Mosaics of projective planes*

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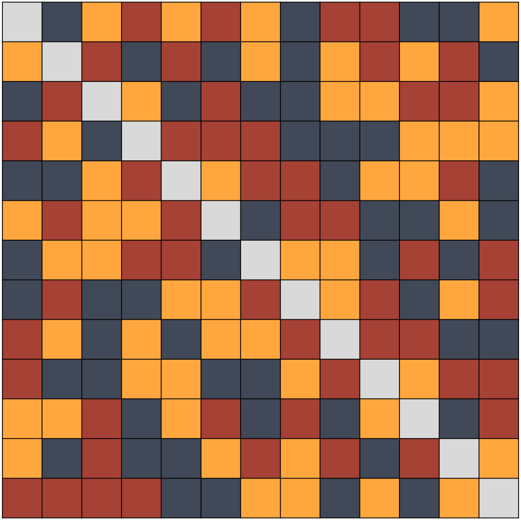
An example of a mosaic



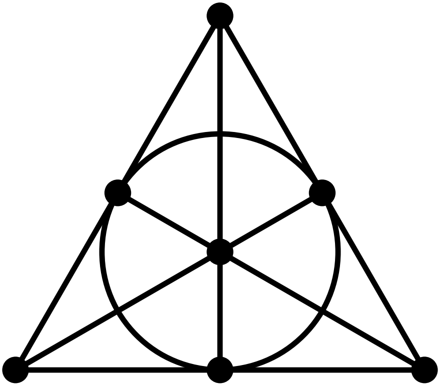
An example of a mosaic

$$\begin{bmatrix}
0 & 2 & 3 & 1 & 3 & 1 & 3 & 2 & 1 & 1 & 2 & 2 & 3 \\
3 & 0 & 1 & 2 & 1 & 2 & 3 & 2 & 3 & 1 & 3 & 1 & 2 \\
2 & 1 & 0 & 3 & 2 & 1 & 2 & 2 & 3 & 3 & 1 & 1 & 3 \\
1 & 3 & 2 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\
2 & 2 & 3 & 1 & 0 & 3 & 1 & 1 & 2 & 3 & 3 & 1 & 2 \\
3 & 1 & 3 & 3 & 1 & 0 & 2 & 1 & 1 & 2 & 2 & 3 & 2 \\
2 & 3 & 3 & 1 & 1 & 2 & 0 & 3 & 3 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 2 & 3 & 3 & 1 & 0 & 3 & 1 & 2 & 3 & 1 \\
1 & 3 & 2 & 3 & 2 & 3 & 3 & 1 & 0 & 1 & 1 & 2 & 2 \\
1 & 2 & 2 & 3 & 3 & 2 & 2 & 3 & 1 & 0 & 3 & 1 & 1 \\
3 & 3 & 1 & 2 & 3 & 1 & 2 & 1 & 2 & 3 & 0 & 2 & 1 \\
3 & 2 & 1 & 2 & 2 & 3 & 1 & 3 & 1 & 2 & 1 & 0 & 3 \\
1 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 2 & 3 & 2 & 3 & 0
\end{bmatrix}$$

An example of a mosaic

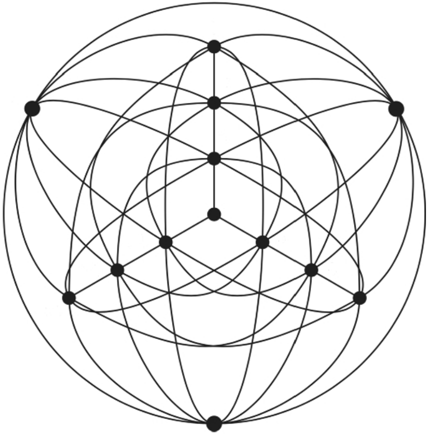
$$\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Finite projective planes



$PG(2,2)$ or symmetric 2-$(7,3,1)$ design

Finite projective planes



$PG(2,3)$ or symmetric 2-$(13,4,1)$ design

Picture source: M. T. Mohan, *p-almost Hadamard matrices and λ-planes*, J. Algebraic Combin. **55** (2022), no. 1, 89–108.

A mosaic of projective planes

$$\begin{bmatrix}
0 & 2 & 3 & 1 & 3 & 1 & 3 & 2 & 1 & 1 & 2 & 2 & 3 \\
3 & 0 & 1 & 2 & 1 & 2 & 3 & 2 & 3 & 1 & 3 & 1 & 2 \\
2 & 1 & 0 & 3 & 2 & 1 & 2 & 2 & 3 & 3 & 1 & 1 & 3 \\
1 & 3 & 2 & 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\
2 & 2 & 3 & 1 & 0 & 3 & 1 & 1 & 2 & 3 & 3 & 1 & 2 \\
3 & 1 & 3 & 3 & 1 & 0 & 2 & 1 & 1 & 2 & 2 & 3 & 2 \\
2 & 3 & 3 & 1 & 1 & 2 & 0 & 3 & 3 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 2 & 3 & 3 & 1 & 0 & 3 & 1 & 2 & 3 & 1 \\
1 & 3 & 2 & 3 & 2 & 3 & 3 & 1 & 0 & 1 & 1 & 2 & 2 \\
1 & 2 & 2 & 3 & 3 & 2 & 2 & 3 & 1 & 0 & 3 & 1 & 1 \\
3 & 3 & 1 & 2 & 3 & 1 & 2 & 1 & 2 & 3 & 0 & 2 & 1 \\
3 & 2 & 1 & 2 & 2 & 3 & 1 & 3 & 1 & 2 & 1 & 0 & 3 \\
1 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 2 & 3 & 2 & 3 & 0
\end{bmatrix}$$

A mosaic of projective planes

$$
\begin{bmatrix}
0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 \\
0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\
2 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 \\
2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 2 \\
2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 & 0 \\
2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\
0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 2 & 0 \\
0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0
\end{bmatrix}
$$

A mosaic of projective planes

$$\begin{bmatrix}
0 & 0 & 3 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\
3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 3 \\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 \\
0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 \\
3 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 3 & 3 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\
0 & 3 & 0 & 3 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 \\
3 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 3 & 0 & 3 & 0
\end{bmatrix}$$

Mosaics of combinatorial designs

O. W. Gnilke, M. Greferath, M. O. Pavčević, *Mosaics of combinatorial designs*, Des. Codes Cryptogr. **86** (2018), no. 1, 85–95.

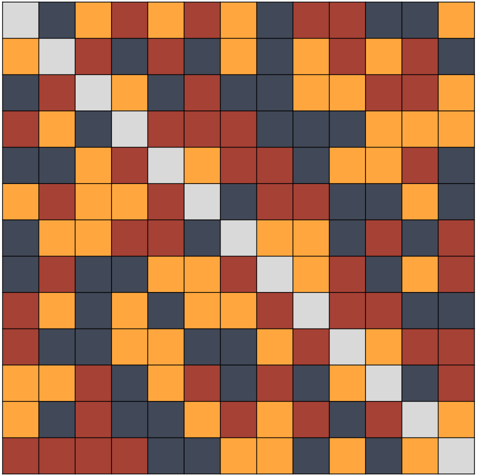
Definition.

Let t_i-(v, k_i, λ_i), $i = 1, \ldots, c$ be parameters of combinatorial designs, all with v points and b blocks and $\sum_{i=1}^{c} k_i = v$. A mosaic with parameters

$$t_1\text{-}(v, k_1, \lambda_1) \oplus \cdots \oplus t_c\text{-}(v, k_c, \lambda_c)$$

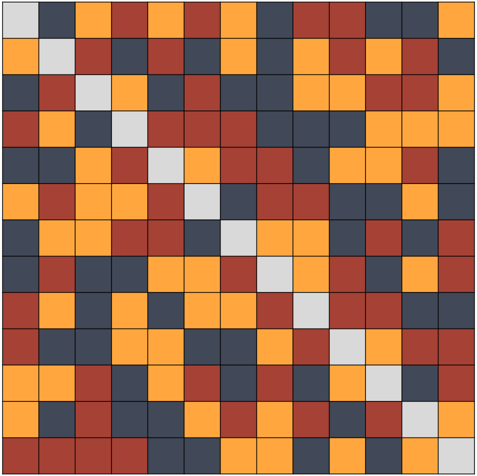
is a $v \times b$ matrix with entries from $\{1, \ldots, c\}$ such that the entries i represent incidences of a t_i-(v, k_i, λ_i) design, for $i = 1, \ldots, c$. Here, c is the number of colors and the matrix is also called a c-mosaic.

A mosaic of 2-(13, 4, 1) designs



$$2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1)$$

A mosaic of 2-(13, 4, 1) designs



$$2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,1,0)$$

A mosaic of 2-(13, 4, 1) designs

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,1,0)$$

A mosaic of 2-(13, 4, 1) designs

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

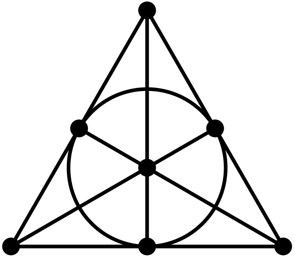
$$\lambda(v-1) = k(k-1)$$

A mosaic of 2-$(13, 4, 1)$ designs

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

$$\lambda(v-1) = k(k-1) \quad \Rightarrow \quad v = k\left(\frac{k-1}{\lambda} + \frac{1}{k}\right)$$

Difference sets



Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$D = \{0, 1, 3\} \subseteq \mathbb{Z}_7 = G$

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \{0, 1, 3\} \subseteq \mathbb{Z}_7 = G$$

$$\operatorname{dev} D = \{D + x \mid x \in G\}$$

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$D = \{0, 1, 3\} \subseteq \mathbb{Z}_7 = G$

$\operatorname{dev} D = \{D + x \mid x \in G\}$

$\{x - y \mid x, y \in D, x \neq y\}$ covers each element of $G \setminus \{0\}$ exactly λ times

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

Definition.

A tiling of an additively written group G is a family of pairwise disjoint (v, k, λ) difference sets $\{D_1, \dots, D_c\}$ such that $D_1 \cup \dots \cup D_c = G \setminus \{0\}$.

Difference sets

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$D = \{0, 1, 3\} \subseteq \mathbb{Z}_7 = G$

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$\{x - y \mid x, y \in D, x \neq y\}$ covers each element of $G \setminus \{0\}$ exactly λ times

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Theorem.

The development of a tiling of G with (v, k, λ) difference sets is a mosaic of symmetric designs. It has G as an automorphism group acting regularly on the rows and columns.

Tiling groups with difference sets

Example. A tiling of $\mathbb{Z}_{31} = \{0, \ldots, 30\}$ with $(31, 6, 1)$ difference sets:

$$D_1 = \{1, 5, 11, 24, 25, 27\}$$

$$D_2 = \{2, 10, 17, 19, 22, 23\}$$

$$D_3 = \{3, 4, 7, 13, 15, 20\}$$

$$D_4 = \{6, 8, 9, 14, 26, 30\}$$

$$D_5 = \{12, 16, 18, 21, 28, 29\}$$

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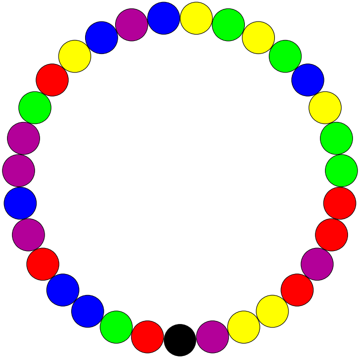

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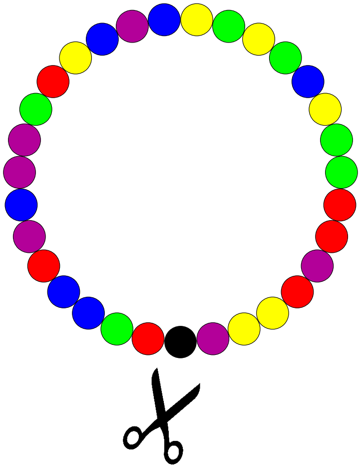
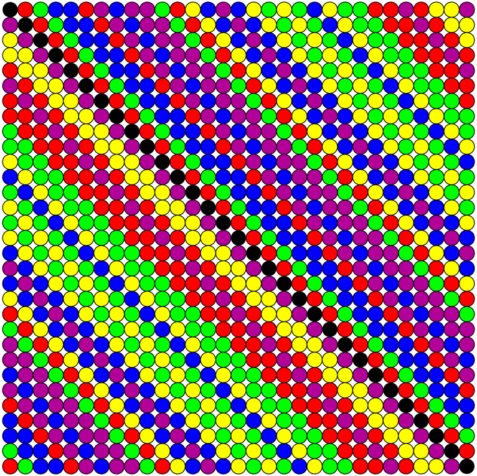
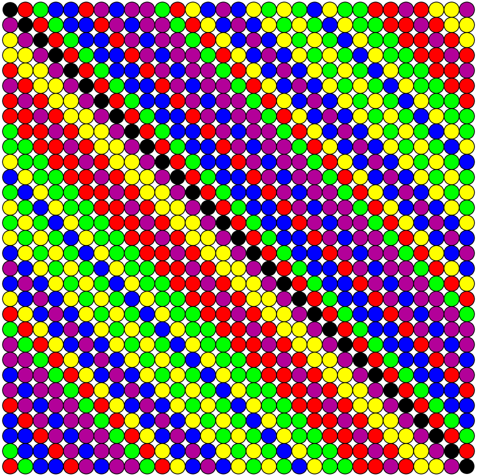


Tiling groups with difference sets

Tiling groups with difference sets



Tiling groups with difference sets



$2\text{-}(31,6,1) \oplus 2\text{-}(31,6,1) \oplus 2\text{-}(31,6,1) \oplus 2\text{-}(31,6,1) \oplus 2\text{-}(31,6,1) \oplus 2\text{-}(31,1,0)$

Tiling groups with difference sets

https://www.imaginary.org/gallery/difference-bracelets

A model of the (31,6,1) bracelet

Difference bracelets cannot be built without the black bead, representing the identity element of the group. A slightly larger faceted bead was used here.

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More Galleries

Mosaics of projective planes

For which orders q are there mosaics of projective planes?

$$(q^2+q+1, q+1, 1) \oplus \cdots \oplus (q^2+q+1, q+1, 1) \oplus (q^2+q+1, 1, 0)$$

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A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

q	2	3	4	5	7	8	9	…
Tiling	✓	✗	✗	✓	✓	✓	?	…
Mosaic								…

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q	2	3	4	5	7	8	9	...
Tiling	✓	✗	✗	✓	✓	✓	?	...
Mosaic	✓			✓	✓	✓		...

Mosaics of projective planes

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q	2	3	4	5	7	8	9	…
Tiling	✓	✗	✗	✓	✓	✓	?	…
Mosaic	✓	✓		✓	✓	✓		…

V. Krčadinac, *Small examples of mosaics of combinatorial designs*, preprint, 2024. https://arxiv.org/abs/2405.12672

Mosaics of projective planes

For which orders q are there mosaics of projective planes?

$$(q^2+q+1,q+1,1)\oplus\cdots\oplus(q^2+q+1,q+1,1)\oplus(q^2+q+1,1,0)$$

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q	2	3	4	5	7	8	9	...
Tiling	✓	✗	✗	✓	✓	✓	?	...
Mosaic	✓	✓	?	✓	✓	✓	?	...

V. Krčadinac, *Small examples of mosaics of combinatorial designs*, preprint, 2024. https://arxiv.org/abs/2405.12672

Mosaics of $PG(2,3)$

Number of incidence matrices for $q = 3$:

$$\frac{13! \cdot 13!}{|Aut(PG(2,3))|} \approx 6.90452 \cdot 10^{15}$$

Mosaics of $PG(2,4)$?

Number of incidence matrices for $q = 4$:

$$\frac{21! \cdot 21!}{|Aut(PG(2,4))|} \approx 2.15797 \cdot 10^{34}$$

Tiling groups with planar difference sets

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

q	v	Group	#DS	Tiling	Group	#DS	Tiling
2	7	$\mathbb{Z}_7$	14	✓			
3	13	$\mathbb{Z}_{13}$	52	✗			
4	21	$\mathbb{Z}_{21}$	42	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_7$	294	✗
5	31	$\mathbb{Z}_{31}$	310	✓			
7	57	$\mathbb{Z}_{57}$	684	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{19}$	12 996	✓
8	73	$\mathbb{Z}_{73}$	584	✓			
9	91	$\mathbb{Z}_{91}$					
11	133	$\mathbb{Z}_{133}$					
13	183	$\mathbb{Z}_{183}$			$\mathbb{Z}_3 \rtimes \mathbb{Z}_{61}$		

Tiling groups with planar difference sets

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

q	v	Group	#DS	Tiling	Group	#DS	Tiling
2	7	$\mathbb{Z}_7$	14	✓			
3	13	$\mathbb{Z}_{13}$	52	✗			
4	21	$\mathbb{Z}_{21}$	42	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_7$	294	✗
5	31	$\mathbb{Z}_{31}$	310	✓			
7	57	$\mathbb{Z}_{57}$	684	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{19}$	12 996	✓
8	73	$\mathbb{Z}_{73}$	584	✓			
9	91	$\mathbb{Z}_{91}$	1092	✗			
11	133	$\mathbb{Z}_{133}$	4788	✗			
13	183	$\mathbb{Z}_{183}$	7320	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{61}$	446 520	?

Tiling groups with planar difference sets

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

q	v	Group	#DS	Tiling	Group	#DS	Tiling
2	7	$\mathbb{Z}_7$	14	✓			
3	13	$\mathbb{Z}_{13}$	52	✗			
4	21	$\mathbb{Z}_{21}$	42	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_7$	294	✗
5	31	$\mathbb{Z}_{31}$	310	✓			
7	57	$\mathbb{Z}_{57}$	684	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{19}$	12 996	✓
8	73	$\mathbb{Z}_{73}$	584	✓			
9	91	$\mathbb{Z}_{91}$	1092	✗			
11	133	$\mathbb{Z}_{133}$	4788	✗			
13	183	$\mathbb{Z}_{183}$	7320	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{61}$	446 520	?

K. Tabak, *Normalized difference set tiling conjecture*, J. Combin. Des. **26** (2018), no. 10, 505–513.

Tiling groups with planar difference sets

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

q	v	Group	#NDS	Tiling	Group	#DS	Tiling
2	7	$\mathbb{Z}_7$	2	✓			
3	13	$\mathbb{Z}_{13}$	4	✗			
4	21	$\mathbb{Z}_{21}$	2	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_7$	294	✗
5	31	$\mathbb{Z}_{31}$	10	✓			
7	57	$\mathbb{Z}_{57}$	12	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{19}$	12 996	✓
8	73	$\mathbb{Z}_{73}$	8	✓			
9	91	$\mathbb{Z}_{91}$	12	✗			
11	133	$\mathbb{Z}_{133}$	36	✗			
13	183	$\mathbb{Z}_{183}$	40	✗	$\mathbb{Z}_3 \rtimes \mathbb{Z}_{61}$	446 520	?

K. Tabak, *Normalized difference set tiling conjecture*, J. Combin. Des. **26** (2018), no. 10, 505–513.

Tiling cyclic groups with Singer difference sets

q	v	Group	#NDS	Tiling
16	273	$\mathbb{Z}_{273}$	12	✗
17	307	$\mathbb{Z}_{307}$	102	✗
19	381	$\mathbb{Z}_{381}$	84	✗
23	553	$\mathbb{Z}_{553}$	156	✗
25	651	$\mathbb{Z}_{651}$	60	✗
27	757	$\mathbb{Z}_{757}$	84	✗
29	871	$\mathbb{Z}_{871}$	264	✗
31	993	$\mathbb{Z}_{993}$	220	✗
32	1057	$\mathbb{Z}_{1057}$	60	✗
37	1407	$\mathbb{Z}_{1407}$	264	✗
41	1723	$\mathbb{Z}_{1723}$	574	✗
43	1893	$\mathbb{Z}_{1893}$	420	✗
47	2257	$\mathbb{Z}_{2257}$	720	✗
49	2451	$\mathbb{Z}_{2451}$	252	✗
53	2863	$\mathbb{Z}_{2863}$	816	✗
59	3541	$\mathbb{Z}_{3541}$	1180	?

The End

Thanks for your attention!