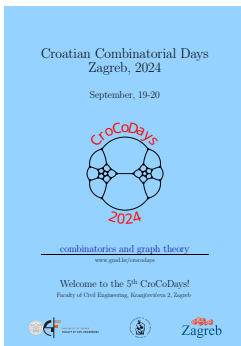


Flower Graphs

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Joint work in progress with G. Devillez, G. Gévay and J. Goedgebeur

Semiregular permutations, semiregular subgroups

- Let $\Gamma \leq \mathbb{S}_X$ be a permutation group acting on a set X of cardinality n .
- A nontrivial group element $\alpha \in \Gamma$ is **semiregular** if $\alpha = (\dots)(\dots)\cdots(\dots)$ is a product of, say k disjoint cycles, $k < n$, of equal length, say m .
- A subgroup $\Gamma' \leq \Gamma$ is **semiregular** if all orbits have the same size.

Semiregular automorphisms of a simple graph

- G – **simple graph** of order n , i.e. $n = |VG|$
- $\text{Aut } G$ – **group of automorphisms**, acting on its vertex set VG .
- **Automorphism** $\alpha \in \text{Aut } G$ – permutation of VG that preserves adjacency.
- Nontrivial $\alpha \in \text{Aut } G$ is a **semiregular automorphism** if α acts semiregularly on the vertex set VG of a simple graph G .

- **Note:** For a semiregular α with k orbits of size m we have $\alpha^m = 1$ and $n = km$. Hence $m > 1$ and k, m both divide n .

Definition

Graph G that admits a semiregular automorphism α is **polycirculant**. More precisely, if $\alpha^m = 1$ and $mk = n$, then G is a **k -circulant**.

- For $k = 1$ we get **circulants**,
- for $k = 2$ we get **birculants**,
- for $k = 3$ we get **trirculants**,
- for $k = 4$ we get **tetraculants**, etc.

Motivation: Polycirculant conjecture

Conjecture (Marušič, 1981)

Every vertex-transitive graph is polycirculant.

Still partially open.

Nowadays, research in the opposite direction became popular.

Problem

Among polycirculants classify vertex-transitive graphs.

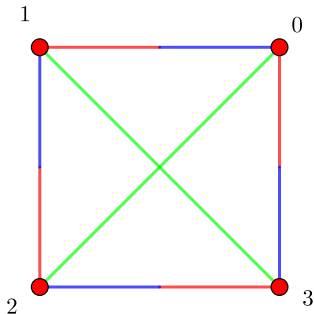
- On the one hand one can restrict attention to some particular class of polycirculants, say cubic tetracirculants.
- On the other hand one can replace the property of vertex-transitivity by some other property, such as arc-transitivity, hamiltonicity, etc.
- These variations make the study of polycirculants interesting and important, even if they are not vertex-transitive,

Induced action on edges and arcs.

A graph automorphism may be viewed as acting on edges and arcs.

Semiregular automorphism (with vertex orbits of size m) acts semiregularly also on the set of arcs. If m is odd it also acts semiregularly on the set of edges. However, if $m = 2m_0$ is even, some edge orbits may have size m_0 .

Example



- K_4 is a circulant.
- Automorphism $\alpha = (0123)$ is (semi)regular on the vertex set with 1 orbit of size 4.
- On the edge set it has two orbits, blue-red of size 4 and green orbit of size 2.
- It is semiregular on the arc set with 3 orbits, blue, red and green of size 4.

Proposition

An automorphism of a simple graph that acts semiregularly on the vertex set with orbits of size m also acts semiregularly on the arc set with orbits of size m .

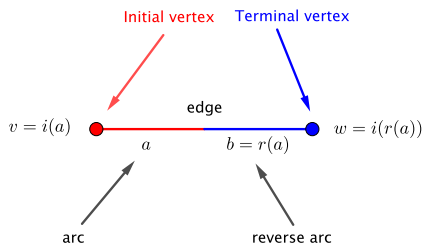
Proposition

An automorphism of a simple graph that acts semiregularly on the vertex set with size m also acts on the edge set such that orbits are of size m or $m/2$.

Proposition

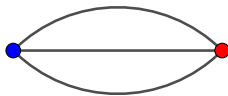
An automorphism of a simple graph that acts semiregularly on the arc set need not to act semiregularly on the vertex set.

Vertices, Edges, and Arcs in a Simple Graph



- An **arc** a has the **initial vertex** $v = i(a)$ and the **reverse arc** $b = r(a)$.
- An **edge** is composed of a pair of **reverse arcs**.

Automorphisms for Multigraphs (parallel edges allowed) are more complicated.



- This (multi)graph, with three parallel edges between two vertices is usually called the Θ -graph and is denoted by Θ_3 .
- There are only 2 permutations of its vertices, however $|\text{Aut } \Theta_3| = 12$.
- We need a more sophisticated model for such graphs and their automorphisms.

Definition

Let $G = (V, A, i, r)$ be a structure, with

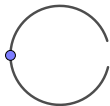
- vertex set V ,
- arc set A ,
- initial vertex mapping $i : A \rightarrow V$,
- involutory reverse arc mapping $r : A \rightarrow A$,

Then G is called a **pregraph**.

Edges: links, loops, and semiedges in Pregraphs

- Recall that an **edge** is composed of a pair of **reverse arcs**.
-

- An edge with both initial vertices distinct is a **link** or **proper edge**.
- An edge with both initial vertices the same is a **loop**.
- An edge with both initial arcs the same is a **semiedge**.



Definition

Let $G = (V, A, i, r)$ be a pregraph.

An automorphism α is a pair of bijections:

$$\alpha_V : V \rightarrow V$$

$$\alpha_A : A \rightarrow A$$

such that $i(r(\alpha(a))) = \alpha(i(r(a)))$

Graph G is ...

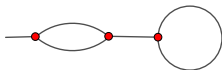
- **simple**, if it has no loops, parallel edges (multi-edges) or semiedges. (Available in SageMath)
- **general**, if it has no semiedges. (Available in SageMath)
- **pregraph** (may have loops, parallel edges or semiedges). (NOT available in SageMath)



simple cubic graph



general quartic graph



cubic pregraph

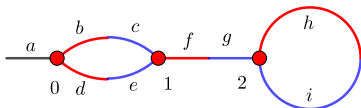
Valence (degree), regular graphs

- The number of arcs with the same initial vertex v is the **valence** of v .
- Graph is **regular** if all vertices have the same valence k . Regular graphs of valence $k = 3$ are **cubic**, for $k = 4$, **quartic**, etc.

Engineering project No. 1.

Pregraphs admit a natural combinatorial description that is suitable for computer representation.

- Note that $|\text{Aut } G| = 4$ for the pregraph on the right.

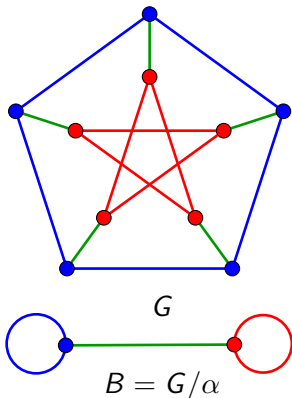


a	b	c	d	e	f	g	h	i	a	arc
a	c	b	e	d	g	f	i	h	$r(a)$	reverse arc
0	0	1	0	1	1	2	2	2	$i(a)$	initial vertex

Project

Implement pregraphs in Python/SageMath. Take special care of drawing loops and semiedges.

Polycirculants and their quotients.

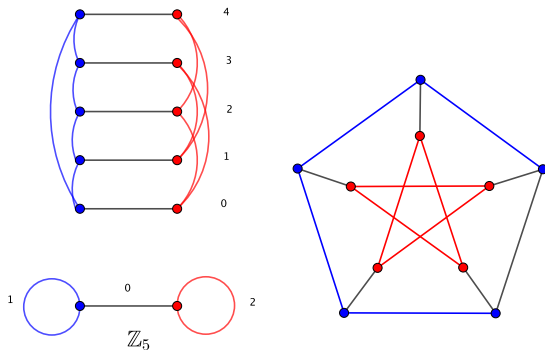


For any polycirculant G and semiregular α we define the **quotient graph** $B = G/\alpha$ in an obvious way.

Even for simple graphs G the quotient may have parallel edges, loops and even semiedges.

- One can prove that the projection $p : G \rightarrow B$ is a **local isomorphism**.
- **However, not every local isomorphism arises from a semiregular automorphism.**

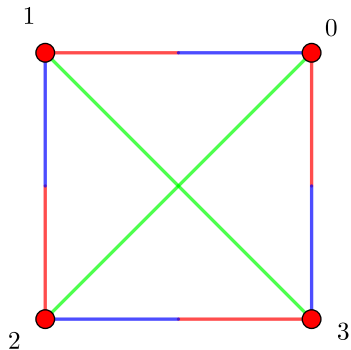
Polycirculants can be described by cyclic voltage graphs.



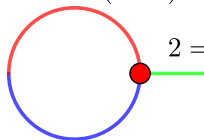
Two-vertex voltage graph defines the Petersen graph.

Loops vs semiedges in the quotient.

Example



$$-1 = 3 = (0321)$$


$$\mathbb{Z}_4$$

$$\alpha = (0123)$$

Brief review of voltage graphs.

For any semiregular group Γ acting on the vertices and arcs of a simple graph G :

- Construct the quotient pregraph B defined on the vertex- and arc-orbits.
- By choosing an arbitrary vertex on each vertex orbit construct the transversal and assign identity to it. Since the action of Γ on each orbit is regular, the choice of transversal uniquely defines the labelling of vertices of G by group elements in such a way that on each orbit each group element appears exactly once.
- Take any arc a of G leading from vertex u (labeled by g) via reverse arc to vertex v (labeled by h) and label it by $w = g^{-1}h$.
- Since the labels w are constant within the arc orbit they may be used to label the arcs in the quotient. Moreover, the arc labels of reverse arcs are inverse to each other. The quotient graph is an arc-labeled **voltage pregraph** B .
- Usually we direct the edges of B and keep only the label of the source arc.

The projection $p : G \rightarrow B$ is called a **regular covering projection** and the opposite construction that constructs G from the voltage graph is called a **lift**.

Voltages assigned to semiedges of B are involutions since a semiedge with identity voltage would lift to semiedges. Also, if the group Γ is cyclic, then there is at most one semiedge per vertex of a voltage graph. For other groups more than one semiedge per vertex are possible. If the order of Γ is odd, no semiedges are possible.

By renowned Vizing's theorem the chromatic index, i.e. edge-chromatic number $\chi'(G)$ of a simple graph G is bounded $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$.

The graphs with $\chi'(G) = \Delta(G)$ are called **class one** whilst the graphs with $\chi'(G) = \Delta(G) + 1$ are **class two**. Even for cubic graphs, the class two graphs are very rare. Martin Gardner called such graphs with some connectivity conditions **snarks**. There are several definitions of snarks.

Nowadays a snark is usually defined as a cyclically 4-edge-connected cubic graph of girth five or more of class two. That means it has no 3 edge-coloring and there can be no subset of three or fewer edges, the removal of which would disconnect the graph into two subgraphs each of which has at least one cycle.

Flower snarks

Discovered by Rufus Isaacs in 1975. One of the oldest snarks is the so-called **Flower Snark J_5** that has been generalized to Flower Snarks J_k , k odd by Isaacs in 1975. Until then only five snarks were known.

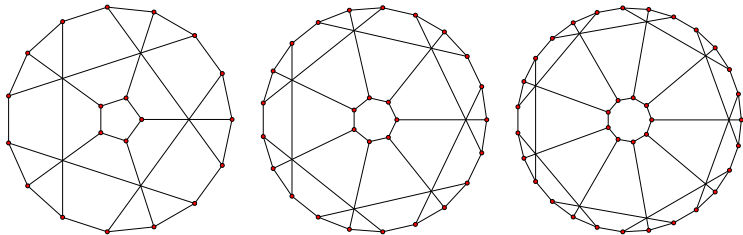


Figure: The smallest flower snarks J_5 , J_7 , J_9 .

Description of Flower snarks

The description of Flower snarks J_m follows a clear pattern.

- There are four types of vertices.
- The inner rim is composed of vertices $x_i, i \in \mathbb{Z}_m$.
- Together with the adjacencies $x_i \sim x_{i+1}$ they form a cycle graph. Addition is taken in \mathbb{Z}_m .
- The spokes $x_i \sim y_i$ connect the inner rim to the outer rim where the vertices $y_i, i \in \mathbb{Z}_m$ are located.
- There are two more sets of m vertices on the outer rim: $u_i, v_i, i \in \mathbb{Z}_m$ attached to y_i by edges $y_i \sim u_i$ and $y_i \sim v_i$.
- To complete the description of J_m we have to add the edges $u_i \sim v_{i+1}$ and $v_i \sim u_{i+1}$. By selecting $m = 5, 7, 9, \dots$ the snarks J_5, J_7, J_9, \dots are obtained.

Flower graphs

Other values of m , $m \geq 3$ may be used, producing cubic graphs J_m that are not snarks. J_3 is of class II but has girth 3, while J_m , m even are of class I. The smallest cases of these graphs are depicted in Figure 2. These graphs may be generalised even further by introducing three parameters $a, b, c \in \mathbb{Z}_m$.

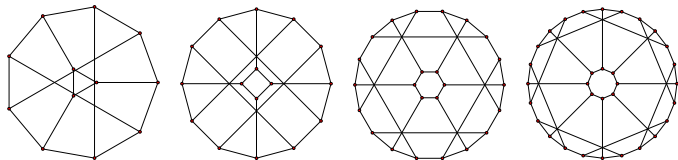


Figure: The smallest Flower graphs that are not snarks J_3, J_4, J_6, J_8 .

Flower graphs – formal definition

Let $J(m; a, b, c)$ be the graph with vertices $x_i, y_i, u_i, v_i, i \in \mathbb{Z}_m$ and edges:

$$x_i \sim x_{i+c}$$

$$x_i \sim y_i$$

$$y_i \sim u_i$$

$$y_i \sim v_i$$

$$u_i \sim v_{i+a}$$

$$v_i \sim u_{i+b}$$

Again, all additions of indices is done in \mathbb{Z}_m .

Flower graphs as tetracirculants over the parachute graph.

The above scheme is conveniently and concisely recorded by a directed graph on 4 vertices, known as a **voltage graph** depicted in this Figure.

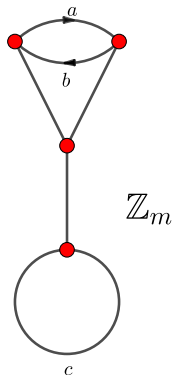


Figure: The Parachute (directed) voltage graph $\tilde{J}(m; a, b, c)$ describes exactly the Flower graph $J(m; a, b, c)$.

Flower graphs – Connectivity

We first correct an omission from a folklore belief that was also stated in a recent paper. For instance, the Flower graph $J(6; 2, 3, 2)$ depicted in Figure 4 is connected even if neither 2 nor 3 is relatively prime with 6. This example contradicts a proposition from that paper where it is required for graph to be connected that at least one a, b or c be relatively prime with m .

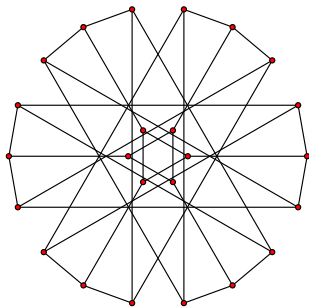


Figure: The graph $J(6; 2, 3, 2)$ is connected.

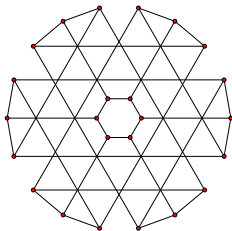
Proposition

Let $\gcd(m, a, b, c) = \delta$ and let $m = \delta m'$, $a = \delta a'$, $b = \delta b'$, $c = \delta c'$. The Flower graph $J(m; a, b, c)$ is composed of δ isomorphic copies of the connected Flower graph $J(m'; a', b', c')$. Moreover, $J(m; a, b, c)$ is connected if and only if $\gcd(m, a, b, c) = 1$.

Flower graphs – Bipartiteness

Proposition

A connected Flower graph $J(m; a, b, c)$ is bipartite if and only if m is even and all a, b, c are odd.



$J(6; 1, 3, 1)$ is connected bipartite graph of girth 6. Hence it is a Levi graph of a (12_3) configuration.

This proposition generalizes to arbitrary voltage graphs, provided we select zero voltages along the edges of a spanning tree. The proofs of both propositions assume that the vertices of the covering graph are placed on m layers, indexed by elements from \mathbb{Z}_m in such a way that the whole spanning tree is placed in the same layer.

One can tell the value of girth of $J(m; a, b, c)$.

Proposition

The girth g of a connected flower graph $J(m; a, b, c)$ is equal to:

- $g = 1$ if and only if $c = 0$
- $g = 2$ if and only if $g > 1$ and $2c = 0$ or $a + b = 0$.
- $g = 3$ if and only if $g > 2$ and $3c = 0$ or $a = 0$ or $b = 0$.
- $g = 4$ if and only if $g > 3$ and $4c = 0$ or $2(a + b) = 0$.
- $g = 5$ if and only if $g > 4$ and $5c = 0$.
- $g = 6$ if and only if $g > 5$ and $6c = 0$ or $2a = 0$ or $2b = 0$ or $a \pm c = 0$ or $b \pm c = 0$.
- $g = 7$ if and only if $g > 6$ and $7c = 0$ or $a \pm 2c = 0$ or $b \pm 2c = 0$.
- $g = 8$ if and only if $g > 7$.

Arithmetic is in \mathbb{Z}_m .

Generating Flower graphs - Isomorphism

When generating $J(m; a, b, c)$, there are some obvious restrictions on the parameters:

$$0 \leq a \leq b \leq m - a$$

$$1 \leq c < m/2$$

Triple (a, b, c) satisfying the conditions above will be called in **basic form**.

We have to consider only **canonical form** of (a, b, c) . For any $k \in \mathbb{Z}_m^*$ consider (ka, kb, kc) , transform it to the basic form and keep only the minimal triple. For a given m we consider triples (a, b, c) and (a', b', c') **equivalent** if and only if they have the same canonical form.

Arithmetic automorphisms and Ádam-like conjecture for Flower graphs.

Note that any two Flower graphs $J(m; a, b, c)$ and $J(m, a', b', c')$ with equivalent triples (a, b, c) and (a', b', c') are **arithmetically isomorphic**.

However, it may be the case that two different canonical triples give rise to isomorphic graphs. In other words, two Flower graphs may be isomorphic without being arithmetically isomorphic. Such a pair would give a counterexample to Ádam-like conjecture for Flower graphs.

We have checked this by computer. Indeed, the Ádam-like conjecture for Flower graphs fails. The smallest counterexample has been found for $m = 16$ and this is the only counterexample up to $m = 20$.

Enumeration of Flower graphs $J(m; a, b, c)$, $3 \leq m \leq 20$

m	(a)	(b)
3	1	1
4	2	2
5	4	4
6	11	11
7	9	9
8	18	18
9	21	21
10	36	36
11	25	25
12	76	76
13	36	36
14	75	75
15	88	88
16	92	91
17	64	64
18	167	167
19	81	81
20	194	194

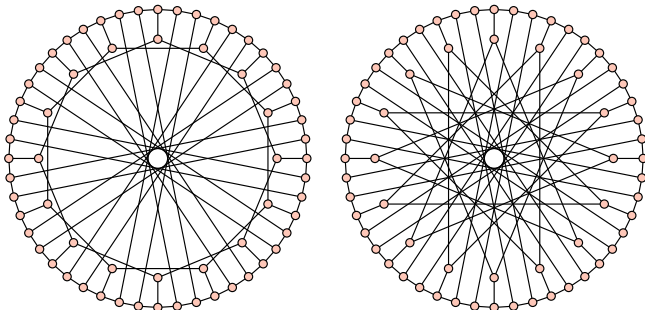
(a) ... Number of Flower graphs up to arithmetic isomorphism.

(b) ... Number of Flower graphs up to isomorphism.

Remark

The first counterexample appears for $m = 16$. The entries for $m = 16$ differ by one.

The graphs $J(16; 1, 7, 2)$ and $J(16; 1, 7, 6)$



The graphs $J(16; 1, 7, 2)$ and $J(16; 1, 7, 6)$ are isomorphic but not arithmetically isomorphic. So far this is the only known counterexample.

Generating Flower graphs $J(m; a, b, c)$, $3 \leq m \leq 11$

We have generated all non-isomorphic Flower graphs up to $m = 11$, however, the list is too big to be included here. We list for each girth only one example of smallest graphs.

m	a	b	c	girth
3	1	1	1	3
4	1	1	1	4
5	1	1	1	5
6	1	1	1	6
7	1	2	3	7
11	1	2	4	8

Flower graphs of girth 8 exist

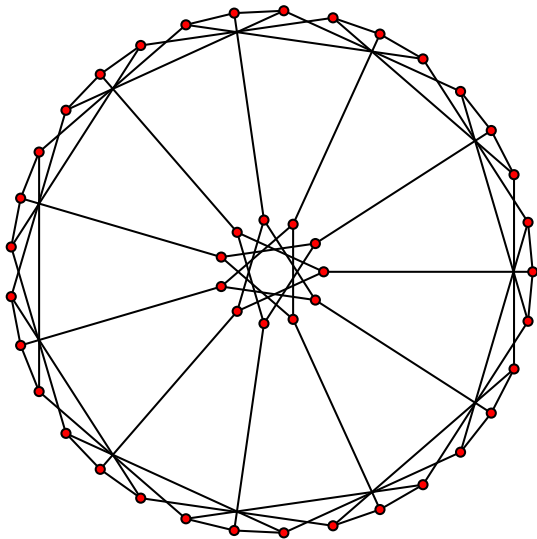


Figure: $J(11; 1, 2, 4)$ is the smallest Flower graph of girth 8.

Two questions about symmetry

Question

What can one say about the automorphism group of Flower graphs?

Question

What can one say about the polycirculant nature of Flower graphs?

What can one say about the automorphism group of Flower graphs?

- Every Flower graph with parameter m admits a dihedral symmetry. Its automorphism group contains \mathbb{D}_m as subgroup.
- It has three, two or one vertex orbit.

Let $J_m = J(m; 1, 1, 1)$.

- All graphs J_m , m odd have three orbits..
- All graphs J_m , m even have two orbits..
- Most three-orbit flower graphs $J(m; a, b, c)$ have small automorphism groups: either \mathbb{D}_m or \mathbb{D}_{2m} . It seems no other Flower graph has one of these automorphism groups.

One-orbit alias vertex-transitive connected Flower graphs up to $m = 20$

m	a	b	c	girth	f	group	arc-trans?
6	1	3	1	6	12	$S_4 \times S_3$	True
6	2	3	1	6	4	$C_2 \times S_4$	False
9	1	3	4	7	4	$((C_2 \times C_2) : C_9) : C_2$	False
12	1	3	1	6	4	$GL(2,3) : C_2$	False
12	1	6	2	6	4	$GL(2,3) : C_2$	False
12	1	9	5	8	12	$((C_3 \times SL(2,3)) : C_2) : C_2$	True
12	2	3	5	7	4	$(C_2 \times S_4) : C_2$	False
12	3	4	1	8	4	$(C_2 \times S_4) : C_2$	False
15	1	6	7	7	4	$C_5 : S_4$	False
18	1	3	5	8	4	$C_2 \times (((C_2 \times C_2) : C_9) : C_2)$	False
18	1	12	4	8	4	$C_2 \times (((C_2 \times C_2) : C_9) : C_2)$	False
18	2	15	1	7	4	$C_2 \times (((C_2 \times C_2) : C_9) : C_2)$	False

- Among connected vertex-transitive Flower graphs up to $m = 20$ only two are arc-transitive: the Nauru graph $G(12, 5)$ and the ADAM graph $G(24, 5)$.

Definition (Sabidussi (1958))

Graph G is a **Cayley graph** for group Γ if $\Gamma \leq \text{Aut } G$ acts regularly (=semiregularly with one vertex orbit) on the vertex set of G .

Question

Are there any vertex-transitive Flower graphs that are not Cayley graphs?

There are well-known examples of vertex-transitive graphs that are not Cayley; e.g. $G(5, 2)$ - Petersen graph or $G(12, 2)$ Dodecahedral graph.

Two problems on vertex-transitive Flower graphs

- Among connected vertex-transitive Flower graphs up to $m = 20$ only two are arc-transitive: the Nauru graph $G(12, 5)$ and the ADAM graph $G(24, 5)$.

Problem

Classify vertex-transitive, connected Flower graphs.

Not sure how hard it is. Subproblem of Classify vertex-transitive connected tetracirculants.

Solved for tricirculants by Potočnik and Toledo (2020).

Problem

How many connected Flower graphs are arc-transitive?

Should follow from classification of arc-transitive tetracirculants by Freluh and Kutnar (2013).

What can one say about the polycirculant nature of Flower graphs?

- Some Flower graphs have only the parachute graph as a quotient.
- Some Flower graphs are bicirculants. For instance, $J(6; 1, 3, 1)$ is isomorphic to the generalized Petersen graph $G(12, 5)$, the Nauru graph and $J(12; 1, 9, 5)$ is isomorphic to $G(24, 5)$, the ADAM graph.

Both Nauru and Adam graph are polycirculants in at least two ways. A natural problem arises:

Problem

Given a Flower graph $J(m; a, b, c)$, in how many ways it can be represented as a polycirculant?

Polycirculant signature of Flower graphs.

m	a	b	c	orbits	(semireg, polycirc)	polycirc signature
3	1	1	1	[1, 1, 2]	(1, 1)	3
4	1	1	1	[3, 1]	(9, 6)	$2^4 4^2$
4	1	2	1	[1, 1, 2]	(4, 3)	$2^2 4^1$
5	1	1	1	[1, 1, 2]	(1, 1)	5^1
5	1	1	2	[1, 1, 2]	(1, 1)	5^1
5	1	2	1	[1, 1, 2]	(1, 1)	5^1
5	1	2	2	[1, 1, 2]	(1, 1)	5^1
6	1	1	1	[3, 1]	(12, 9)	$2^4 3^2 6^3$
6	1	1	2	[1, 1, 2]	(9, 7)	$2^4 3^1 6^2$
6	1	2	1	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	1	2	2	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	1	3	1	[4]	(35, 12)	$2^4 3^2 4^2 6^3 12^1$
6	1	3	2	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	1	4	1	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	1	4	2	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	2	2	1	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$
6	2	3	1	[4]	(20, 8)	$2^4 3^1 4^2 6^1$
6	2	3	2	[1, 1, 2]	(5, 4)	$2^2 3^1 6^1$

- orbits ... list of vertex orbits sizes, expressed as multiples of $2m$.
- (semireg, polycirc) ... (number of non-conjugate semiregular subgroups, number of non-conjugate semiregular cyclic subgroups)
- polycirc signature ... formal product of factors k^r where r represents the number of non-conjugate semi-regular \mathbb{Z}_k in the automorphism group..

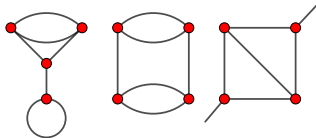
Nauru graph vs. ADAM graph

Nauru $G(12, 5)$

$2^4 3^2 4^2 6^3 12^1$

bicirculant

tetracirculant in three ways

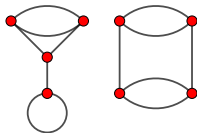


ADAM $G(24, 5)$

$2^3 3^2 4^2 6^3 8^2 12^2 24^1$

bicirculant

tetracirculant in two ways



Adjacency list with coordinates, a new format in the HoG

In this format, the first line should contain only the number of vertices of the graph. Next, each line describes the position of a vertex and its neighbourhood. Vertices are given in increasing index starting from zero. The values are separated by spaces. The line starts with two floating point numbers separated by a space. These are the coordinates of the vertex in the plane with the first one being the x-coordinate and the second one being the y-coordinate. This is followed by a list of all sequence numbers of the vertices that are connected by an edge to the reference vertex. Sequence numbers are expressed in decimal notation and are separated by spaces. The resulting file is a text file consisting of ASCII characters only.

In case of importing a new drawing to the database, the given drawing will be transformed to occupy a square between coordinates -1.5 to 1.5. The aspect ratio will be preserved. If the graph contains more than one vertex, make sure the area spanned by the drawing is large enough (i.e., that the vertices are not all at the same place). **(House of Graphs)**

The interface in Sage that outputs a graph suitable for HoG

```
def to_HoG(gr, fname="fname.txt", digs=6):  
    """After relabelling of vertices, graph gr  
    is stored in file fname as 'adjacency list  
    with coordinates' for House of Graphs  
    where digs determine precision for coordinates."""  
    n = gr.order(); pos = gr.get_pos()  
    verts = gr.vertices()  
    dic = {x:i for i,x in enumerate(verts)}  
    with open(fname,"w") as f:  
        print(n,file =f)  
        for i,v in enumerate(verts):  
            (x,y) = pos[v]  
            print(N(x,digits=digs),N(y,digs = digits),  
                  end = " ",file  
                  =f)  
            lin = [dic[u] for u in gr.neighbors(v)]  
            for t in lin:  
                print(t, end = " ",file=f)  
        print(file=f)
```

New drawing of Nauru graph $J(6, 1, 3, 1) = G(12, 5)$ for the House of Graphs

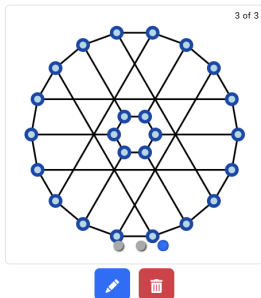
24

```
1.00000 0.000000 6 1 5
0.500000 0.866025 0 7 2
-0.500000 0.866025 1 8 3
-1.000000 0.000000 2 9 4
-0.500000 -0.866025 3 10 5
0.500000 -0.866025 0 4 11
5.00000 0.000000 0 18 12
2.50000 4.33013 1 19 13
-2.50000 4.33013 2 20 14
-5.00000 0.000000 21 3 15
-2.50000 -4.33013 16 22 4
2.50000 -4.33013 17 23 5
4.69846 1.71010 6 19 21
0.868241 4.92404 7 20 22
-3.83022 3.21394 8 21 23
-4.69846 -1.71010 18 22 9
-0.868241 -4.92404 19 23 10
3.83022 -3.21394 18 20 11
4.69846 -1.71010 6 17 15
3.83022 3.21394 16 7 12
-0.868241 4.92404 17 8 13
-4.69846 1.71010 9 12 14
-3.83022 -3.21394 10 13 15
0.868241 -4.92404 16 11 14
```

Graph 1234

Name: Nauru Graph

Submitted by MathWorld



Adjacency Matrix

```
0111000000...
1000000000...
1000000000...
1000000000...
0000000001...
0000000000...
0000000001...
0000000000...
0000000000...
0000000000...
0000000000...
0000101000...
...
```

Adjacency List

```
1: 2 3 4
2: 1 19 21
3: 1 20 23
4: 1 22 24
5: 10 13 14
6: 11 12 15
7: 10 12 18
8: 13 15 16
9: 11 14 17
10: 5 7 19
...
```

New drawing of ADAM graph $J(12, 1, 9, 5) = G(24, 5)$ for the House of Graphs



The House of Graphs

Search

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Graph

Meta-
directory

Publications

Help

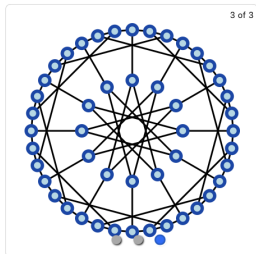


Tomaž Pisanski ▾

Graph 36514

Name: n/a

Submitted by House of Graphs



Adjacency Matrix

```
0111000000...  
1000000000...  
1000000000...  
1000000000...  
0000000111...  
0000000000...  
0000000000...  
0000100000...  
0000100000...  
0000100000...  
...
```

Adjacency List

```
1: 2 3 4  
2: 1 43 45  
3: 1 44 47  
4: 1 46  
48  
5: 8 9 10  
6: 11 12  
13  
7: 14 15  
16  
8: 5 17 23  
9: 5 19 26  
10: 5 20  
28  
...
```

Download this graph in the selected format: ?

Graph6 ▾

Download

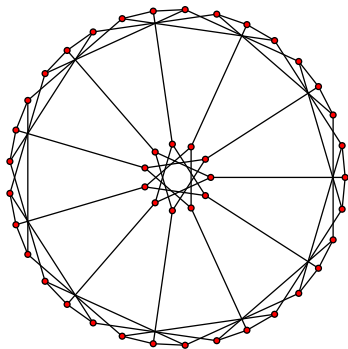
See also: [supported formats](#)

Related graphs:

- [Line graph](#) →

Engineering project No. 2.

Sometimes polycirculants drawn by a computer program are not optimal. For instance, the inner radius of the Flower graphs on the right is too small. We need a graph editor that would allow the user to drag the whole inner circle and not just each vertex separately.



Project

Upgrade an existing graph editor, such as the one of HoG that would allow dragging the whole orbits.

Thank you!