5th Croatian Combinatorial Days Zagreb, September 2024

Book of Abstracts

Impressum

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Contributed talks

Label propagation algorithm for detecting communities in directed acyclic networks

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When dealing with a complex system, sometimes dividing it into smaller parts enables gaining a better insight into the organization of the system and its functioning. Therefore, community detection is one of the fundamental problems in complex networks theory.

Intuitively, a community is a cohesive group of vertices that are more densely connected to each other than to vertices in other communities. One of the aspects that makes community detection demanding is that the precise definition of the community depends on the domain in which it is observed. The situation is further complicated by the directionality of the edges. Since every directed acyclic network has a topological order of vertices, we are interested in finding communities which can also be topologically ordered.

In other words, if the vertices are topologically ordered is such a way that $x_1 < x_2 < \dots < x_n$, we are interested in dividing the network into communities $C_1, C_2, \dots, C_k$ in such a way that

$$\text{if } x_i < x_j,\, x_i \in C_i, x_j \in C_j \text{ then } C_i < C_j \text{ or } C_i = C_j.$$

Here, we present heuristic algorithm based on the label propagation process and modularity optimization in order to obtain an ordered set of communities.

Two new constructions for nut graphs – chemical and otherwise

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A nut graph is a simple graph for which the adjacency matrix has a single zero eigenvalue such that all non-zero kernel eigenvectors have no zero entry. If the isolated vertex is excluded as trivial, nut graphs have seven or more vertices; they are connected, non-bipartite, and have no leaves.

In this talk, we present two new constructions that can be seen as generalisations of the well-known bridge and subdivision constructions for making larger nut graphs from smaller by adding only degree-2 vertices. The new Cut and Frustration Constructions seem to have escaped notice so far in the extensive literature of nut graphs.

This is a joint work with Patrick W. Fowler.

Construction of extremal Type II $\mathbb{Z}_8$-codes via doubling method

Sara Ban Martinović
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The subject of this talk is a construction of new extremal Type II $\mathbb{Z}_8$-codes using doubling method.

Extremal Type II $\mathbb{Z}_8$-codes are a class of self-dual $\mathbb{Z}_8$-codes with Euclidean weights divisible by 16 and the largest possible minimum Euclidean weight for a given length.

We introduce a doubling method for constructing a Type II $\mathbb{Z}_{2k}$-code of length n from a known Type II $\mathbb{Z}_{2k}$-code of length n. Based on this method, we develop an algorithm to construct new extremal Type II $\mathbb{Z}_8$-codes starting from an extremal Type II $\mathbb{Z}_8$-code of type $(\frac{n}{2}, 0, 0)$ with an extremal $\mathbb{Z}_4$-residue code and length $24, 32$ or 40.

We construct at least ten new extremal Type II $\mathbb{Z}_8$-codes of length 32 and type $(15, 1, 1)$. Extremal Type II $\mathbb{Z}_8$-codes of length 32 of this type were not known before. Moreover, the binary residue codes of the constructed extremal $\mathbb{Z}_8$-codes are optimal $[32, 15]$ binary codes.

This is a joint work with Sanja Rukavina.

Szeged and Mostar root–indices of graphs

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Topological indices play a crucial role in network theory, providing a quantitative description of network structures. The field originated with the well-known Wiener index, which led to the development of other distance-based and/or degree-based indices such as the edge-Wiener, Schultz, and Gutman indices. The corresponding graph polynomials, including the Hosoya, edge-Hosoya, Schultz, and Gutman polynomials, have also been extensively studied in the literature.

In this talk we examine various distance-based root-indices of graphs, derived as unique positive roots of modified graph polynomials. Specifically, we focus on the Szeged polynomial, the weighted-product Szeged polynomial, the weighted-plus Szeged polynomial, and the Mostar polynomial. We provide closed-form expressions for these polynomials in the context of several basic graph families and, as a result, present closed formulas for certain root-indices. We also examine the convergence of sequences of these root-indices and discuss their general properties. Additionally, we present numerical results that analyze the discrimination power, correlations, structure sensitivity, and abruptness of the root-indices, offering interpretations of these findings.

This is a joint work with M. Dehmer, N. Tratnik, and P. Žigert Pleteršek.

Early termination criterion for polyhedral enumeration

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The computation of the facets of a polytope up to some group action is a key algorithmic step of many computations. One prominent example is the computation of the perfect forms in dimension n.

The method of choice for this is the Adjacency Decomposition Method and its extension, the Recursive Adjacency decomposition Method. The method works, but sometimes it computes more than is needed, and early termination criterion allows one to considerably accelerate the enumeration when possible.

The basic result is the Balinski theorem on the k-connectivity of the skeleton of a graph. However, the more performant criterion, the better. We will explore in this presentation what is effectively possible, and what we could hope for with this kind of approach.

D_n-Specht Ideals

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For a group $G \in \{S_n, B_n, D_n\}$, a G-Specht Ideal is the ideal generated by all G-Specht Polynomials of a given shape that can be understood via integer partitions. The zero-sets of these ideals encode combinatorial types of points in them which are again integer partitions. Using previous work for the case $G = S_n$ and $G = B_n$, we investigate the poset of D_n-Specht ideals with respect to inclusion and how it is related to the before mentioned integer partitions.

This is a joint work with Sebastian Debus.

The connectivity dimension of a graph

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In this talk, we introduce a new graph parameter, called the connectivity dimension of a graph. This parameter is inspired by the notion of the metric dimension of a graph and can be seen as a measure of heterogeneity of a graph. Among other fundamental properties, we study how to construct graphs on n vertices having connectivity dimension k, for given n and k. We also show that deciding whether the connectivity dimension of a graph is bounded by a given constant is NP-complete. We conclude by raising a couple of open questions.

This is a joint work with Kurt Klement Gottwald.

On equienergetic regular graphs by means of their spectral distances

Irena M. Jovanović
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Let G be a simple graph of order n whose eigenvalues with respect to the adjacency matrix $A = A(G)$ are $\lambda_1(G) \geq \lambda_2(G) \geq \cdots \geq \lambda_n(G)$. The *energy* $E(G)$ of G is defined as $E(G) = \sum_{i=1}^{n} |\lambda_i(G)|$. Two graphs with the same number of vertices are said to be *equienergetic* if they have the same energy. By *spectral distance* $\sigma(G_1, G_2)$ of two non-isomorphic graphs G_1 and G_2 of order n we mean the Manhattan distance between their adjacency spectra, i.e., $\sigma(G_1, G_2) = \sum_{i=1}^{n} |\lambda_i(G_1) - \lambda_i(G_2)|$.

Relatively recently, a problem of identifying spectral properties of graphs which are equienergetic with their complements has been raised in the literature. Based on the fact that the spectral distance $\sigma(G, K_n)$ between a n-vertex regular graph G and the complete graph K_n is equal to the energy $E(\overline{G})$ of the complement $\overline{G}$ of the graph G, a necessary and sufficient condition for a regular graph to be equienergetic with its complement is stated. Due to this result, strongly regular graphs equienergetic with their complements are characterized, and equienergetic regular graphs with respect to some graph operations are considered.

In additon, the computed spectral distance $\sigma(G, K_n)$ served to disprove one of the conjectures related to the spectral distances of graphs. In that way, it turns out that certain connections between the energies of graphs and their spectral distances can be useful in considering various questions connected with spectral properties of graphs.

This is a joint work with Emir Zogić.

Minimum-cost double Roman domination problem

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Double Roman Domination (DRD) is a combinatorial optimization problem defined on undirected graphs. It can be seen as the task of assigning various service providers (such as military units, fire departments, or ambulances) to selected locations within a specified area. The traditional literature on DRD equates the total service expense with the total number of providers required. This paper, however, introduces a new 'minimum-cost' variant of DRD that is broader and more realistic than the standard version, acknowledging that service costs can vary by location. In this context, each vertex of the graph is assigned a cost representing the expense of maintaining one service provider at that specific site. In this work we show that the minimum-cost DRD problem is NP-hard, applicable not just to general graphs but also to numerous specific graph classes. A dynamic programming algorithm is then developed, demonstrating that the problem can be solved in linear time when the graph is a tree. Finally, for general graphs, a heuristic solution is proposed, utilizing a combination of greedy methods and local search. The effectiveness of both algorithms is assessed through experimental evaluations.

This is a joint work with Robert Manger.

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Projective curves and generalized rascal triangles

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We start with four integers $a, b, c, d \in \mathbb{Z}$, and define generalized rascal triangle with $T_{a,b,c,d}(n,k) = a + bk + cn + dkn, \quad n, k \geq 0$.

For these triangles we have two generating rules:

- rascal-like multiplication rule

$$T(n,k) = \frac{T(n-1,k)T(n,k-1) + D}{T(n-1,k-1)}, \quad \text{with } D = ad - bc, \tag{1}$$

- rascal-like addition rule

$$T(n,k) = T(n,k-1) + T(n-1,k) - T(n-1,k-1) + d. \tag{2}$$

We give a combinatorial interpretation of the difference between the Pascal's triangle and generalized rascal triangle as the dimensions of graded pieces of homogeneous ideal of a smooth nondegenerate projective curve. We represent the values of the Hilbert polynomial of a family of curves $\mathcal{C} \subset \mathbb{P}^n$ as entries in generalized rascal triangles generated for degree and genus of the curve given as linear functions of projective dimension n.

2									
1	1								
0	2	0							
−1	3	3	−1						
−2	4	6	4	−2					
−3	5	**9**	9	5	−3				
−4	6	**12**	**14**	12	6	−4			
−5	7	15	**19**	19	15	7	−5		
−6	8	18	24	26	24	18	8	−6	
−7	9	21	29	33	33	29	21	9	−7

Figure 1: $T_{2,0,n-1}(n,k)$ for curves of degree $2n-1$ and $g = n-1$

This is a joint work with Helena Koncul.

Triangle of numbers arisen from modular forms

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Let $\Gamma_0(N) \subset SL_2(\mathbb{Z})$ be the congruence subgroup

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) : c \equiv 0 \pmod{N} \right\}.$$

In our previous work we have computed bases of homogeneous polynomials of degree d that vanish on basis of $S_m(\Gamma_0(N))$, cuspidal modular forms of even weight $m \geq 2$ for various $t := \dim S_m(\Gamma_0(N))$. We have found two formulas for calculating the dimension of the basis of homogeneous polynomials vanishing on cuspidal forms of certain degree d and number of variables t depending on the genus g.

The dimensions for genus 0 appear to be the number sequence A124326 from OEIS database which can be obtained as a difference of Pascal's triangle A00731 and rascal triangle A077028. We have found a new way of generating number sequence A124326 combining the Pascal's rule and multiplication table and investigated the found formulas for every $g \in \mathbb{N}_0$ and the triangle numbers arisen from them regardless to the bounds in number theory for the possible interpretation of the subject.

0
0 0
0 0 0
0 0 0 0
0 0 **1** 0 0
0 0 **3** **3** 0 0
0 0 **6** **10** **6** 0 0
0 0 **10** **22** **22** **10** 0 0
0 0 **15** **40** **53** **40** **15** 0 0
0 0 **21** **65** **105** **105** **65** **21** 0 0

Figure 2: number sequence A124326

This is a joint work with Iva Kodrnja.

Mosaics of projective planes

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Mosaics of combinatorial designs were introduced in [3]. A related concept are colored designs, introduced in [1]. Mosaics of symmetric designs can be obtained from tilings of groups with difference sets [2]. In this talk we will explain the relevant definitions and focus on mosaics of symmetric designs with $\lambda = 1$, i.e. projective planes. An interesting small example not coming from a group tiling was recently discovered [4].

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A Problem of Colored Perfect Matchings Inspired by Quantum Physics

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At the CroCoDays 2018 meeting, I presented a then-new graph-theory problem directly inspired by our work in quantum physics [1, 2]. Now, six years later, despite some interesting progress in numerics [3], combinatorics [4, 5], and computational complexity [6, 7], the problem remains largely unsolved. The problem concerns certain properties of vertex coloring induced by the perfect matchings of a graph. A solution to this problem will be rewarded with a prize of 3000 Euros.

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Perfect matching in graphs obtained from simplicial complex of tilings

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This paper presents a connection between polyomino shapes, tilings, simplicial complexes, simplicial complexes of tilings, and graphs obtained from the simplicial complexes of tilings. Simplicial complexes of tilings are introduced in mathematics in [1]. These complexes are obtained with tilings of the given region R with a finite set of polyomino shapes Σ, where region R is a grid of finite region or grid of topological surfaces. We consider the regular placement of k polyomino shapes on square grid M. Here a regular placement is every placement without overlaps, and it is denoted with $K_P(M_{m\times n})$, where P presents polyomino shapes, M presents a grid in the plane or a grid on topological surfaces, and m, and n dimension of grid. We will consider perfect matchings in graphs $G(K_P(M_{m\times n}))$ which contain all 0-simplexes and 1-simplexes from a simplicial complex of tilings $K_P(M_{m\times n})$.

References

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This is a joint work with Đorđe Baralić.

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A Comprehensive Overview of the Graovac-Ghorbani Index of a Graph

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The Graovac-Ghorbani index is a new version of the atom-bond connectivity index. This topological index, associated with chemical structures, provides a compact and efficient means of describing structural formulas, facilitating the study and prediction of structure-property relationships in organic compounds. The graovac-Ghorbani index has a predictive potential compared to analogous descriptors. It is applied in the pharmaceutical industry to model both the boiling point and melting point of molecules. Although the Graovac-Ghorbani index was introduced 14 years ago, its mathematical properties have been explored to a limited extent due to the complexity of its formula. We will present the key results on the Graovac-Ghorbani index of a graph and outline some open problems.

Characterizing bipartite distance-regularized graphs with vertices of eccentricity at most 4

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Consider a bipartite distance-regularized graph with color partitions Y and Y' where all vertices in partition Y exhibit an eccentricity denoted as D. In this talk we will characterize bipartite distance-regularized graphs, having $D \leq 4$, in relation to the incidence structures they represent.

This is a joint work with Blas Fernández and Sanja Rukavina.

Generalized Continuant Polynomials

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In this talk we present a pair of dual identities for continuant polynomials that imply some well known summations. It includes the upper and parallel summation of binomial coefficients. We generalise continuants to find analogues of these duals, with many more consequences and implications to tridiagonal matrices and quivers, among the other structures. We introduce a variation of Euler's tiling interpretation of continuants to prove these identities.

A New Theorem from The Number Theory and its Application for a 3-adic Evaluation for Large Schröder Numbers

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We present a new theorem from the number theory (up to our knowledge) motivated by the binomial theorem. By using this theorem and central Delannoy numbers, we give another proof of a recently discovered formula for 3-adic evaluation of large Schröder numbers.

Walks on tiled boards

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Many articles deal with tilings with various shapes. Another very frequent problem in combinatorics is to examine walks on graphs or on grids. We combine these two things and give the numbers of the shortest walks crossing the tiled $(1 \times n)$ and $(2 \times n)$ square grids by covering them with squares and dominoes. We describe these numbers not only recursively, but also as rational polynomial linear combinations of Fibonacci numbers.

On hyperfibonacci and hyperlucas numbers and their weighted sums

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We first study the hypersequences $\left(a_n^{(r)}\right)_{n\in\mathbb{N}_0}$, $r \in \mathbb{N}_0$, of an arbitrary sequence $(a_n)_{n\in\mathbb{N}_0}$. Then we apply the results to the Fibonacci sequence and the Lucas sequence and obtain some new results on the hyperfibonacci and hyperlucas numbers. After that, we investigate weighted sums of the type $\sum_{k=0}^{n} k^\ell a_k^{(r)}$, $\ell \in \mathbb{N}_0$, and derive a recurrence relation and its solution. This solution depends on the expression $\sum_{k=0}^{m}(-1)^k\binom{m}{k}(k+n+1)^\ell$. We derive some old and new properties of the generalized expression $\sum_{k=0}^{m}(-1)^k\binom{m}{k}(kx+y)^\ell$ and apply the results to the two sequences as above. In this way we obtain known and new formulas for weighted Fibonacci and Lucas sums, also known as 'Ledin and Brousseau's summation problems'.

Flower graphs

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Flower graphs constitute an interesting family of tetracirculant graphs that contain the well-known family of Flower snarks. We present some basic facts about Flower graphs. We explore the polycirculant nature of Flower graphs to draw them in the plane with non-trivial symmetry. The same approach can be applied to some other polycirculant graph families. Recently, the House of Graphs, the leading database of interesting graphs, introduced the possibility of inputting and outputting graphs together with vertex coordinates. This enables users to upload graphs drawn with the help of computer programs.

This is a joint work with G. Devillez, G. Gévay, and J. Goedgebeur.

Resonance graphs of linear phenylenes

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A recently introduced family of graphs called metallic cubes generalizes Fibonacci cubes, a famous and well-investigated family of graphs. Metallic cubes proved to be a natural generalization since they maintain most of their desirable properties. One of these properties is that Fibonacci cubes can be represented as resonance graphs of fibonaccenes. In this talk, we present the family of phenylenes and represent metallic cubes as their resonance graphs.

This is a joint work with Tomislav Došlić.

Combinatorial Settlement Model: Resistance to Predators and Altruists

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We study an extension of a combinatorial settlement model on rectangular grids introduced previously by P., Šebek, and Žubrinić. Recall that a configuration $C\colon [m] \times [n] \to \{0, 1\}$ (where we interpret 0 as an unoccupied lot, and 1 as an occupied lot) was called admissible if each occupied lot was adjacent to at least one unoccupied lot to its east, south, or west, i.e. not all of its sun-facing sides are blocked from the sunlight. If, additionally, no further lots could be occupied whilst keeping the configuration permissible, it was called maximal.

The extension suggested by Tomislav Došlić involves introducing two new types of agents: predators — who are willing to occupy any currently unoccupied lot that is not blocked from sunlight, regardless of whether this renders the configuration impermissible, and altruists — who are willing to occupy any unoccupied lot, even if it is blocked from sunlight, whilst still taking care not to completely block sunlight from reaching any other previously occupied lot.

In this talk, we will discuss the structure of maximal (admissible) configurations that are resistant to predators, altruists, or both. This talk is based on our preprint arXiv:2401.01225.

This is a joint work with Tomislav Došlić, Stjepan Šebek and Josip Žubrinić.

On the Petersen Coloring Conjecture

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The Petersen Coloring Conjecture is a well known classical conjecture in graph theory, and it is known to imply few other classical conjectures such as Ford-Fulkerson Conjecture and (5,2)-cycle-cover Conjecture. A normal k-edge-coloring of a cubic graph G is a proper k-edge-coloring of G such that for every edge e of G the number of distinct colors on the four edges incident to e is either two or four (and never three). The Petersen Coloring Conjecture is equivalent to the statement that every bridgeless cubic graph has a normal 5-edge-coloring. Vizing's theorem implies that every cubic graph has either a 3-edge-coloring or a 4-edge-coloring. Since every 3-edge-coloring is trivially normal, to establish that the Petersen Coloring Conjecture holds it is sufficient to consider only snarks, i.e., cubic graphs which require 4 colors for a proper edge-coloring. The most general known method of constructing snarks is a superposition. We consider a class of superpositioned snarks, where a snark G is superpositioned along a cycle C by one of the two particular supervertices and superedges obtained from any snark H by "cutting out"two nonadjacent vertices. For such superpositions we provide two sufficient conditions under which a normal 5-edge-coloring of the underlying snark G can be extended to a superposition. The first sufficient condition applies to superpositions by any hypohamiltonian snark H used as superedge, but only for some of the possible ways of connecting them. The second sufficient condition applies to all Flower snarks H used as superedge.

This is a joint work with Riste Škrekovski.

Selected topics on Wiener index

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The Wiener index is defined as the sum of distances between all unordered pairs of vertices in a graph G, i.e.

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u, v).$$

The Wiener index, first introduced in 1947 by Harold Wiener, was originally designed to estimate the boiling points of paraffin. It has become one of the most renowned and extensively studied topological indices, and continues to be a vibrant area of research.

This talk presents some directions of research that are proposed in the survey

M. Knor, R.Š., A. Tepeh, *Selected topics on Wiener index*, published in Ars Math. Contemp.

We have compiled answers to some questions, provided additional insights into the topic of extremal values of the Wiener index in various contexts, and introduced a range of intriguing problems.

Combinatorics and artificial intelligence

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Application of artificial intelligence can be found in almost all areas of human life. Mathematics as a science has not remained an indispensable part of artificial intelligence's (AI) attempt to rule it. In this paper, we will describe the importance, role and strength of mathematics in the application of AI in combinatorics. We will give an overview of problems that AI cannot solve and for which it does not have sufficiently developed intelligence, which shows the power and importance of mathematics, but also of combinatorics as a scientific discipline.

This is a joint work with Edin Liđan.

Geometry of point particles. Symbolic computer verification of the Atiyah's Conjecture for five points in the Euclidean plane

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In 2001 Sir M. F. Atiyah formulated a conjecture $C1$ and later with P. Sutcliffe two stronger conjectures $C2$ and $C3$. These conjectures, inspired by physics (spin-statistics theorem of quantum mechanics), are geometrically defined for any configuration of n points in the Euclidean three space. The conjecture $C1$ is proved for $n = 3$ in [1] and for $n = 4$ in[2], and $C1 - C3$ in [3]. After two decades we succeeded in verifying $C1$ for arbitrary five points in the Euclidean plane. The computer symbolic certificate produces a new remarkable universal ('hundred pages long') positive polynomial invariant (for any five planar points), in terms of newly discovered shear coordinates. This refines the original Atiyah's conjecture and we are optimistic for its verification for n greater than five (less optimistic variant ... 'It remains a conjecture for 300 years (like Fermat)', see Atiyah: Edinburgh Lectures 2010). In 2013. Atiyah's conjectures were put on the new list of nine open problems [4] (hopefully easier than the remaining six Clay millennium problems!).

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Character type constructions and 3-dimensional Hadamard cubes

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We prove that in order to construct a 3-dimensional Hadamard cube using a composition of character and a map on an abelian group G of even order, it is necessary to construct a piecewise defined function, or in other words, it is impossible that such a function is of a type $\chi \circ \psi$, where $\psi : G^3 \to G$ and χ is a nontrivial character.

This is a joint work with Vedran Krčadinac and Mario-Osvin Pavčević.

Some notes on generalized Pell graphs

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Fibonacci cubes were introduced as a model for interconnection networks. These graphs have found numerous applications elsewhere and are also extremely interesting in their own right. The n dimensional Fibonacci cube Γ_n is obtained by removing vertices in n-dimesional hypercube that have two consecutive 1s in its binary labeling. The number of vertices of Fibonacci cube Γ_n is F_{n+2} where F_n is the n-th Fibonacci number. It is interesting to study new graphs whose vertices are counted by Fibonacci-like numbers.

In this talk, we will explore generalized Pell graphs Π_n whose vertices are all Pell strings that are words over the alphabet $T = \{0, 1, 22\}$ and two vertices are adjacent whenever one of them can be obtained from the other by replacing a 0 with a 1 (or vice versa), or by replacing a factor 11 with 22 (or vice versa). The order of Π_n is P_{n+1}, where P_n is the n-th Pell number defined by the recurrence relation $P_n = 2P_{n-1} + P_{n-2}$ for $n \geq 2$ with initial terms $P_0 = 0$, $P_1 = 1$. We present some basic properties associated with them.

This is a joint work with Sandi Klavžar and Vesna Iršič.

Resonance Graphs and Daisy Cubes – Part I

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We characterize all plane bipartite graphs whose resonance graphs are daisy cubes and therefore generalize related results on resonance graphs of benzenoid graphs, catacondensed even ring systems, as well as 2-connected outerplane bipartite graphs. Firstly, we show that if G is a plane elementary bipartite graph other than K_2, then its resonance graph $R(G)$ is a daisy cube if and only if the Fries number of G equals the number of finite faces of G, which in turn is equivalent to G being homeomorphically peripheral color alternating. Next, we extend the above characterization from plane elementary bipartite graphs to all plane bipartite graphs and show that the resonance graph of a plane bipartite graph G is a daisy cube if and only if G is weakly elementary bipartite and every elementary component of G other than K_2 is homeomorphically peripheral color alternating.

This is a joint work with Simon Brezovnik, Zhongyuan Che, and Petra Žigert Pleteršek.

Symmetric inequalities on side lengths of triangles and simplices

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There are many trivial symmetric inequalities for trinagle's side lengths. They can be obtained by multiplying all three triangle inequalities, or from non negativity of the area of the triangle expressed by Heron's formula, etc. Similar claims hold for a tetrahedron and all of its six faces and non negative volume expressed in terms of edge lengths, as well as for higher dimensional simplices and Cayley-Menger determinants and so on.

But by refining the Euler inequality between circumradius and inradius we get some not quite obvious inequalities among edge lengths of a simplex. In the case of a triangle, by general principles we can get analogoues in spherical and hyperbolic plane geometries. The concept of Crelle's subsimplex enables us to get recursive algorithm to any dimension for Euclidean simplices.

Finally, we briefly discuss on the Grace-Danielsson-Egan inequality about the upper bound of the distance between the incenter and the circumcenter of a simplex, proved very recently by geometric-combinatorial methods. Conjectures on these results will also be presented in the sense of the refined Euler's inequality.

Resonance Graphs and Daisy Cubes – Part II

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Assume that G is a homeomorphically peripheral color alternating graph with inner dual G^* and resonance graph $R(G)$. We first establish a bijection between the set of maximal hypercubes of the resonance graph $R(G)$ and the set of maximal independent sets of the inner dual G^*, where G^* is a tree and isomorphic to the τ-graph of $R(G)$. A novel characterization on when resonance graphs are daisy cubes follows naturally. Furthermore, the characterization provides a binary code labelling for the vertex set of a resonance graph $R(G)$ as a daisy cube with respect to the set of maximal independent sets of the inner dual G^* of G. We then show that a daisy cube with at least one edge is a resonance graph of a plane bipartite graph if and only if its τ-graph is a forest. As applications of our main theorems, interesting results are obtained for Fibonacci cubes and Lucas cubes which are special types of daisy cubes. We conclude with two algorithms which provide a binary code labelling for the vertex set of a resonance graph as a daisy cube.

This is a joint work with Simon Brezovnik, Zhongyuan Che, and Niko Tratnik.

Application of cycles related graphs to dual-ring type of network topology

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Graph theory is today an extremely diverse field with wide applications, and graphs have proven to be an excellent tool for modeling systems that emphasize connections and relationships between objects. In graph theory, matching is a fundamental concept used to describe a set of edges without common vertices. Understanding this is essential for solving problems involving efficient routing and resource allocation. One of the applications is network design. Dual-ring topology is a new network configuration where each device is connected to two others, creating two concentric circles for information flow. This design offers a backup route for communication and ensures consistent network stability and resilience. Representation of this topology in graph theory is called a book graph. We establish connections between this topology and book graphs and enumerate maximal matchings, calculate the saturation number and the centrality of this graph.

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Notes