = -F^h - Z = 10 \text{ kN}$$

vrijednosti ostalih unutarnjih sila:

$$M_J = -1 \cdot F^h = -20 \text{ kN}$$

$$M_K = 1 \cdot A^h = 20 \text{ kN}$$

$$T_{F^+} = -F^h = -20 \text{ kN}$$

$$T_{H^-} = N_G = 10 \text{ kN}$$

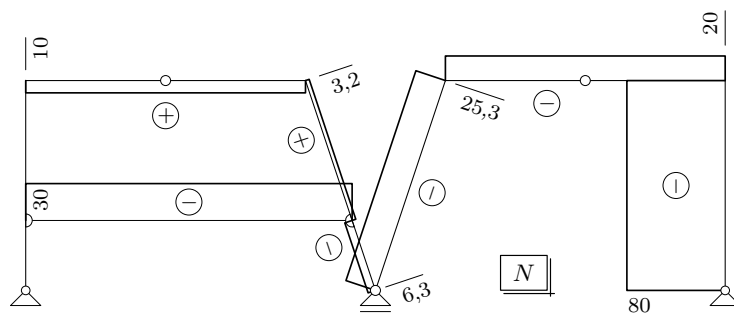
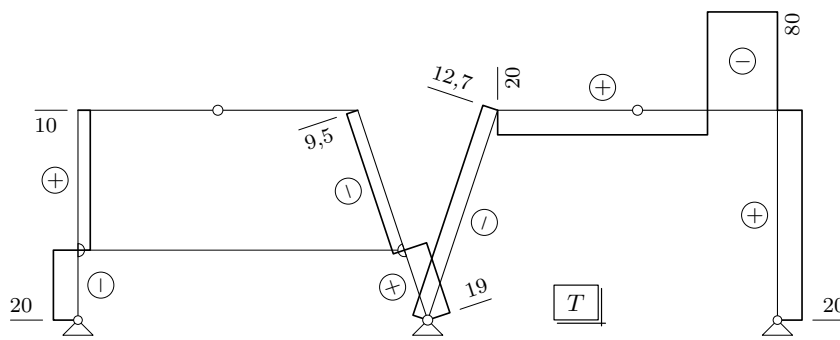
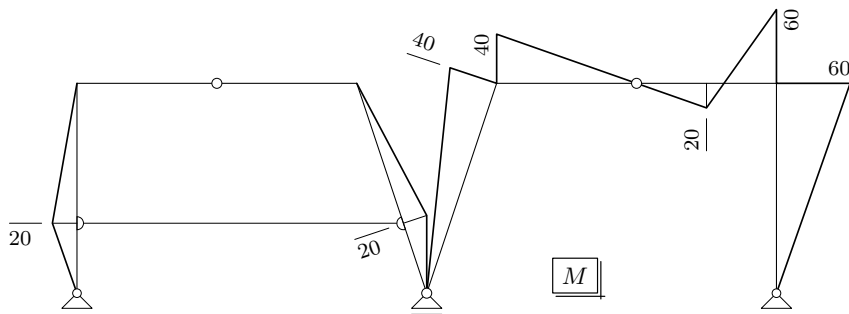
$$T_{\bar{A}^+} = \frac{3}{\sqrt{10}} A^h = 6\sqrt{10} \text{ kN}$$

$$N_{\bar{A}^+} = -\frac{1}{\sqrt{10}} A^h = -2\sqrt{10} \text{ kN}$$

$$T_{I^-} = -\frac{3}{\sqrt{10}} N_G = -3\sqrt{10} \text{ kN}$$

$$N_{I^-} = \frac{1}{\sqrt{10}} N_G = \sqrt{10} \text{ kN}$$

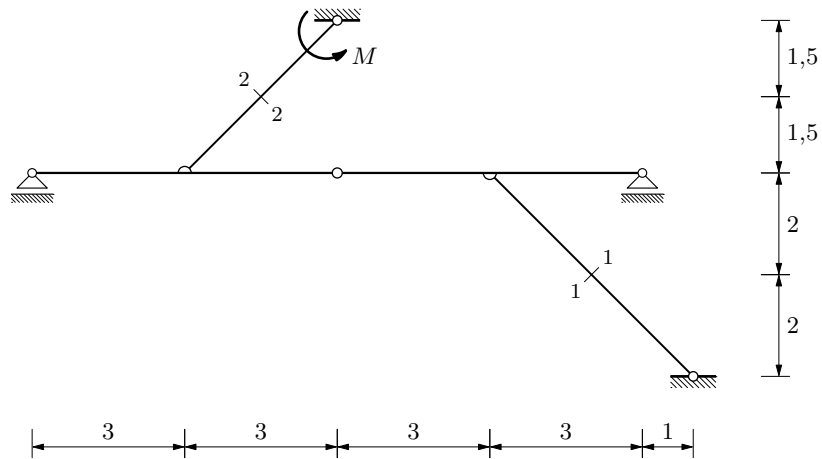
dijagrami:



Zadatak 3.

Grafičkim postupkom odredite vrijednosti unutarnjih sila u presjecima 1-1 i 2-2!

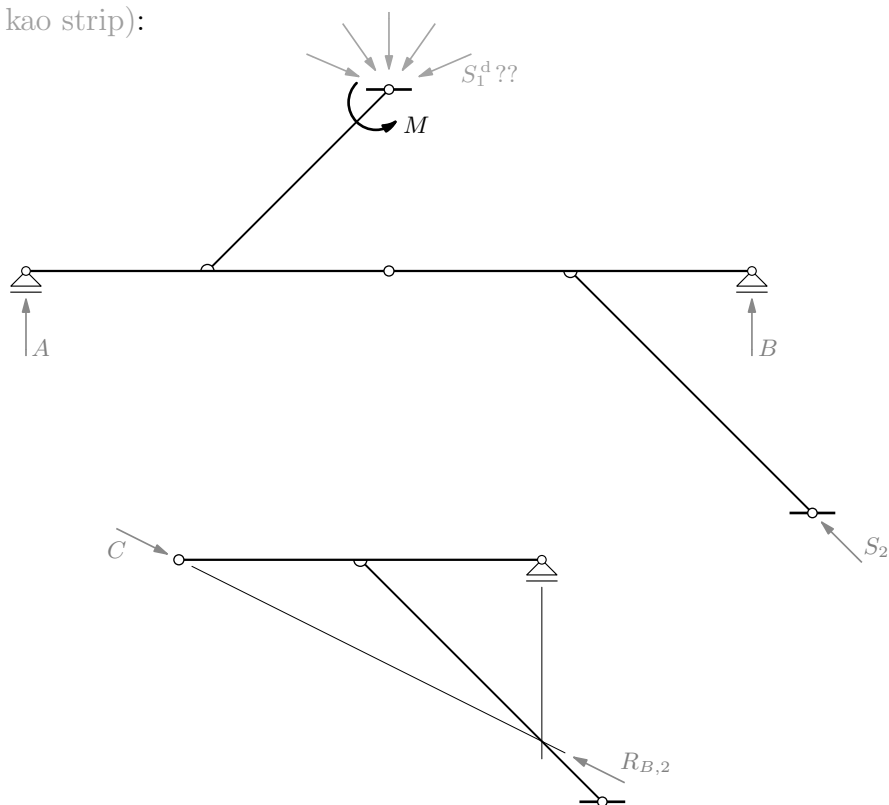
$M = 100 \text{ kNm}$ (neposredno ispod ležajnoga zgloba)

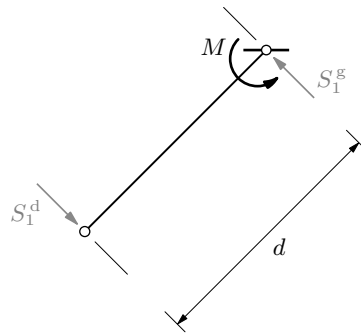
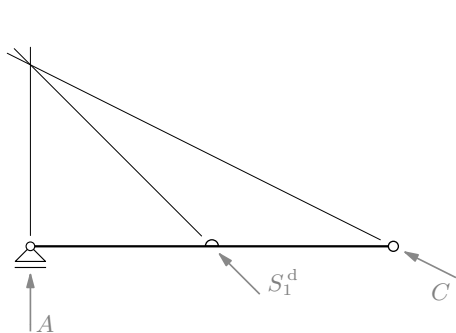


(Analogni je zadatak, sa silom kao opterećenjem, riješen i objašnjen na predavanju o grafičkim postupcima na nosačima koji su srodni trozglobnim nosačima. Crteži s predavanja dostupni su u odjeljku *Poduprta greda (2)* na stranici 3 datoteke <http://master.grad.hr/nastava/gsgs1/sl/lng1.pdf>.)

Način prvi:

reakcije (nešto kao strip):





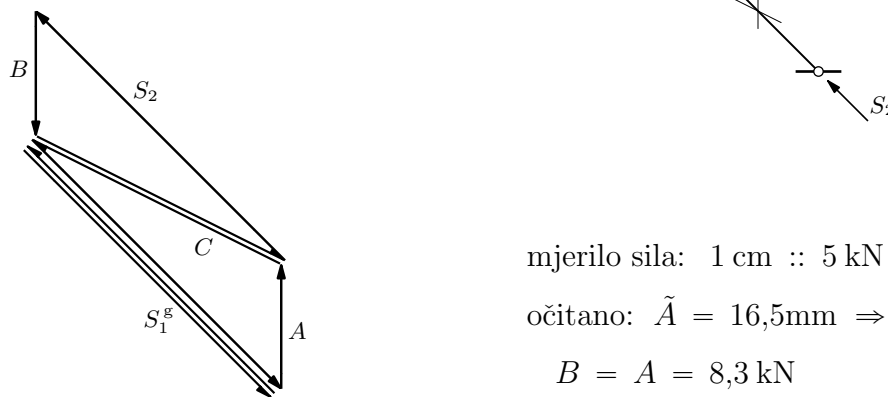
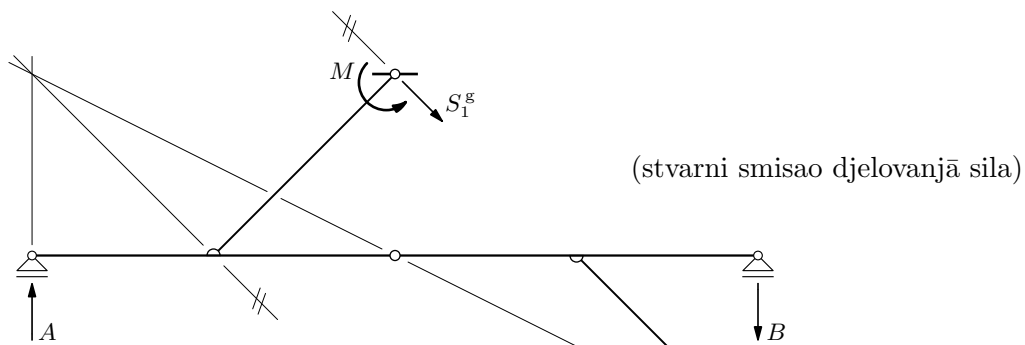
mjerilo duljina: 1 cm :: 1,25 m

očitano: $\tilde{d} = 34,5 \text{ mm} \Rightarrow d = 4,3 \text{ m}$

(ili, točnije, $d = \sqrt{3^2 + 3^2} = 4,24264 \text{ m}$)

$$S_1^d = S_1^g = \frac{M}{d} = \frac{100}{4,3} = 23,3 \text{ kN}$$

(smisao djelovanja suprotan od nacrtanoga)



mjerilo sila: 1 cm :: 5 kN

očitano: $\tilde{A} = 16,5 \text{ mm} \Rightarrow A = 8,3 \text{ kN}$

$$B = A = 8,3 \text{ kN}$$

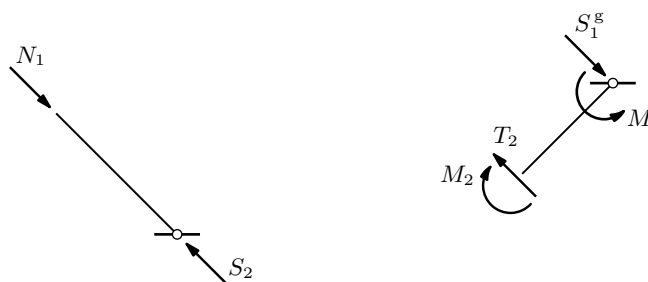
$$S_2 = S_1^g = 23,3 \text{ kN}$$

(djelomična provjera:

\vec{S}_2 i \vec{S}_1^g djeluju na istome pravcu \Rightarrow moment je uravnotežen (samo) spregom sila \vec{A} i \vec{B} :

$$A = B = \frac{100}{12} = 8,3 \approx 8,3 \text{ kN})$$

vrijednosti sila u presjecima:



$$N_1 = S_2 = 23,3 \text{ kN}$$

$$T_2 = S_1^g = 23,3 \text{ kN}$$

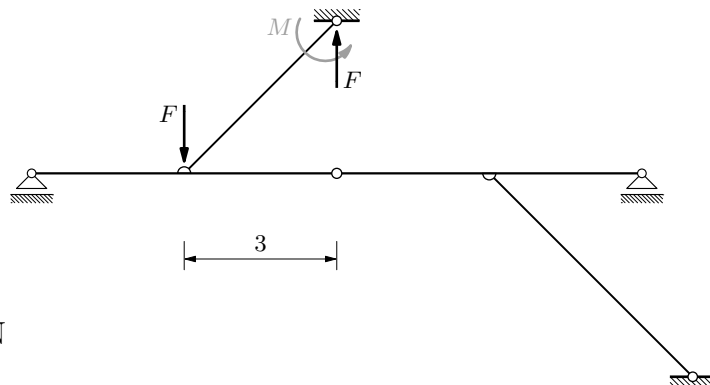
$$M_2 = M - \frac{d}{2} S_1^g = 100 - \frac{4,3}{2} \cdot 23,3 = 49,9 \simeq 50 \text{ kNm}$$

(točna je vrijednost 50 — vrijednost se linearno mijenja od 100 neposredno ispod ležaj-noga zgloba do 0 neposredno iznad zglobnoga spoja s gredom)

smisao djelovanja sila i smisao vrtnje momenta kao na slici

Način drugi:

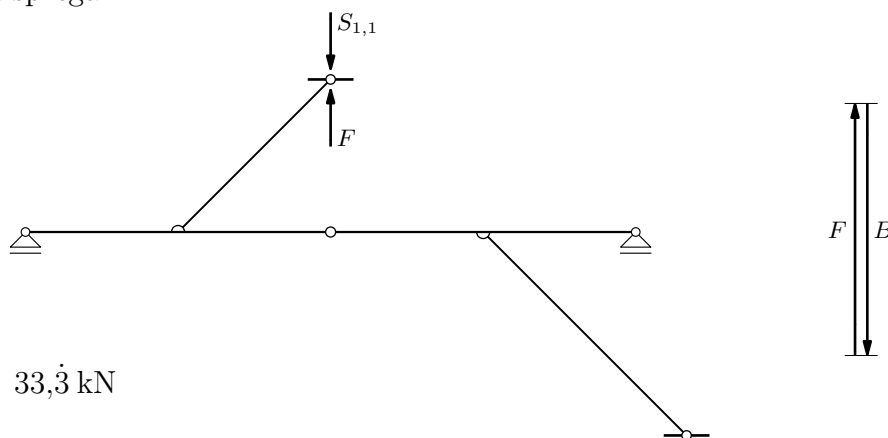
„pretvaranje” opterećenoga štapa u zglobni štap:



$$F = \frac{M}{3} = 33,3 \text{ kN}$$

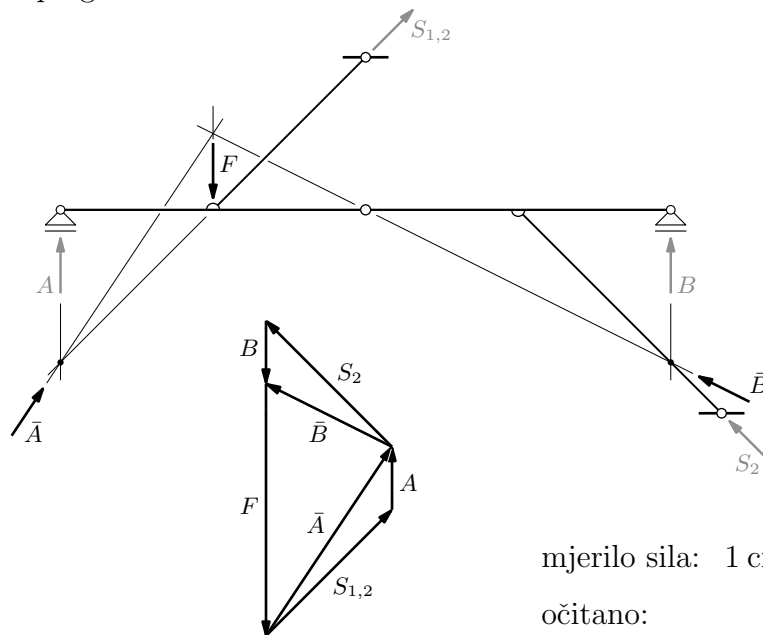
reakcije:

za desnu silu sprega:



$$S_{1,1} = F = 33,3 \text{ kN}$$

za lijevu silu sprega:



mjerilo sila: 1 cm :: 10 kN

očitano:

$$\bar{A} = 30 \text{ kN}$$

$$S_{1,2} = 23,5 \text{ kN}$$

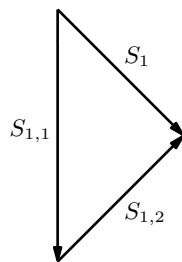
$$A = 8,25 \text{ kN}$$

$$\bar{B} = 18,5 \text{ kN}$$

$$S_2 = 23,5 \text{ kN}$$

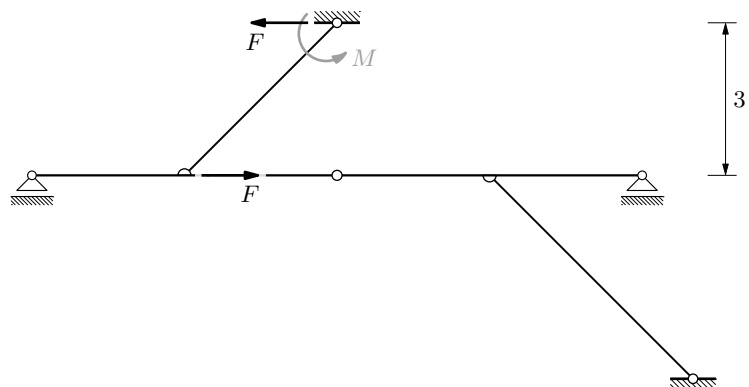
$$B = 8,25 \text{ kN}$$

$$S_1 = 23,5 \text{ kN}$$



vrijednosti sila u presjecima: kao u načinu prvom

kao zamjena momenta može se odabrati *bilo koji* spreg sila pravci djelovanja kojih prolaze zglobovima štapa (koji nije zglobni štap):

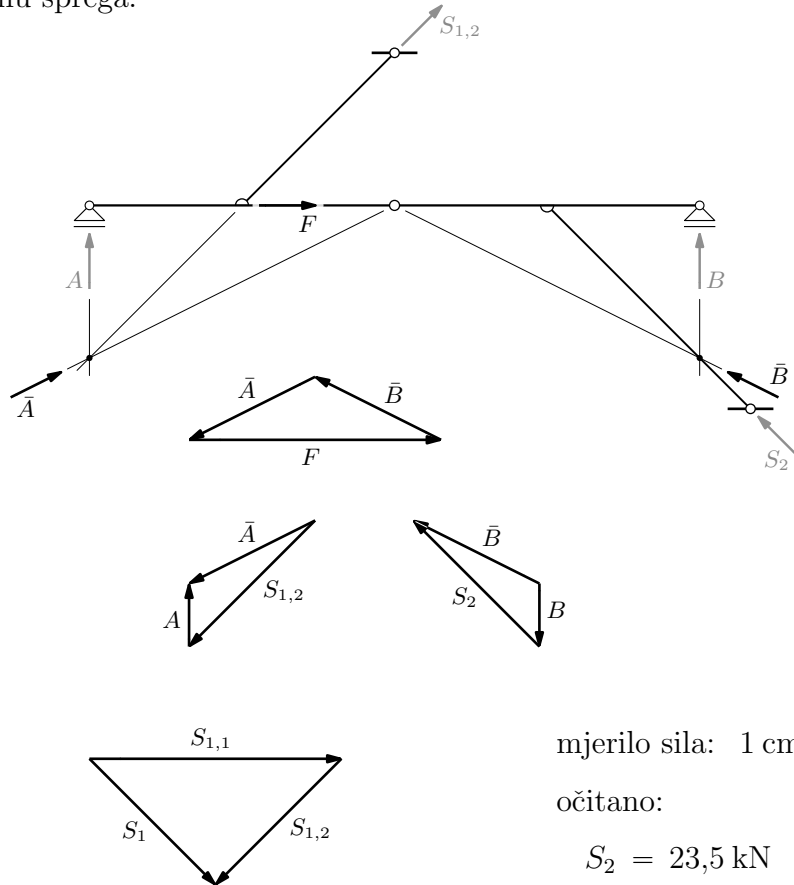


$$F = \frac{M}{3} = 33,3 \text{ kN}$$

reakcije:

za gornju silu sprega: $S_{1,1} = F = 33,3 \text{ kN}$ (isti intenzitet i pravac djelovanja, suprotan smisao)

za donju silu sprega:



mjerilo sila: 1 cm :: 10 kN

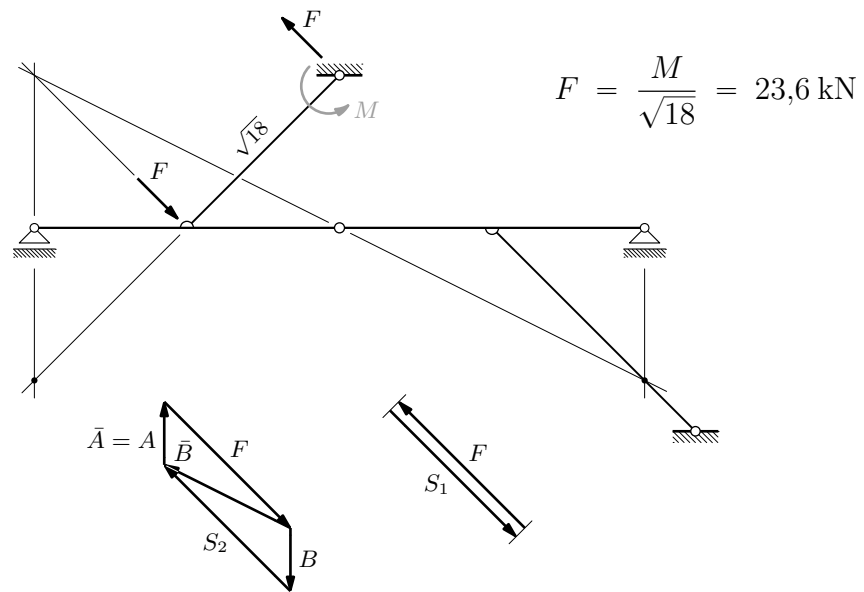
očitano:

$$S_2 = 23,5 \text{ kN}$$

$$S_1 = 23,5 \text{ kN}$$

vrijednosti sila u presjecima: kao u načinu prvom

ili zamjena momenta spregom sila okomitih na os štapa (koji nije zglobni štap):



očitano:

$$S_2 = 23,6 \text{ kN}$$

$$S_1 = 23,6 \text{ kN}$$

a vrijednosti sila u presjecima, naravno, ponovno kao u načinu prvom