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Geometric Considerations About Seemingly Wrong Tilt of Crescent Moon

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ABSTRACT

The following phenomenon is well-known and again and again appears as an unanswered question in literature and on internet platforms: If you see moon and sun in the sky at the same time, then the (bisector of the) crescent moon in most cases does not seem to be precisely directed at the sun. Particularly at sunset, when you would expect the bisector of the crescent moon to be horizontal, it mostly points upwards. To "prove" that, photos that seem to support this view are displayed. In this paper it is shown by means of geometry what the "wrong moon tilt" is all about and that an explanation is to be found in the nature of central or normal projections (photography is basically a central projection, at an extremely long focal length it is approximately a normal projection). The paper also deals with the reason why the seemingly wrong tilt is subjectively felt. The path of the light from the sun to the moon is in any case displayed straight (apart from minor deviations due to refraction close to the horizon), except one takes photos with a fish-eye lens.

Key words: Moon tilt, Terminator, Normal Projection

MSC 2010: 51N05, 51P05, 51M04

Geometrijska razmatranja o naizgled krivom nagibu polumjeseca

SAŽETAK

Sljedeći je fenomen dobro poznat i često se u literaturi i na internetu pojavljuje kao pitanje bez odgovora: Ako na nebu istovremeno vidite mjesec i sunce, onda simetrala polumjeseca u većini slučajeva ne izgleda usmjerena točno prema suncu. Posebno u doba sunčeva zalaska, kad očekujete da simetrala polumjeseca bude horizontalna, ona je uglavnom usmjerena prema gore. Da bismo to "dokazali" prikazujemo fotografije koje podržavaju takav pogled. U ovom se članku posredstvom geometrije objašnjava pojava "krivog nagiba mjeseca", a rješenje se nalazi u prirodi centralnog ili ortogonalnog projiciranja (naime, fotografija je temeljno centralna projekcija, ali kod izuzetno velike žarišne duljine približno je ortogonalna projekcija). U članku se također razmatra razlog zašto se naizgled krivi nagib subjektivno osjeća. Svjetlosna putanja od sunca do mjeseca u svakom se slučaju prikazuje ravno (osim kod malih odstupanja koja se javljaju zbog loma svjetlosti u blizini horizonta), i osim kod fotografija slikanih sa širokokutnim objektivom.

Ključne riječi: nagib mjeseca, ograničenje, ortogonalna projekcija

1 Introduction, Motivation

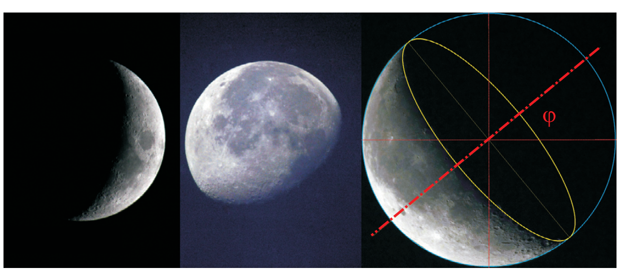

Figure 1: *Three different close-ups of the crescent moon. The outer edge appears circular, the border of the shadow (the picture of the terminator) elliptical.*

The crescent moon is composed (with an approximately equal share) of the outline of the part illuminated by the sun and the picture of the bordering line between shadow and illuminated area (in astronomy also called terminator [9]) (Fig. 1 left). In this paper we will (for the sake of simplicity) also call the more than half full moon as "crescent moon", even though the crescent is only noticeable when the moon is less than half full (in this case either the illuminated or the dark part form a crescent, see Fig. 1 middle). Roughly speaking, it is a geometrical figure, whose outline (at normal or central projection, when the center of the moon is aimed at) is approximately made out of a half circle or half ellipse (mountain ranges on the moon do not play a vital role with regard to the outline, regarding the bordering line between shadow and illuminated area they lead – due to the flatly incoming light – to noticeable deviations). This is why the following applies:

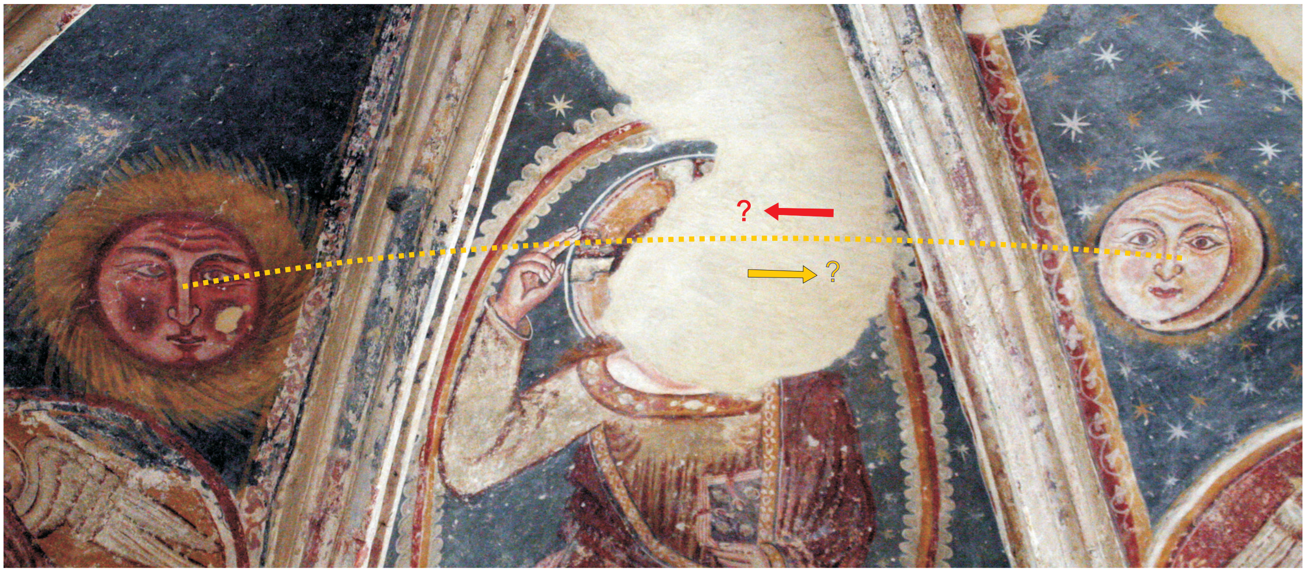

Figure 2: *Tele-photo of a fresco from the 14th century (St. Lawrence's church, Požega, Croatia). The yellow dashed line and the arrows were added.*

Lemma: *When having a normal or central projection with the center of the moon on the principal projection ray, the minor axis of the picture ellipse of the terminator of the moon is the line of symmetry of the moon crescent. As we intuitively focus on the center of the moon when looking at it with the naked eye, we always see the crescent symmetrical with regard to this axis.*

So it makes sense to take the minor axis of the ellipse in order to be able to fix with it or to measure the tilt of the crescent moon (Fig. 1 right).

Probably everyone who gazes in admiration at the waxing moon in the afternoon or at sunset or at the waning moon at sunrise, at one time asks himself whether this bisector when extended runs through the sun point.

Let us have a closer look at Fig. 2, where sun and moon are displayed next to each other. Apart from the fact that strictly speaking a waning and not a waxing moon should have been painted, this over 600-year-old painting is outstanding in various respects: First the outline of moon and sun are not exactly circular but actually elliptical (in particular the outline of the moon), secondly the moon crescent is not exactly symmetrical and thirdly the illumination caused by the sun does not seem to come from one side. The crescent is – as often described – slightly tilted. Additionally one could add that the painting is produced on an approximately spherical ceiling so that the distortion in the original is even stronger than it seems on the photo. In the course of this paper we will show that all these phenomena are surprisingly linked to the "wrong tilt" of the moon crescent[1].

2 Photographs on which sun and moon are shown at the same time

Sun and moon have both a diameter of half a degree on the firmament.This is an optical angle that can be completely captured by our eyes – the "external branch" of our brain – without moving the eye apple. In order to be able to photograph sun and moon in a way that they both fill the picture, one needs a focal length of about 2000 mm. The angular distance sun - moon is now always at least several degrees (otherwise there is new moon or the crescent of the moon is so thin that it cannot be seen with the naked eye). At half moon the deviation is 90°, which already requires a distinct wide-angle lens (20 mm focal length). Until the full moon is reached the angle increases up to 180°, so that both sun and moon can only be photographed at the same time with special fish eye lenses that are definitely not linear.

The in geometry common perspective projection is a central projection of the space onto a plane. The same process is relevant for photography, where the projection center is the center of the lens system and the projection plane is the light sensitive sensor plane. The central projection is linear, which means straight lines – for example light rays – are portrayed straight. This also applies to wide-angle photography in good approximation (The quality of wide-angle lenses is often determined by this criterion).

[1]Let us mention in passing: The face of the moon seems to be rather female, the face of the sun rather male. That is for someone who is a native speaker of English or Romance language obvious. In Germanic languages, however, the moon is male and the sun female.

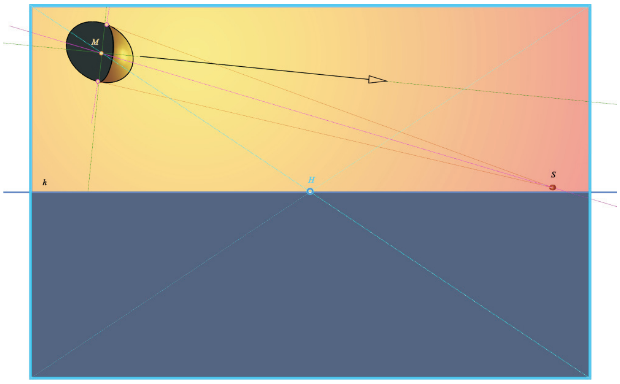

Figure 3: *"Photograph" showing sun and moon, simulated by means of the computer, so that the moon diameter could be enlarged. Two ellipses form the boundary of the moon crescent. The minor axis of the picture ellipse of the terminator (black with arrow) is directed above the sun point. The blue frame symbolizes the sensor plane.*

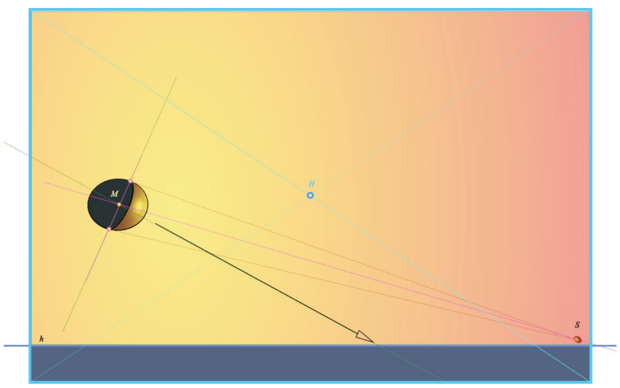

Figure 4: *Another "photo" of the same situation, this time with a different camera tilt. The minor axis of the picture of the terminator is now directed below the sun point.*

The outline of a sphere that fully lies in front of the observer is – under the condition of such a projection – generally an ellipse whose main axis runs through the main point H (the intersection of the optical axis with the sensor plane, hence planimetrically speaking the intersection of the diagonals of the chips) (Fig. 3 and 4, [8],[1]).

A general circle in front of the observer (in our case the terminator) is displayed as an ellipse whose axes in general do not go through the main point. There is just the following exception: If the axis of the circle (the perpendicular of the circular plane in the circle's center point) hits the principal ray (the optical axis), the minor axis of the picture ellipse runs through H ([3]) for reasons of symmetry. As the axis of the terminator is the direction of the light rays, the following applies

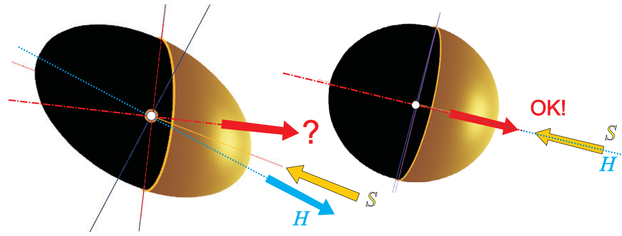

Figure 5: *If you want to enlarge details taken from a wide-angle photo even more, it becomes even more obvious: The two ellipses (the outline of the sphere and the picture of the terminator) generally have no common axis of symmetry. If the picture of the connecting straight line to the sun runs through the main point H of the picture, the crescent is however symmetrical and is directed at the sun indeed.*

Theorem 1: *In a photograph where not the center of the moon is focused at, the moon crescent has an elliptical boundary on both sides and is in general not symmetrical. Only if the light ray goes through the center point of the moon in the picture through the main point H (the center point of the sensor), both ellipses have a common line of symmetry (the minor axis of the terminator and the main axis of the outline ellipse). The direction of the minor axis of the picture of the terminator therefore is not directed at the sun point in general. If H lies below the connecting line of moon point and sun point, the minor axis is directed above the sun point; is H above, the minor axis is directed below the sun point.*

As clear as this theorem seems to be in geometry: On the wide-angle photo of the moon it can hardly be noticed that we are dealing with a double elliptical crescent, because the crescent appears naturally so small that this circumstance does not become obvious in the photo.

If one photographs sun and moon simultaneously with a wide angle lens, one should consider that the connecting straight line between sun point and moon point runs through the center of the picture, for example by positioning the moon point in one corner and the sun point in the opposite corner (Fig. 6).

If one photographs under the condition of a bigger distance between sun and moon point with a fish eye lens (Fig. 7), we get a distortion of straight lines: fish eye lenses are not linear and in general increase the above mentioned deviations upwards or downwards. Yet, due to rotation symmetry of the lenses around the optical axis, straight lines, which meet the optical axis, are portrayed as straight radial rays. Circles are portrayed oval but not elliptical. Therefore the following applies:

Figure 6: *Photo with 20 mm focal length (optical angle about 90°). Despite extremely high resolution (22 megapixel) the moon (top, right) also appears blurred in the magnification (framed white) due to the small size in the picture, but obviously elliptically distorted.*

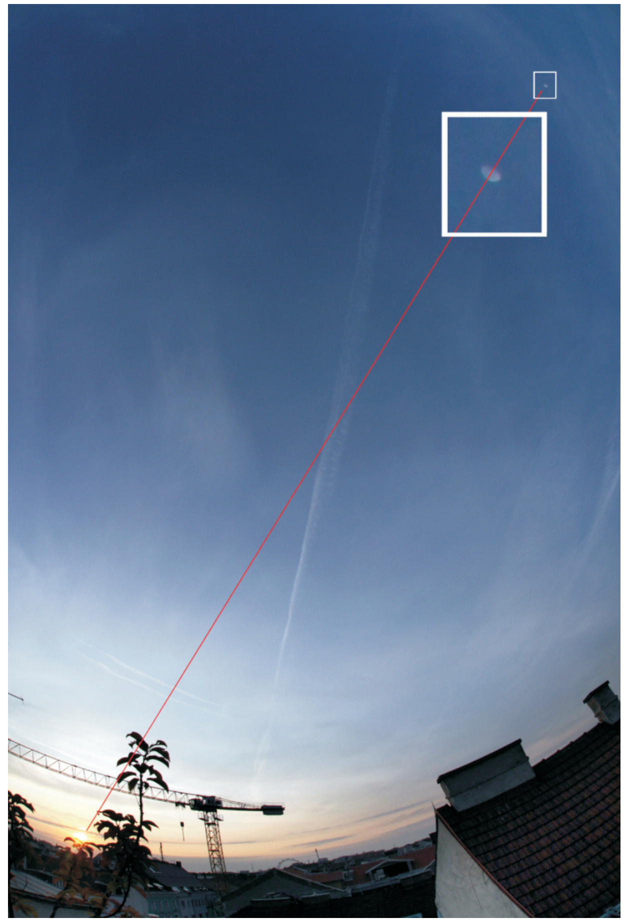

Figure 7: *Similar situation, but a few days later when the angle distance between sun and moon point has extremely increased. That is why a fish eye lens (15mm) had to be used.*

Theorem 2: *If taking photos with a fish eye lens and if it is not explicitly the moon center focused at, the moon crescent appears as being framed by not elliptical ovals on both sides. The crescent is only then symmetrical and aims at the sun point if the connecting line between sun point and moon point is a radial ray through the middle of the picture.*

In Fig. 7 sun and moon were photographed in a way that sun point and moon point are approximately on a picture diagonal. The (natural not visible) sun ray through the moon center is colored red and is also displayed straight in the fish eye picture. The crescent that is limited by two ovals has the red line as a line of symmetry. (Note: In the picture also the vapor trail of a plane is visible. This vapor plane is displayed – in contrast to the horizon – almost straight, because it is approximately radial.)

3 Close-up of the moon and the seemingly wrong tilt of the crescent

We now deal with close-ups of the moon, which can only be produced by means of an efficient telephoto lens. Here the moon center is automatically moved into the center of the picture. The respective representation is an extreme magnification of the center of an ordinary perspective and in good approximation a normal projection. The outline of the moon sphere is circular, the terminator elliptical. The center of this ellipse is the projection of the moon center. The vertices of the image ellipse lie diametral on the sphere's outline: The connecting line of these points in space has "principal position", hence is parallel to the sensor plane. The minor axis is orthogonal to the connection of the two vertices (Fig.1 right) and according to the theorem of the right angle normal projection of the circle axis ([7], [2], [17]). Hence, the following applies:

Theorem 3: *When dealing with close-ups of the moon, we almost have a normal projection. The minor axis of the picture ellipse of the terminator is the normal projection of the light ray through the moon center. The points of intersection of the circum-circle and the terminator (the end points of the crescent) are the vertices in the picture.*

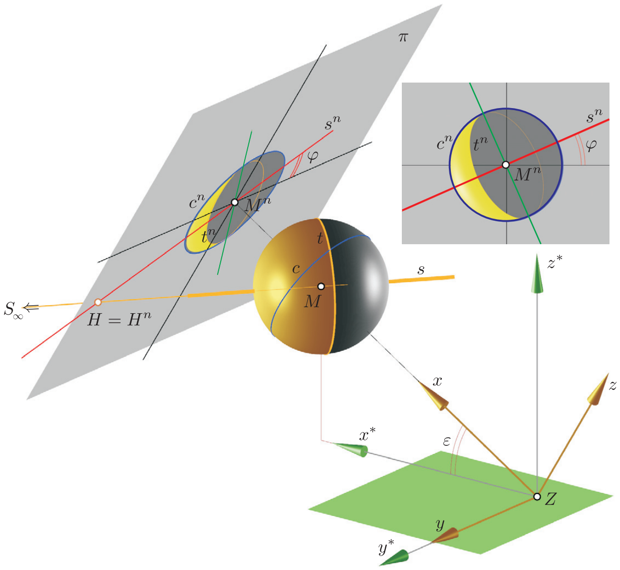

Figure 8: *Normal projection of the moon onto the sensor plane* π*. The sun rays direction* s *turns into direction* s^n*, the outline* c *of the moon turns into the circle* c^n*, the terminator* t *into the crescent edge* t^n*.* s^n *is the minor axis of the ellipse* t^n*.*

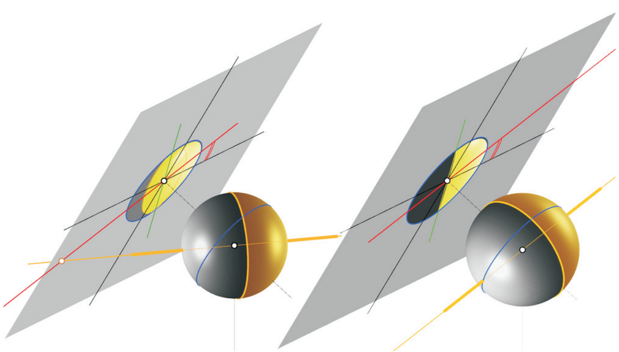

Figure 9: *The same crescent tilt* φ *as in Fig. 8, yet at 3/4-moon or half moon (sun rays parallel to the picture plane* π*).*

If you photograph the moon in a way that the lower part or frame of the camera (and hence the border of the rectangular sensor) is horizontal, then one can measure the rotation angle of the picture ellipse well. According to Theorem 3 this angle can practically have any size between -90° and +90° and can only be determined when the length of the minor axis (half minor axis b), hence indirectly the "thickness" of the crescent, is taken into consideration as well:

Theorem 4: *Only the simultaneous interpretation of direction and length of the minor axis of the picture ellipse of the terminator makes it possible to reconstruct the light rays in space.*

For better understanding, some special cases are to be mentioned (here b again defines the length of the half minor axis and r defines the moon radius):

- Having a half moon ($b = 0$) the sun rays are necessarily parallel to the sensor plane (Fig. 9 right). At sunrise or sunset the line of symmetry of the crescent is therefore horizontal, no matter how high the moon is in the sky.
- At new or full moon ($b = r$) the sun rays have the direction of the optical axis (= direction of the projection). The tilt of the circle that has mutated into an ellipse is undefined.
- If one defines the quarter and three quarter moon a week after full moon or new moon, then $b = r/\sqrt{2} \approx 0,7r$. One comes from the picture to the direction to the sun by imagining the minor axis as normal projection of an axis tilted by 45° (Fig. 8 or Fig. 9 left).

Now we want to derive a formula for the tilt of the minor axis or "bulbousness" of the ellipse (a comparable formula can also be found in [5]). Imagine the lens center is the origin of a Cartesian coordinate system. For matters of simplicity we define the optical axis as x-axis, the horizontal direction of the sensor plane as y-direction and the line of steepest slope of the sensor plane as z-direction. The direction to the moon is also given through $\vec{m} = (1,0,0)$, the one to the sun is defined by the direction vector $\vec{s} = (s_x, s_y, s_z)$. Its normal projection onto the sensor plane has the components $\vec{s}_n = (0, s_y, s_z)$. We measure the angle φ to the y-axis by scaling / normalizing $\vec{s}_n$ and multiplying by $\vec{y} = (0,1,0)$:

$$\cos\varphi = \vec{s}_n \cdot \vec{y} = s_y / \sqrt{s_y^2 + s_z^2} \qquad (1)$$

Moon and sun coordinates (in horizontal polar coordinates given through their azimuth angles α, α^* or their difference $\delta = \alpha^* - \alpha$ and elevation angle ε and ε^*) may be at disposal (for example by means of relevant software). Let us look at the Cartesian coordinates in a coordinate system which has the same y-axis, whose z-axis is however vertical: There the direction vector to the sun has the components

$$\vec{s}^* = (s_x^*, s_y^*, s_z^*) = (\cos\varepsilon^* \cdot \cos\delta, \cos\varepsilon^* \cdot \sin\delta, \sin\varepsilon^*) \qquad (2)$$

Relative to the coordinate system that is twisted around the elevation angle ε the following applies:

$$\vec{s} = (s_x^* \cos\varepsilon + s_z^* \sin\varepsilon, s_y^*, -s_x^* \sin\varepsilon + s_z^* \cos\varepsilon) \qquad (3)$$

If we again insert (2) in (3), φ can be calculated directly by inserting

$$s_y = \cos\varepsilon^* \sin\delta,\ s_z = -\cos\varepsilon^* \cos\delta \sin\varepsilon + \sin\varepsilon^* \cos\varepsilon \quad (4)$$

in (1). The tilt ψ of the sun rays to the picture plane is equivalent to the complementary angle to the negativ x-axis $-\vec{m}$, determined through

$$\cos\psi = -\vec{s}\cdot\vec{m} = -s_x/\sqrt{s_y^2 + s_z^2} \quad (5)$$

with $s_x = \cos\varepsilon^* \cos\delta \cos\varepsilon + \sin\varepsilon^* \sin\varepsilon$.

This cosine value is also a measure for the thickness of the picture of the terminator.

4 Human perception of the direction of light rays from sun to moon

In order to be able to perceive sun and moon at the same time, human beings have to move their head (or at least roll the eyes balls when keeping the head stiff). After it was proved through various experiments that human beings can only perceive quite small optical angles in one complete picture (and then almost perceives a normal projection), the brain has to do the job of gathering all the individual impressions gained by moving the eye balls. Here only a limited "impression similar to a photo" can be created: One cannot fix together individual photos showing single parts of an object without manipulation[2]. Most likely a spherical picture develops which must be interpreted by the brain by comparing optical angles. Looking at straight lines (without any other straight line as a reference) is in particular at optical angles of over 90° a rather deceitful venture. One cannot even compare the simultaneous observation of sun and moon with observing a vapor trail of an airplane about which one knows that it runs parallel to the base plane (which is rarely the case considering the sun-moon condition, most likely at moonrise and simultaneous sunset but then one cannot notice any tilt of the moon crescent).

If one wants to assess the tilt of the crescent, one automatically and necessarily refers to the picture of the moon, which one gets through direct sighting, and this is – according to the considerations made in Section 3 – tilted like the normal projection of the sun rays.

In the following we want to deal with frequent claims and questions arising in connection with the moon tilt:

1. A simple but didactically helpful animation about the "development" of the moon crescent can be found on [13]. Pictures and animations that are much more demanding and only comprehensible with some previous knowledge can be found on [14].

2. The so-called AUBERT's phenomenon claims that one must turn and instinctively swing the head when trying to perceive larger angle areas. This is sometimes used as the only explanation for the phenomenon of the "wrong tilt" of the moon crescent ([11]). According to what we have heard so far, this is however not true.

3. In [4] it is assumed that the direction of the normal projection of the sun rays is equivalent to the tangent on the great circle in the sky that connects moon M and sun S and that has the observer Z as the center. This is – planimetrically speaking – correct, because the mentioned great circle obviously lies in the optical plane MSZ and appears projective, hence in the picture it cannot be differentiated from the sun ray SM at that moment. Having a normal projection on the picture plane, the optical plane is projective and the great circle always appears as a straight line. One could therefore apply the following trick in order to estimate the tilt of the projection of the terminator: One points with the stretched arm at the moon crescent and turns the arm to the sun in the optical plane. The direction in which the index finger starts indicates the tilt. This trick makes us suppose that the light ray in the sky is crooked, which is, however, not true. As mentioned it can only be compared to the situation *in the beginning*.

   Unfortunately many participants in internet discussions are often "tempted to rely on the crooked line". Didactically speaking, it therefore does not seem useful to introduce the great circle at the beginning of the explanation of this phenomenon: The tilt is only to be determined by the effect of the central and normal projections and by considering various preconditions. Consequently the explanation of the moon tilt given in [16] is not correct as far as it relies on the crooked line, whereas the explanation in [6] is wrong due to the fact that the effect of the perspective is not taken into consideration.

[2] For example for a "panorama photo" stripes that actually come from the picture centers of various photos are usually fixed together. The margins of these stripes must be contracted in order to compensate for the perspectively caused enlargement. After having fixed the stripes together, the picture that is framed by various crooked lines is cut into a rectangle ([2]).

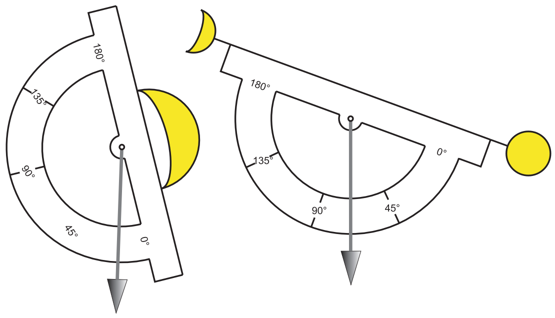

Figure 10: *“True” and “expected” tilt of the moon crescent.*

4. On internet pages people differentiate between “true” and “expected” moon tilt, which is explained by a figure comparable to Fig. 3. There the expected tilt is seen as a projection of the connection moon - sun on a vertical plane which owns – as a normal – the angle bisector of the outlines of moon and sun ([5], [16]). If one agrees on certain requirements (e.g. that the connecting line of sun and moon runs through the center of the picture and both points have the same distance from the margin of the picture; additionally that the picture plane is tilted if the moon is not quite low), this “expected” tilt can be defined mathematically and can be compared with the tilt at normal projection (there called “true tilt”) (Fig. 10). The difference of the angles can then be named “supposed mistake”.

5. It would be tempting to assume that the line of symmetry of the moon crescent indicates the motion direction of the crescent. This is however at least not the case if the path of the moon is steeper or less steep than the one of the sun.

   Is, for example, as in Fig. 11 (moonset in Vienna on the 25th of October 2009, at about 10pm) the moon path flatter by about 10° than the sun path, then due to the different heights of the paths e.g. at half moon a clear deviation downwards is noticable[3].

   In order to test our Formula (4) and the respective formula in [5] (there “Formel 2”) [4], we insert the values $\delta = 83,1^\circ$, $\varepsilon = 2,6^\circ$ and $\varepsilon^* = -47,5^\circ$ and get according to both formulas $\varphi \approx -48^\circ$. This value is determined graphically correct to one degree for the middle position of the moon in Fig. 11.

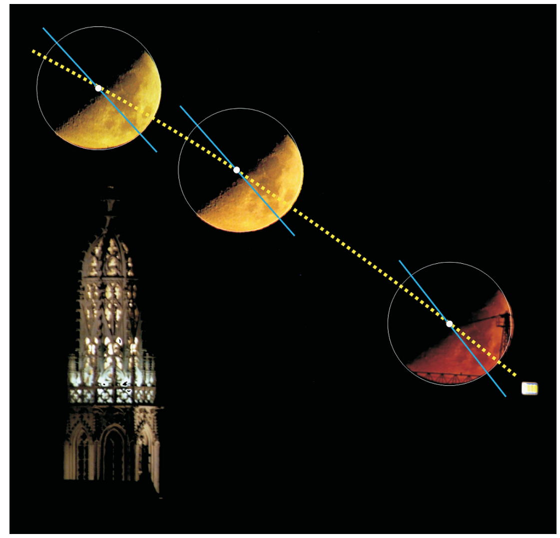

Figure 11: *Set of the waxing half moon. On that day the path of the moon was flatter than the path of the sun. That is why the crescent is directed more downward.*

The fact that the moon does not always culminate at the same elevation angle as the sun can be easily realized with the educational (and simplified) simulation [10]. Here we do not deal with small deviations (as the one of the path planes of the earth or the moon, which is at the most $\pm 5^\circ$), but with enormous differences: The full moon moves in the winter so high as the sun in the summer and the other way round (difference in the elevation angle approximately 47°). At equinox we have a deviation of about $\pm 23,5^\circ$ at half moon, whereas new and full moon culminate approximately as high as the sun.

5 Summary

When having a close-up or when a human being is the observer, the moon crescent appears as a symmetrical object which is restricted by a half circle and a half ellipse with the moon center point as center. The minor axis of the half ellipse shows into the direction of the normal projection of the sun rays on the projection plane – and therefore generally not “to the sun”. Only when also considering the length of the minor axis, the position of the sun can be determined. If you photograph sun and moon at the same time with a wide-angle lens, then the moon crescent is in general restricted by two ellipses that have no shared line

[3] In Fig. 11 two peculiarities are worth mentioning: First, the moon – such as the sun – is colored red briefly before reaching the horizon and secondly, the usually circle shaped outline appears flatter. Both are consequences of the refraction of the flatly incoming light into the gradually becoming more dense atmosphere of the earth.

[4] To determine the equatorial coordinates [15] was used, which – by means of [12] – were changed into horizontal coordinates.

of symmetry. That is why we cannot speak of a direction of the moon crescent in case of a wide-angle photograph.

If sun point and moon point are on one straight line through the center of the photo, the moon crescent is symmetrical and the line of symmetry is directed at the sun point. Something similar applies to photographs with fisheye lenses. Theories which want to explain the phenomenon of the "wrong moon tilt" with crooked light rays are incorrect. Exclusively geometrical characteristics of different kinds of projections are involved.

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