Resonance graphs of linear phenylenes

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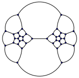
(joint work with Tomislav Došlić)

5th Croatian Combinatorial Days
Zagreb, Croatia



September 19–20, 2024

Hypercubes

Alphabet $\mathcal{T} = \{0, 1\}$. Adjacency $0 \leftrightarrow 1$

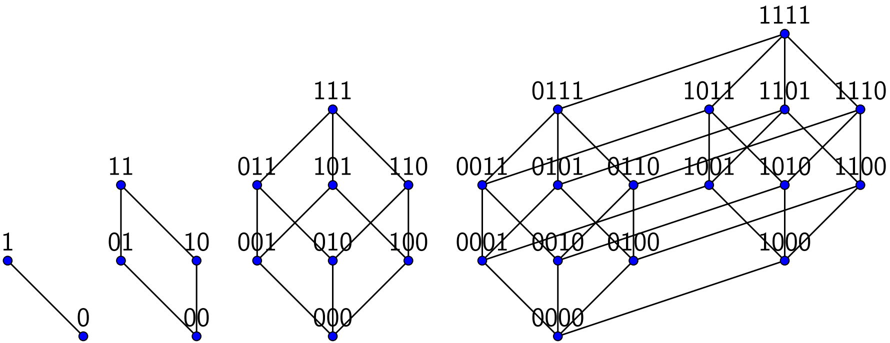


Figure: Hypercubes Q_1, Q_2, Q_3 and Q_4.

Fibonacci cubes

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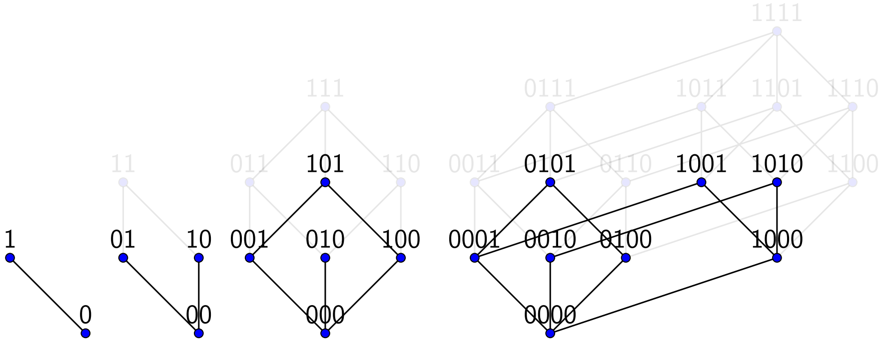

Figure: Fibonacci cubes $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4.

Metallic cubes

Alphabet $\mathcal{S}^a = \{0, 1, 2, \ldots, a-1, a\}$.

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211203,

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211203, 130202

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Graph Π_n^a:

$V(\Pi_n^a) = \mathcal{S}_n^a$.

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Graph Π_n^a:

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For $\alpha = \alpha_1 \cdots \alpha_n$ and $\beta = \beta_1 \cdots \beta_n$ we define

$$\overline{h}(\alpha, \beta) = \sum_{k=1}^{n} |\alpha_k - \beta_k|.$$

Then α and β are adjacent if and only if $\overline{h}(\alpha, \beta) = 1$.

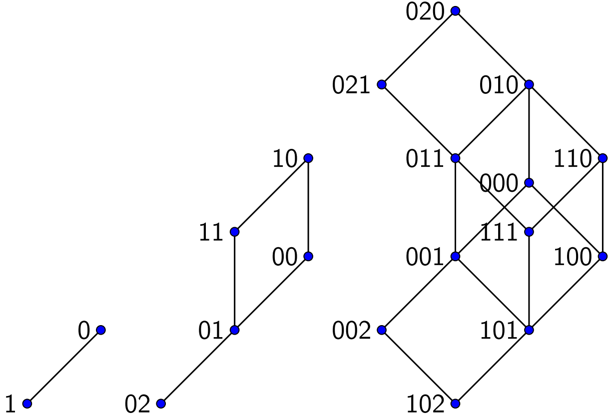


Figure: Graphs Π_1^2, Π_2^2 and Π_3^2.

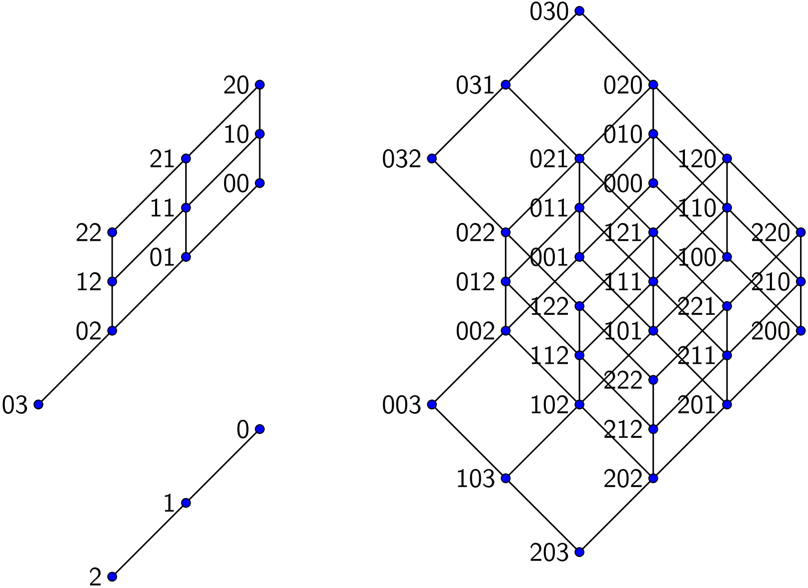


Figure: Graphs Π_1^3, Π_2^3 and Π_3^3.

Resonance graphs

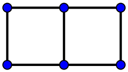
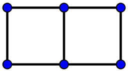


Figure: Ladder graph L_3.

Resonance graphs



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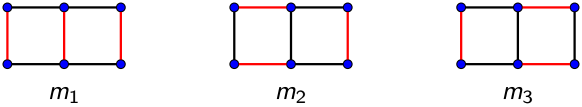
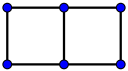


Figure: All perfect matching of the ladder graph L_3.

Resonance graphs



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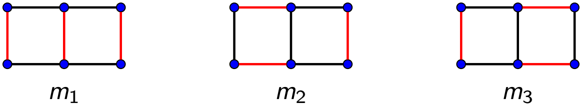

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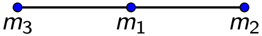


Figure: The resonance graph of L_3.

Fibonaccenes

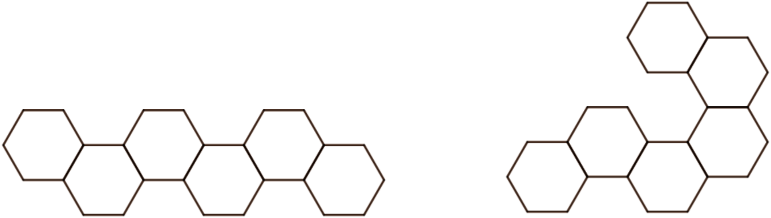
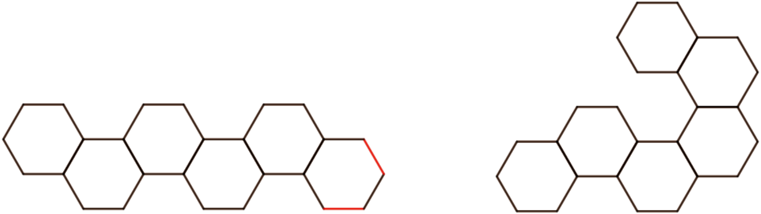


Figure: Fibonaccenes.

Fibonaccenes [1]



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[1] S. Klavžar and P. Žigert Pleteršek, Fibonacci Cubes are the Resonance Graphs of Fibonaccenes, *Fibonacci Quart.* 43 (3), 2005

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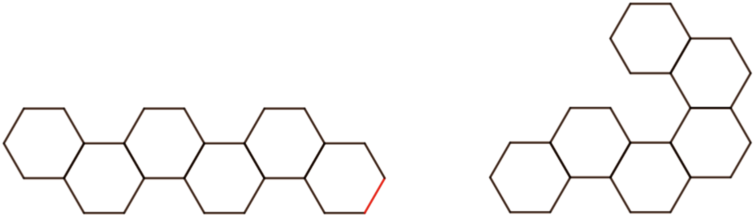

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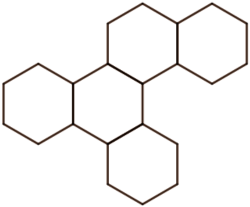
Benzenoids and phenylenes



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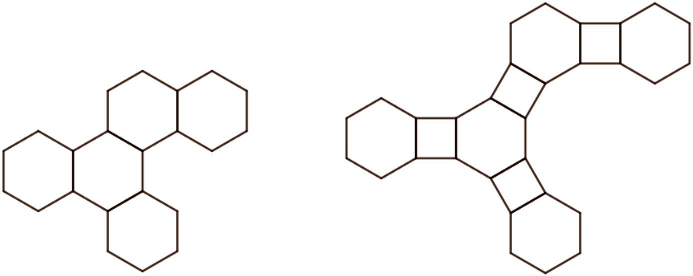

Figure: Benzenoid and phenylene

Generalized phenylenes

Let $a \geq 1$.

$a = 1$

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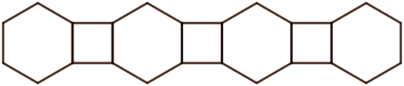

$a = 2$

Generalized phenylenes

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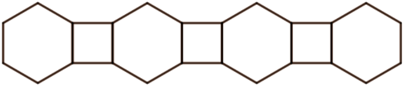

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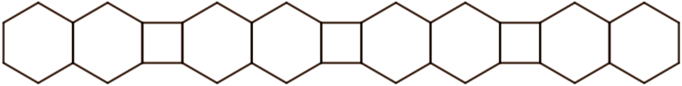


$a = 3$

Metallic cubes are the resonance graphs of (generalized) phenylenes

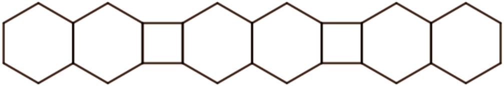
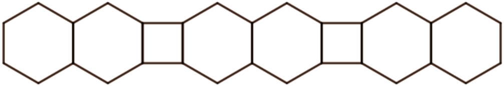


Figure: Generalized phenylene P_3^3.

Metallic cubes are the resonance graphs of (generalized) phenylenes



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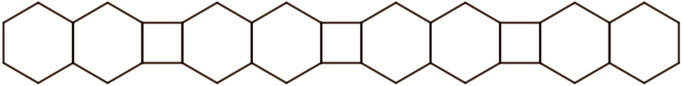
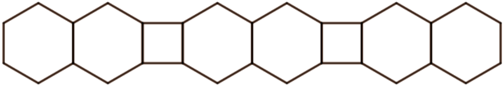


Figure: Generalized phenylene P_4^3.

Metallic cubes are the resonance graphs of (generalized) phenylenes



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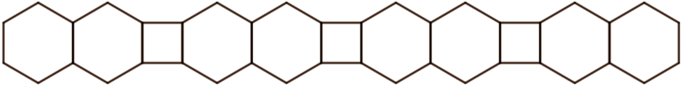

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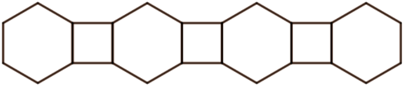


Figure: Phenylene P_4^2.

Metallic cube Π_n^a is the resonance graphs of generalized phenylene P_n^a.

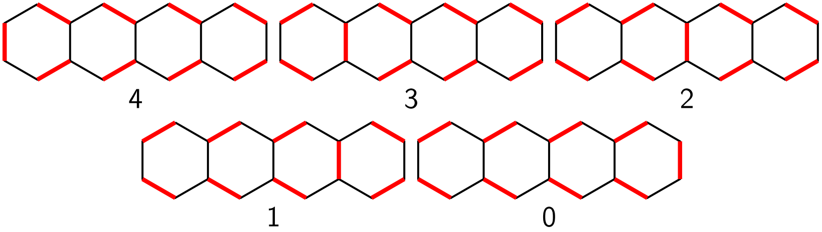


Figure: All perfect matching of the hexagonal chain with 4 hexagons.

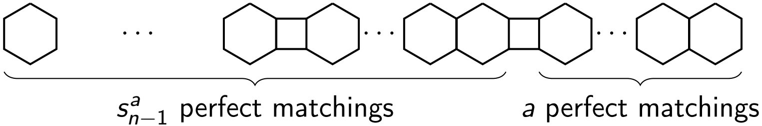
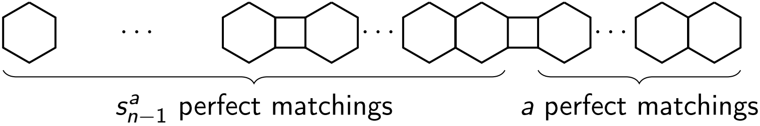


Figure: There are no horizontal edges of the quadrilateral in the perfect matching.



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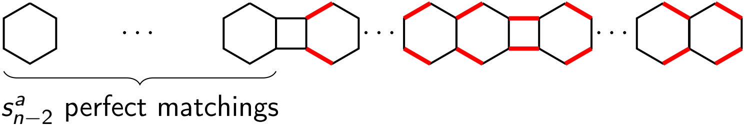
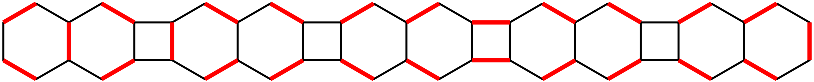


Figure: The perfect matching in the phenylene where both horizontal edges of the last quadrilateral are in the matching.



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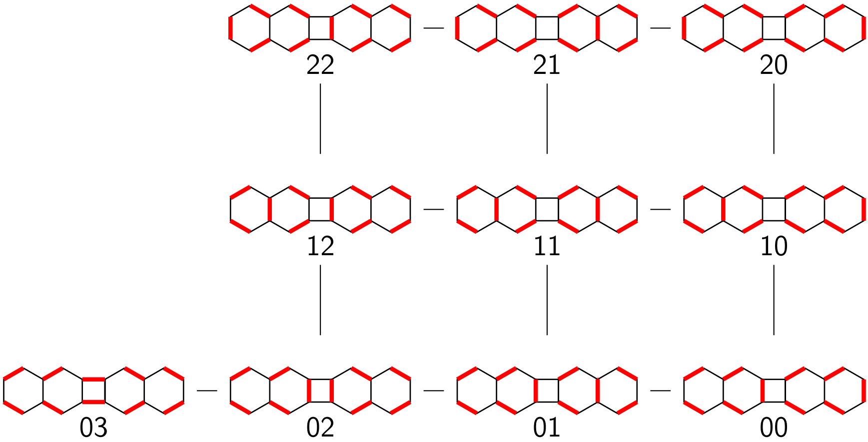

Figure: Metallic cube Π_2^3 as resonance graph of P_2^3.

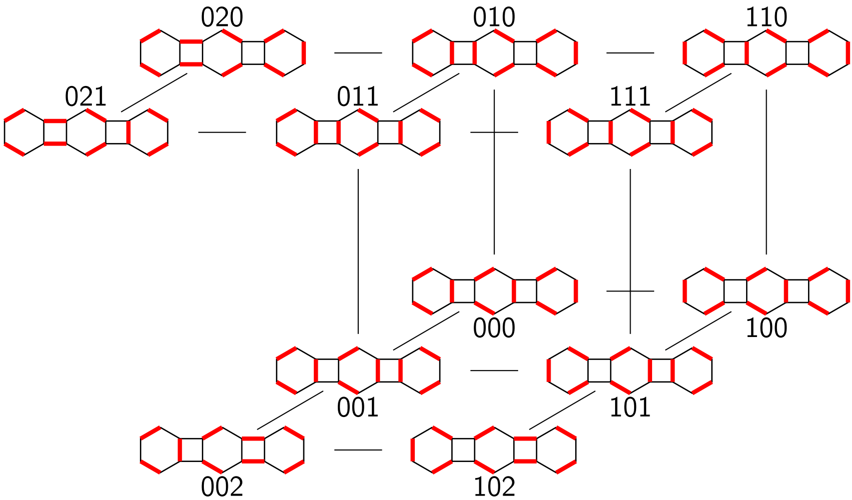

Figure: Metallic cube Π_3^2 as resonance graph of P_3^2.

Thank you for your attention!