APPLICATION OF CYCLES RELATED GRAPHS TO DUAL-RING TYPE OF NETWORK TOPOLOGY

IVANA ZUBAC

UNIVERSITY OF MOSTAR

INTRODUCTION

- Graph theory is today an extremely diverse field with wide applications.
- Graphs have proven to be an excellent tool for modeling systems that emphasize connections and relationships between objects.
- If we pay attention, we will notice that the problems studied by graph theory are actually all around us.
- We will give an interesting example of the application of graph theory in computing, specifically computer networks.

DEFINITION

- A graph G(V,E) is a pair of two sets, V and E, V=V(G) being a finite nonempty set and E=E(G) is binary relation defined on V.
- A graph can be visualized by representing the elements of V by vertices and joining pairs of vertices (i,j) by an edge if and only if $(i,j) \in E(G)$. The degree of a vertex in a non-directed graph is defined as the number of links a vertex has with other vertices.
- A path graph is non-branced chain, a tree is a branched structure without cycle. A star is a set of vertices joined in a common vertex. A cycle is a chain which starts and ends in one and the same vertex. A complete graph K_n is the graph of with any two vertices are adjacent. All graphs in this presentation are finite, simple and undirected. Terms not defined here are used in the sense of Harary [Harary F. (1969) Graph Theory, Addison- Wesley, Reading].

CENTRALITY

- Centrality is a crucial concept in graph analytics that deals with distinguishing important nodes (vertices) in a graph. Simply put, it recognizes vertex that are important or central among the whole list of other vertices in a graph.
- There are a large number of different definitions and methods of determination the importance of every vertex, various metrics are taken into consideration to study each vertex from a different perspective.
- The different perspectives of a particular vertex are studied under different indices, which are collectively known as centrality measures. They help in gaining a better grasp of the network and cutting through the chaos while extracting information from a network.
- Degree centrality defines the importance of a vertex based on the degree of that vertex. The higher the degree, the more crucial it becomes in the graph. It's used to find popular individuals, the most connected, individuals who connect quickly in a wider network, or the ones that hold the most information.
- It's one of the simplest of all the centrality measures of vertex connectivity, and is used in transactional data, account activity, etc.

GRAPH LABELING

- A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).
- Labeled graphs serve as useful models for a broad range of applications such as radar, coding theory, astronomy and communication network addressing.
- Various authors have introduced labelings that generalize the idea of a magic square. Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, satisfying some kind of ”constant-sum” property. A vertex-magic labeling is one in which the sum of all labels associated with a vertex is a constant independent of the choice of vertex.
- An antimagic labeling is an edge-labeling of the graph with the integers 1, 2, . . . , e so that the weight at each vertex is different from the weight at any other vertex. The concept of an (a,d)-antimagic labeling as an edge-labeling in which the vertex weights form an arithmetic progression starting from a and having common difference d.

MATCHING

- Matching is a fundamental concept used to describe a set of edges without common vertices. Matchings are used in various applications such as network design (efficient routing and resource allocation), job assignments (assigning jobs to machines or workers), scheduling (optimal scheduling of tasks), chemistry, graph coloring, neural networks in artificial intelligence and more.

- The cardinality of M is called the size of the matching. As the matchings of small size are not interesting, we will be mostly interested in matchings that are as large as possible. A matching M is maximum if there is no matching in G with more edges than M. The matching M is perfect if each vertex of G is incident with an edge of M, perfect matchings are obviously also maximum matchings.

- There is another way to quantify the idea of "large" matchings. A matching M in G is maximal if no other matching in G contains it as a proper subset. Obviously, every maximum matching is also maximal, but the opposite is generally not true.

- Maximal matchings are much less researched (structural and enumerative). The cardinality of any smallest maximal matching in G is the saturation number of G (denote s(G)). The saturation number provides an information on the worst possible case.

NETWORK TOPOLOGY

- Network topology refers to the arrangement and interconnection of various components within a computer network, including nodes (computers, switches, routers) and links (wired or wireless connections). It defines how these components are connected and interact with each other.
- The structure of a network topology determines how data is transmitted, affecting the network's performance, reliability, and scalability. An efficiently designed topology can reduce cable costs, enhance data transfer speeds, and improve network reliability.
- For organizations, choosing the right topology is a key part of network planning, as it affects both the operational efficiency and the ease of future expansion.
- The landscape of network topology is diverse, offering various configurations, each with its unique characteristics and suitability for different network scenarios.

TYPES OF NETWORK TOPOLOGY

The primary types of network topology include:

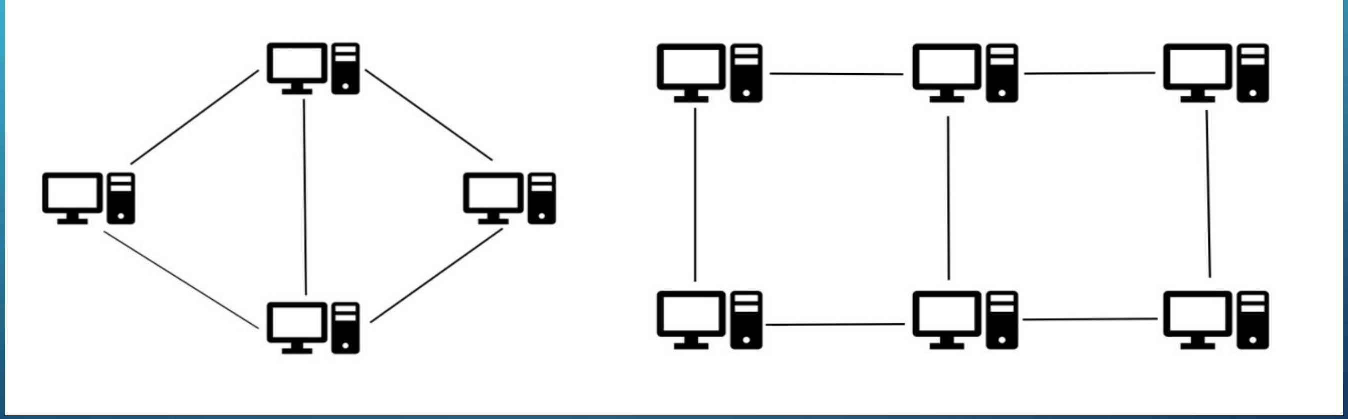
- Point-to-Point: A simple topology that connects two nodes directly (in graph theory represent by path graph),
- Bus Topology is a network type in which every computer and network device is connected to a single cable (caterpillar)
- Star: Features a central connection point that links individual nodes (star graph)
- Ring: Each node is connected to two other nodes, forming a circular data path (cycle graph)
- Tree: A variation of the star network, where branches connect multiple star networks (tree graph)
- Mesh: Offers a high level of redundancy, with each node having a connection to several other nodes (usualy complete graph)
- Hybrid: Combines two or more different types of topologies.

DUAL - RING TOPOLOGY

- **Dual-ring topology** is a network configuration where each device is connected to two others, creating two concentric circles for information flow. This design offers a backup route for communication – if one circle fails, the second one ensures consistent network stability and resilience. Such a system boosts the efficiency of data transmissions.
- Use cases of dual-ring topology: corporate networks (uninterrupted communication between departments, even if one network path experiences failure), metropolitan area networks (offers a fail-safe in case of a single-path disruption), university campus networks (supports a reliable network across multiple buildings, it ensures that network issues in one place don't disrupt connectivity in others), industrial settings (can be crucial for process control systems, it guarantees continuous data flow for monitoring and controlling machinery), public transportation (its communication systems employ this topology to guarantee reliable information exchange, that's a critical safety measure for transit operations).

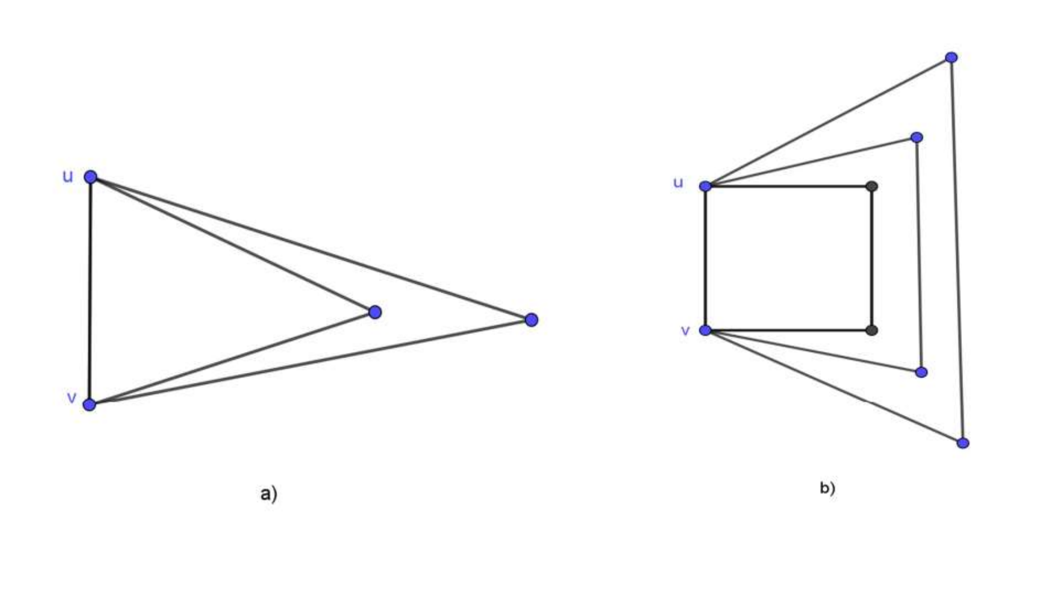
DUAL - RING TOPOLOGY [2]

- The most suitable context for testing the properties of this topology is the graph theory. A dual-ring topology can be represented by a cycle related graph, more precisely a book graph.
- We represent the components, nodes (computers, switches, routers) by vertices and links (wired or wireless connections) by edges of certain graphs.



BOOK GRAPH WITH 3 OR 4-GONAL PAGES

- A book graph B=B(n,r) with r-gonal pages is a graph constructed using n copies of a cycle of length r that share exactly one common edge.
- Network connection with a dual-ring topology can be represented by a book graph with a cycle length of 3 or 4.



MAXIMAL MATCHINGS

- We start by counting all maximal matchings in B(n,3). Let φ_n denote the total number of maximal matchings in B.
- **Proposition 1.** The number of maximal matching in B(n,3) is equal $\Phi_n=n(n-1)+1$, $n\geq 4$.
- ***Proof.*** Let us label common edge with uv (fig.2.a). Then is exatly one maximal matching in B that cover edge uv. If edge uv is not cover by matching then it is necessary that the vertices u and v are covered with the edges of which they are ends. If we do not include the edge uv, we have n paths of length 2, and the total number of maximal pairings in them is n(n-1).
- **Proposition 2**. The sequence φ_n is given by the recurrence
  - $\Phi_n=\varphi_{n-1}+\varphi_{n-2}+\varphi_{n-3}.$
- The initial conditions are $\varphi_0=1$, $\varphi_1=\varphi_2=3$.

SATURATION NUMBER

- **Proposition 3**. Generalized function for Φ_n is equal $\varphi(x) = \frac{1+2x-x^2}{1-x-x^2-x^3}$.
- The asymptotic behavior of is given by $\varphi(x) \sim 0.543689$. In OEIS, this sequence is A214825.
- **Lema 4.** The saturation number of B(n,3) is s(B)=1.
- Evidently, the maximal matching that containing the (common) edge uv is smallest cardinality for all B (n,3).
- All other types of network topology, except a star topology, have a higher saturation number. For the example, the saturation number for the path and cycle are $s(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 2\ (mod 3) \\ \left\lfloor \frac{n}{3} \right\rfloor, \quad otherwise. \end{cases}$ and $s(C_n) = \left\{ \left\lceil \frac{n}{3} \right\rceil \right.$.
- However, star topology is not suitable because there is a high possibility of data loss. The "falling out" of the central “vertices/node” leads to a complete interruption of data transmission and there is no possibility of an alternative path.

B(n,4)

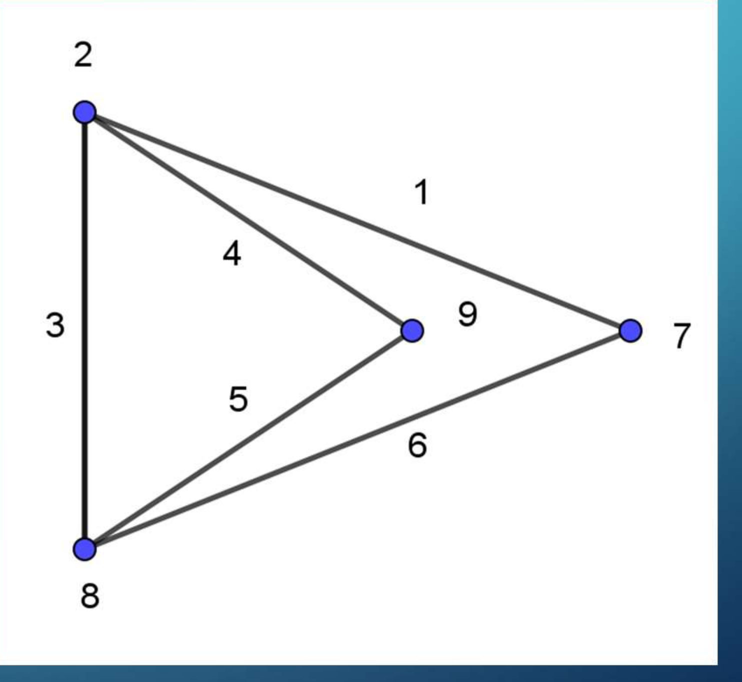
- **Proposition 5.** The number of maximal matching in B(n,4) is equal $\Phi_n = n^2 + 1$, $n \geq 4$.
- Proof. Let us label common edge with uv. Then is exactly one maximal matching in B that cover edge uv. If edge uv is not cover by matching then it is necessary that the vertices u and v are covered with the edges of which they are ends. If we do not include the edge uv, we have n paths of length 3, and the total number of maximal pairings in them is nxn.
- **Proposition 6**. The sequence φ_n is given by the recurrence $\Phi_n = (\varphi_{n-1} + \varphi_{n+1})/2 - 1$.
- Φ_n is one less than the arithmetic mean of its neighbors. In OEIS, this sequence is A002522, one of the interpretations is $a(n) = \varphi_{4(n)}$, where φ_k is the k-th cyclotomic polynomial.

B(n,4)

- **Proposition 7**. Generalized function for Φ_n is equal $\varphi(x) = \frac{1-x+2x^2}{(1-x)^3}$.
- **Proposition 8.** The saturation number of B(n,4) is s(B)=n, for n>1.
- Proof: Evidently, the maximal matching that not containing the (common) edge uv must contain one of the edge of which one edge is u and one edge of which one vertex is v (not from the same cycle), and edges parallel to the common edge in other cycles.

CENTRALITY/VATL

- **Proposition 9**. Degree centrality c_d in book graph of vertices u/v is $c_d\left(\frac{u}{v}\right) = n + 1.$
- All other vertices in graph have a degree centrality 2.
- An example of (10,4)-VATL (vertex-antimagic total labeling) of B(2,3)



FURTHER RESEARCH

- Matching, saturation number, centrality and graph labeling in all B(2,r) and B(n,r), for r even and odd…
- By comparing the results we can come to the conclusion which is better, more circles of length 3 or 4 or two circles with more nodes for less data loss.

Thank you