Professional paper
Accepted 29.04.1997

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3D Polyhedra Scenes and the Triangulation

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ABSTRACT.
One of the possible utilizations of the planar region triangulation is presented in this paper - a part of the visibility solution in the visualization of the general 3D scene consisting of disjoint polyhedra by the means of computer.
Keywords:
triangulation, potential visibility, polyhedra scene visibility, triangle intersection

3D poliedarne scene i triangulacija
SAŽETAK
Jedna od mogućih primjena triangulacije područja u ravnini prikazana je u ovom radu - dio rješenja vidljivosti pri vizualizaciji opće 3D scene koji se sastoji u rastavljanju poliedra uz pomoć računala.
Ključne riječi:
triangulacija, potencijalna vidljivost, vidljivost scene poliedra, presjek trokuta

INTRODUCTION

The term *triangulation* is used here for the decomposition of the connected planar region bounded by closed polygons (not intersecting each other) into elemental regions - triangles. The triangulation gives an opportunity to solve some of the problems in computer geometry of polyhedra in simply and easy understandable way, e.g. visualisation of polyhedra, determination and drawings of their intersections, calculation of their boundary surface, etc, and more, in order to simplify and make this field more understandable and attractive for students it can be used in teaching process at universities as well.

ASSUMPTIONS

The visualization of the scene can be obtained in a user specified view and polyhedra arrangement. The scene is defined by all the faces of disjoint polyhedra. They are filed in attaching the same exterior orientation.

The face orientation determines potential visibility. *Potentially visible* faces (abr. PV faces) are concidered to be the faces of the same orientation as their images in the projection plane [8].

SCENE ANALYSIS

Three different groups of PV faces arrangements can be distinguished, according to their mutual location to the centre of projection:

1. group. PV faces do not overlay; i.e. no PV face is hidden behind another one of the same or other polyhedron.
Solution: All PV faces are displayed.

2. group. PV faces are hidden behind other ones. Their images intersect in a "planar" intersection (bounded planar region containing at least three noncollinear points). PV faces can be ordered in the sequence $\{F_i\}_{i=1}^{n}$ according to their mutual position towards the centre of projection (Fig.1).

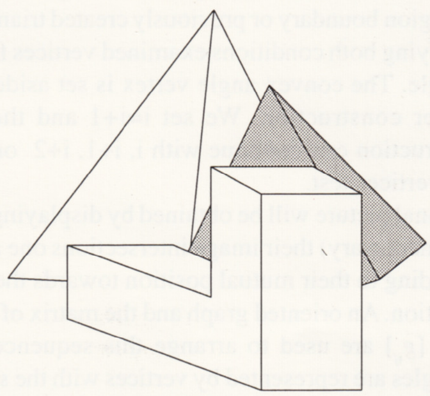

Fig. 1

The mutual position of two plain regions can be determined by any of those region points that lay on one projecting ray (projector) and so produce their image intersection (Fig. 2).

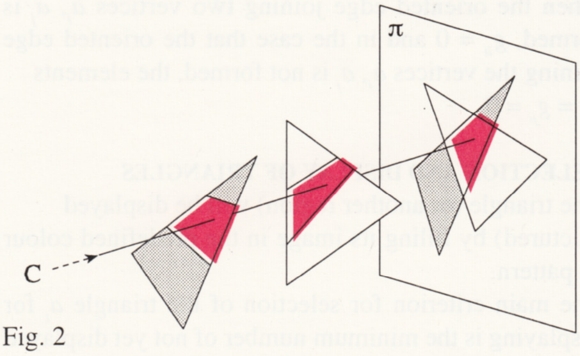

Fig. 2

Solution: PV faces are displayed gradually in the order corresponding to the sequence $\{F_i\}_{i=1}^{n}$.

3. group. Images of PV faces intersect in a "planar" region but faces cannot be ordered in the sequence $\{F_i\}_{i=1}^n$ (Fig. 3).

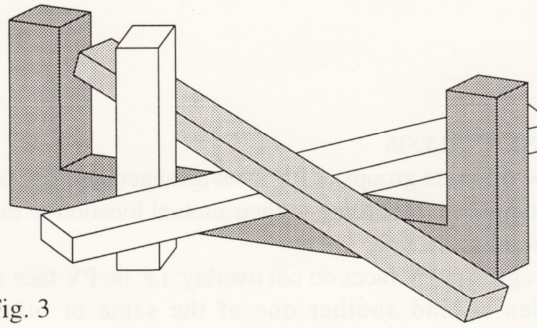

Fig. 3

Solution:: PV faces are triangulated - decomposed into elemental figures - triangles a_i, i=1,..n. For this purpose several algorithms can be used [e.g.3]. Our method is based on the triangle generation test of three suceeding polygon vertices: i, i+1, i+2 [8]. Two conditions are examined:

a) the vertices create the convex angle

b) none of the currently created edges intersect the region boundary or previously created triangle edges.

Satisfying both conditions examined vertices formed the triangle. The convex angle vertex is set aside from the further construction. We set i=i+1 and the triangle construction can continue with i, i+1, i+2 or i+2, i+3, i+4 vertices test.

The final picture will be obtained by displaying triangles or (if necessary) their image intersections one after other according to their mutual position towards the centre of pojection. An oriented graph and the matrix of incidence $\mathbf{G}_n = [g_{ij}]$ are used to arrange this sequence [1], [5]. Triangles are represented by vertices with the same notes a_i, i =1,..n. The directed edge joining two vertices a_i, a_j is formed only if the triangles a_i, a_j are located in two different faces and their images intersect in the "planar" region; the triangle a_i is located closier to the centre of projection than the triangle a_j and so the former overlays the latter (Fig. 2). Corresponding elements in the matrix of incidence $\mathbf{G}_n$ are defined in this way: $g_{ij} = 1$, $g_{ji} = 0$ when the oriented edge joining two vertices a_i, a_j is formed, $g_{ii} = 0$ and in the case that the oriented edge joining the vertices a_i, a_j is not formed, the elements $g_{ij} = g_{ji} = 0$.

SELECTION AND DISPLAY OF TRIANGLES

The triangle (or another region) will be displayed (pictured) by filling its image in the predefined colour or pattern.

The main criterion for selection of the triangle a_i for displaying is the minimum number of not yet displayed triangles laying "under" (or behind) the triangle a_i, i.e. the triangles that are located in the longer distance to the centre of projection than the triangle a_i and have a "planar" intersection with the triangle a_i in images. So these triangles are overlayed by the triangle a_i in the predefined view. This fact is examined in the matrix $\mathbf{G}_n$ via the parameter $S_i = \sum_{j=1}^{n} g_{ij}$ for the current i.

If $S_i = 0$ then non of not yet displayed triangles is located "under" the triangle a_i. The triangle a_i can be displayed and as follows, eliminated from the further process. Corresponding row and column are omitted in the matrix $\mathbf{G}_n$. The next triangle can be examined.

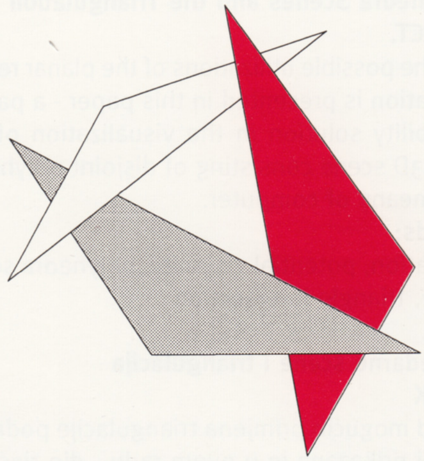

Fig. 4

If $S_i > 0$ for all i = 1,...n no triangle lays "under" the other ones (Fig. 4). Let $S_{min} = min S_i$. Two cases are possible:

1. $S_i = S_{min}$ for unique one i. The triangle a_i is selected.

2. $S_i = S_{min}$ for i of number f, $f > 1$. Let $\{r_k\}_{k=1}^n$ is the sequence of indexes i. Now the triangle a_i will be selected only from the triangles a_{r_k} according to the main criterion (minimum number of not yet displayed triangles a_{r_k} laying "under" the triangle a_i) with respect to the mutual position of triangles a_{r_k} only. If $S_{min} = S_{r_k}$ for all r_k, the triangle will be selected in the comparisn test of the mutual position of triangles in pairs. That triangle in the tested pair will be chosen which is "under" the other one. The selected triangle will be repeatedly compared to the next triangle from the sequence $\{a_{r_k}\}_{k=1}^f$.

Selected triangle a_i can be displayed immediately, and the information about not yet displayed trianges a_j laying "under" the triangle a_i have to be stored, for instance in a row b_i of another matrix $\mathbf{P}_n = [b_{lh}]$ - the matrix of intersections, by setting the elements b_{ij} =1. After each triange a_j displaying the intersection of the triangle a_i, a_j images will be filled in the colour of the triangle a_i (The triangle a_i was displayed before the triangle a_j although the triangle a_i is located over the triangle a_j).If several triangles a_i, $i = r_v$, v = 1,...t, have been already displayed in the time of triangle a_j displaying, (filling in the intersections of triangles a_{r_v}, a_j images will be ordered and realised with respect to the main criterion.

The problem of selecting a triangle is solved in all levels by using the same algorithm that can be recursively called and applied on the current sequence of triangles and so the scene with arbitrary overlaying faces can be visualized.

DETERMINATION OF THE TRIANGLE IMAGES INTERSECTION

Construction of the intersection of the triangle images is based on the region orientation of the boundary polygon vertices [7].

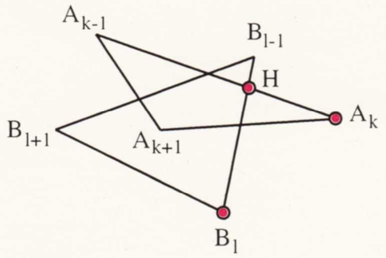

Fig. 5

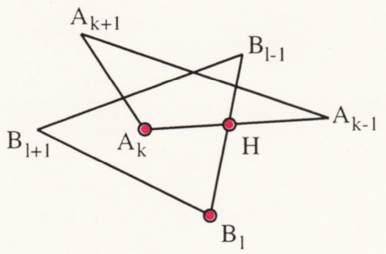

Fig. 6

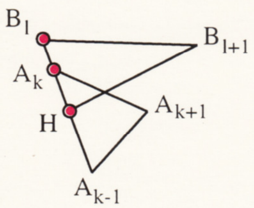

Fig. 7

Let *A*, *B* be the concordantly oriented images of the triangles. The first vertex of the intersection can be an intersection point of two arbitrary sides. The succesor can be found as the intersection point of sides or as the vertex on the boundary of that triangle *A*, for which the three points H, A_k, B_l (H-topical vertex of the intersection, A_k - the successive vertex of the triangle *A* and B_l - the successive vertex of the triangle B (Fig. 5, 6, 7) are in the same orientation as the both triangles *A*, *B*. Having found the succeding vertex equal to the first one the determination of the intersection is finished (Fig. 8).

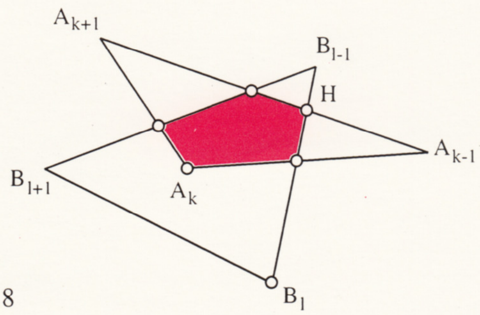

Fig. 8

CONCLUSION

Visibility represents quite an important and big part in the field of computer graphics. There have been a lot of visibility algorithms worked out [6]. They differ in various ways with respect to complexity, universality, the choice of examined phenomena, methods, etc. Our algorithm can be tabled next to the Newell's and Sancha's one [2] in which the polyhedra faces are ordered into the display sequence according to the "z- depth", z denotes the direction of the view. The problem of cyclic overlaying (Fig. 4) is suggested to be solved by the division of the problematic polygon into two parts, that are to be ordered then as new polygons into the sequence. The aim of the paper is to describe the visibility problem solving in displaying the general polyhedra scenes *via the triangulation* of polyhedra faces. This method was elaborated for the purposes of the educational process. It solves the problem in understandable way and enables to visualize polyhedra scenes in any view and arrangement. Even very complex scenes with overlaying of polyhedra faces (rare in the space) are allowed.

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